Optimal Two-Stage Procurement with Private R&D Efficiency*

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Abstract

In this paper, we study the optimal procurement in a two-stage environment with R&D. The principal wishes to procure a product from an agent. At the first stage, the agent can conduct R&D to discover more cost efficient way of production. The agent's efficiency in conducting R&D can be high or low, which is his private information, and his effort level in R&D is unobservable to the principal. Higher R&D effort leads to a better distribution of production cost in the sense of first-order stochastic dominance. At the second stage, the agent's cost of supplying the product is randomly drawn according to the distribution determined by his R&D effort. The cost is again privately observed by the agent. The principal's goal is to minimize the expected procurement cost through a two-stage mechanism. The optimal deterministic two-stage mechanism can be implemented by a menu of two put contracts with different strike prices. If the agent pays a higher premium in the first stage for the put with higher strike price, then he is entitled to charge the higher strike price for supplying the good in the second stage. The efficient type chooses the more expensive put with an efficient strike price, while the inefficient type opts for the less expensive put with a strictly lower (thus inefficient) strike price.

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1 Introduction

Procurement is ubiquitously employed as a cost-efficient way of acquiring goods, services or works from an outside external source. It accounts for a substantial part of the global economy. Averagely,

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around 15 percent of yearly global domestic product (a total amount of over $10 trillion) is spent merely through public procurement that covers in particular military acquisitions as a significant component.[1] It has long been recognized that procurement of new goods/services often involves and stimulates private research and development (R&D) and/or innovations before production and delivery (See Rob [24], Hendricks, Porter, and Boudreau [14], Lichtenberg [18], Rogerson [25], and Tan [28] among others).[2] More recently, Nyiri et al. [21] emphasize the promotion of R&D investment and innovation in Information and Communication Technology (ICT) by public procurement in EU member states. The United Nations Office for Project Services (UNOPS) in its 2013 report[3] also stresses the role played by public procurement on fostering investment in new technology and research in both developed and developing countries.

Cost effectiveness has long been the central issue in procurement design.[4] An established literature has been devoted to designing cost-minimizing acquisition in a variety of environments. It is clear that the contractors' pre-delivery R&D incentive on cost reduction should be fully utilized by the procurers to lower their acquisition costs. To achieve the most cost reduction, an optimal procurement policy must appropriately balance between extracting surplus ex post and providing the right R&D incentive ex ante.[5]

It is typically assumed in the literature that the R&D effort as well as the good/service delivery efficiency (i.e., the production cost) are a contractor's private information. However, the situations are abundant where the R&D efficiency (e.g., the marginal cost of R&D effort) of a contractor is also his private information. R&D activities require both technical facilities and researchers with different expertise and specializations. The competency of the contractor in organizing, coordinating and carrying out a specific R&D task (e.g., the quality of its technical facilities, abilities and experience of its researchers, its efficiency in project management) is usually not observable by the procurer. An immediate implication is that the contractor's R&D incentive must respond to his R&D efficiency. Some interesting issues thus arise for the procurement design. How does this additional dimension of private information of the agent affect his R&D incentive and consequently

[1] See p. 1 in "Supplement to the 2013 Annual Statistical Report on United Nations Procurement: Procurement and Innovation", 2014, www.unops.org. More details are available from *http://en.wikipedia.org/wiki/Procurement* and *http://en.wikipedia.org/wiki/Government_procurement.*

[2] Hendricks, Porter, and Boudreau [14] observe that in the federal auctions for leases on the outer continental shelf (OCS), bidders make private investment to acquire more information before bidding. Abundant empirical evidence in defense procurement indicates that bidders make significant amount of private investments in R&D prior to bidding. Tan [28] provides an example of jet fighter procurement by the U.S. Air Force. Lichtenberg [18] provides more examples about private R&D investment in public defense procurement.

[3] "Supplement to the 2013 Annual Statistical Report on United Nations Procurement: Procurement and Innovation", 2014, www.unops.org.

[4] Rob [24] points out: "... the importance of the cost effectiveness in the acquisition process cannot be overemphasized."

[5] Notably, the contractors' pre-delivery R&D incentive has been carefully incorporated into the analysis in many pioneer studies including Rob [24], Dasgupta [12], Tan [28], Piccione and Tan [23], Arozamena and Cantillon [2] among others.

the optimal design of procurement? In particular, how should the optimal design incorporate this new element in the information flow into the natural dynamics of the procurement process? In this paper, we address these issues by studying the cost minimizing procurement design in a two-stage environment from a dynamic mechanism design perspective.

We consider a two-stage contract between a procurer (she, the principal) and a supplier (he, the agent) in the following environment. The principal wishes to procure a product from the agent which she can acquire from an alternative source at cost c_0.[6] The agent can invest in R&D that improves his endowed production technology and generates a production cost potentially lower than c_0. The agent's ability of conducting R&D, which can be either high (the efficient type) or low (the inefficient type), is his private information. At the first stage, the agent is offered the contract. If he accepts, then he exerts an unobservable effort in R&D. Each effort level leads to a distribution of production cost. Higher effort generates a better distribution in the sense of first-order stochastic dominance. But higher effort costs more given the type of the agent. The first stage involves both adverse selection (the agent's R&D ability is his private information) and moral hazard (the agent's effort is not observed by the principal). Thus, the principal faces a mixed adverse selection and moral hazard problem in the first stage. At the second stage, the production cost is realized and it is again the agent's private information. The contract has to sequentially elicit the agent's private information and provide the right incentive for the agent to exert effort in R&D at the same time. The principal's goal is to design the optimal contract to minimize her expected procurement cost.

We first provide a first-best benchmark analysis for the case where the agent's R&D effort is observable and his private information at both stages is public. It is straightforward that the second stage allocation must be ex post efficient and the R&D effort of both types must be efficient. First stage transfers can be employed to extract all the surplus of each type of supplier.

We then investigate the dynamic pure adverse selection environment where the R&D ability and the production efficiency are private information of the agent but his R&D effort is contractable. We find that the optimal mechanism induces no effort distortion for the efficient type and a downward distortion for the inefficient type compared with the first-best effort levels, while the second stage allocation remains ex post efficient for both types. This effort downward distortion reflects the usual rent extraction-efficiency trade-off in the adverse selection literature. Therefore, as in the first-best benchmark setting, the efficient type invests more in the R&D stage at the optimum. However, diverging from the insight from the dynamic adverse selection literature (e.g., Courty and Li [10] and Eső and Szentes [13]), the second stage allocation is not discriminatory. The intuition is as follows. Although the distribution of the second stage delivery cost (which in turn is determined by the R&D effort) is endogenously determined by the first stage type, it eventually becomes public information as the R&D effort is observable. As a result, the principal can solely rely on the first

[6] c_0 can be equivalently treated as the procurer's value of the product.

stage transfers to maintain the incentive compatibility without sacrificing the ex post efficiency in the second stage allocation.

We now turn to the main environment of interest assuming further that the agent's R&D effort is not observable by the principal. Then this becomes a dynamic mixed adverse selection and moral hazard problem. Since effort is not contractable, the first stage mechanism merely specifies the transfer contingent on the agent's first stage report. In the second stage, the mechanism specifies the trading probability and a transfer, both of which rely on the reports of two stages. The mechanism has to induce both information revelation and effort provision from the agent. We fully characterize the optimal mechanism within the class of deterministic mechanisms.[7] The optimal deterministic two-stage mechanism can be implemented by a menu of two put contracts with different strike prices: one strike price is ex post efficient, the other is strictly lower. The put with higher strike price is more expensive. If in the first stage the agent opts for the put with higher strike price by paying a higher premium, then he is entitled to charge the higher (and efficient) strike price for providing the good in the second stage. At the equilibrium, the efficient type chooses the more expensive put with an efficient strike price, while the inefficient type selects the less expensive put with a strictly lower (thus inefficient) strike price.

Three features of the optimal contract are worth noting. *First*, unlike the first-best benchmark and the pure adverse selection case, the second stage trading cutoff rather depends on the first stage type. We find that the acquisition price for the inefficient type is strictly lower than c_0, while the price for the efficient type remains being ex post efficient (i.e., set at c_0). Therefore, the principal needs to combine the second stage price discrimination together with the first stage transfer to screen and incentivize different types. This results from the fact that when moral hazard is further brought in, the principal now has to pay an additional moral hazard rent to the agent: the efficient type is more tempted to mimic the inefficient type since he can choose the most profitable effort level when he misreports his first stage type as effort is unobservable. Setting an acquisition price lower than c_0 for the inefficient type would make it less profitable for the efficient type to deviate from truthful reporting and thus reduces the rent to the efficient type. A strictly lower acquisition price turns out to be optimal which balances the rent extraction-efficiency trade-off.

Second, the first stage payment to the principal is higher for the efficient type. This result is rather intuitive as with uniform payment for different types, the efficient type definitely has no incentive to mimic the inefficient type due to the discriminatory acquisition price. Therefore, the principal is able to extract more surplus from the efficient type by setting a higher payment.

Third, relative to the first-best outcome, there is no effort distortion for the efficient type, while there is a downward distortion for the inefficient type. Unlike the pure adverse selection case, the

[7] We establish that the optimal first stage mechanism must be deterministic, and there is no loss of generality to focus on deterministic second stage mechanism for the efficient type.

downward distortion is rather due to the discriminatory acquisition price set for the inefficient type, which dampens the incentive of the inefficient type.

Our paper primarily belongs to the literature on procurement design with R&D.[8] Dasgupta [12] considers a two-period sealed bid procurement model. Firms having identical marginal R&D cost make investment in period one, and learn their production costs in period two. The procurer then chooses the reservation price. He finds that when R&D investment is not observable by the procurer and the procurer cannot precommit to the second stage reservation price, the firms underinvest relative to the socially optimal level. When the procurer can precommit to the second stage reservation price or when investment is observable such that the procurer can subsidize investment costs, then firms' investment is higher and can even reach or go beyond the socially optimal level. Tan [28] focuses on an environment where the procurer can precommit to a reservation price in a first or second price auction and bidders' investment decisions are not observable. Tan [28] accommodates both linear and nonlinear R&D cost functions. He finds that the first and second price auctions are equivalent when the R&D cost function is convex. However, this equivalence no longer prevails when the R&D cost function is linear.[9]

Piccione and Tan [23] generalize the analytical framework of Dasgupta [12] and Tan [28] by allowing the selected supplier further reducing the production cost by exerting a second round costly effort. Suppliers' investment decisions are their private information. They find that the full information solution can be achieved by the first and second price auctions when the R&D technology exhibits decreasing returns to scale, if the procurer offers the contract before R&D investment. If the decisions on the procurement mechanism and the level of investment are simultaneous, then the full information solution is not implementable. Bag [4] differentiates from these studies by emphasizing the role of ex ante entry fees, which are first stage transfers. He focuses on the case where the principal makes the offer before unobservable investment decisions. One main insight is that the first stage entry fees together with a second stage efficient auction implement the first-best outcome for the procurer. Arozamena and Cantillon [2] consider a procurement contract of first or second price sealed bid auction without reservation price. They assume that investment is observable and only one firm has the opportunity to invest at a time. They find that firms tend to underinvest, and the first price auction elicits less investment than the second price auction when firms are heterogeneous.

Our paper complements these existing studies by accommodating private R&D efficiency of the supplier and explicitly examining the optimal two-stage procurement in a framework of dynamic

[8] In the early literature of procurement design (e.g., Baron and Myerson [6]), suppliers are assumed to perfectly observe their provision costs. The procurement literature with R&D is comprehensive. This literature is also closely related to the well established regulation literature with cost-reducing investment and that on mechanism design with information acquisition. Here we are unable to provide an exhaustive review due to the space constraint.

[9] Both Dasgupta [12] and Tan [28] look at the issue of endogenous entry from different perspectives.

mechanism design with sequential private information. We find that at the optimum only the inefficient type underinvests regardless of the observability of investment, and the second stage mechanism must discriminate against the inefficient type if investment is unobservable.

Our paper contributes to the growing literature on dynamic mechanism design, which originates from the seminal work of Baron and Besanko [5] in a two-period regulation environment where a regulated firm's private costs evolve over time.[10] Courty and Li [10] and Eső and Szentes [13] demonstrate in different environments on sequential screening with pure adverse selection that at the optimum the second stage mechanism is discriminatory across first stage types. Pavan, Segal, and Toikka [22] provide a general treatment of optimal dynamic mechanism design with pure adverse selection.[11] Our paper studies a mixed adverse selection and moral hazard problem in a dynamic environment where the supplier's R&D efficiency is his first stage private information. Unlike the exogenously given informativeness of the first stage signal in Courty and Li [10] and Eső and Szentes [13], in our paper it is the agent's own effort that determines the distribution of second stage types. In this sense, the informativeness is endogenous. We find that the optimal second stage mechanism is discriminatory if and only if the agent's investment is not observable. In particular, with observable investment, the second stage mechanism is always ex post efficient.

Our paper is most closely related to Krähmer and Strausz [15] who essentially introduce endogenous information acquisition to the monopolistic price discrimination model of Courty and Li [10].[12] Their setting also involves both moral hazard and adverse selection in the first stage and adverse selection in the second stage (if the agent acquires information). The agent decides whether or not to incur a commonly known cost to discover his second stage type, and moral hazard arises because the principal cannot observe the agent's decision. Krähmer and Strausz [15] find that relative to the first-best outcome, adverse selection induces too much/little information acquisition for some low/high first-stage types while moral hazard mitigates/exacerbates the inefficiency of these lower/higher types.[13] When moral hazard is present, the second stage becomes more distorting relative to the pure adverse selection setting. We find similar result in our setting: when moral hazard is further brought in, the second stage acquisition cutoff becomes inefficient for the inefficient type.

On the other hand, there are several important differences on which our work differentiates from Krähmer and Strausz [15]. First, the source of moral hazard is different. In our setting,

[10] Baron and Besanko [5] allows the agent to exert an effort to reduce the production cost in the second stage.

[11] Another strand of literature focuses on the implementation of efficient mechanism, e.g., Bergemann and Välimäki [9], and Athey and Segal [3]. While we focus on cost-minimizing procurement design, the efficient mechanism can be analyzed analogously.

[12] Other related works on information acquisition include Bergemann and Välimäki [7], Ye [29], Crémer, Spiegel, and Zheng [11], Shi [26], Lu and Ye [19], and Aoyagi [1] among many others. See Bergemann and Välimäki [8] for an excellent survey.

[13] The first stage type in Krähmer and Strausz [15] is continuous. They derive the optimal contract within deterministic mechanisms when moral hazard kicks in.

the agent's choice of effort is continuous, which determines the distribution of the second stage type, and the moral hazard arises because the principal cannot observe how much effort the agent exerts. Moreover, in their setting the information acquisition decision does not change the marginal distribution of the second stage type, while in our setting the R&D investment improves the agent's provision cost distribution. Second, the first stage type fully and immediately describes the second stage type distribution in Krähmer and Strausz [15], while our first stage type (i.e., the R&D efficiency) is not a direct measure of the second stage type. As a result, in the pure adverse selection setting, the second stage allocation rule is discriminatory across the first stage type in their setting (thus inefficient), while in our setting the second stage is always efficient in the pure adverse selection problem. A third difference lies in that in their setting moral hazard does not necessarily cause additional agency cost so that the optimal contract of the mixed problem can coincide with that of the pure adverse selection problem, while in our setting this is never the case. This contrast results from the fact that in their setting the agent's action is discrete (acquire information or not) such that the agent has little freedom to manipulate his effort. The agent's incentive of information acquisition can be perfectly aligned with the principal's in the optimal contract of pure adverse selection, in which case moral hazard would not cause additional agency cost. This is not the case in our setting where the agent's effort is continuous such that the agent has much more freedom to manipulate his effort supply.

The rest of the paper is organized as follows. Section 2 introduces the model. We study the optimal mechanism in the benchmark case in section 3. Section 4 characterizes the optimal mechanism for the pure adverse selection setting. In section 5, we solve the optimal mechanism for the mixed problem and discuss the properties of the optimal mechanism. Section 6 provides some concluding remarks. The appendix collects some technical proofs.

2 The model setup

A risk neutral buyer (the principal) wants to procure a product from a risk neutral supplier (the agent). The agent can carry out R&D to improve his production technology. The agent's R&D efficiency, θ, is his private information. Exerting R&D effort $\alpha \geq 0$ costs him $C(\alpha, \theta)$, with $C(0, \theta) = 0$, $C_\alpha > 0$, $C_\theta \geq 0$ with equality only when $\alpha = 0$,[14] $C_{\alpha\theta} > 0$,[15] $C_{\alpha\alpha} \geq 0$ and $C_{\alpha\alpha\theta} \geq 0$.[16] Thus, both cost and marginal cost increase when θ increases, which means that a lower θ is more efficient at conducting R&D. As an example, the cost function $C(\alpha, \theta) = \theta\alpha^r$, $r \geq 1$, satisfies all these conditions.

[14] Note that $C_\theta(0, \theta) = 0$ since we assume $C(0, \theta) = 0$.

[15] Our analysis allows $C_{\alpha\theta}(0, \theta) = 0$.

[16] The assumption that $C_{\alpha\alpha\theta} \geq 0$ is purely for convenience. It ensures the uniqueness of effort provision of the inefficient type in the pure adverse selection setting.

We assume that θ can be either θ_l with probability q or θ_h with probability $1-q$, where $\theta_l < \theta_h$ and $0 < q < 1$. For convenience, the θ_l (θ_h) type is also referred to as the efficient (inefficient) type. The agent's cost c of delivering the product when he exerts R&D effort α is a random variable with a cumulative distribution function $H(\cdot, \alpha)$ defined on support $[\underline{c}, \bar{c}]$ with $0 \leq \underline{c} < \bar{c} \leq \infty$. After the R&D, the delivering cost is privately observed by the agent. The production cost c is incurred by the agent if and only if the trade happens between the procurer and the supplier. The principal however can turn to an outside option of acquiring the good at cost c_0 with $\underline{c} < c_0 < \bar{c}$. Equivalently, c_0 can be treated as the procurer's value of the good.

The timing of the game is as follows.

Time 0: $C(\cdot, \cdot)$, q, $H(\cdot, \cdot)$, and c_0 are revealed by nature as public information. Nature draws θ for the agent. The agent is privately informed about his type θ.

Time 1: The principal offers a contract and she commits to it. If the agent rejects, then the game ends and he obtains the reservation utility which is normalized as zero. If the agent accepts, then he reports his type θ and the first stage contract is executed. The agent decides on his R&D effort α if it is unobservable,[17] and then his delivery cost c is realized according to $H(\cdot, \alpha)$.

Time 2: The agent decides whether to quit. If he quits, the game is over. If he continues, then he reports his delivery cost c. The contract is executed.

We use t to denote the gross transfer (aggregated over two stages) from the principal to the agent. Suppose the agent with type θ exerts effort level α, and the realized cost is c. If the principal acquires the product from the agent, then her procurement cost is t, and the agent's payoff is $t - C(\alpha, \theta) - c$. If the principal does not acquire the product from the agent, then her procurement cost is $t + c_0$, and the agent's payoff is $t - C(\alpha, \theta)$.

The principal's objective is to minimize the expected procurement cost. Formally, this is a two-stage game. In the first stage, the agent with type θ exerts R&D effort α at the cost of $C(\alpha, \theta)$. In the second stage, the cost c is drawn from $H(c, \alpha)$. Both θ and c are the agent's private information. Thus, this is a dynamic game with both moral hazard (when α is unobservable) and adverse selection in the first stage and adverse selection in the second stage.

We assume that the density function $h(c, \alpha)$ (i.e., $H_c(c, \alpha)$) is continuously differentiable in $(c, \alpha) \in [\underline{c}, \bar{c}] \times [0, +\infty)$, and for any c, $H(c, \alpha)$ is a twice continuously differentiable function with respect to α. In addition,[18]

$$H_\alpha(c, \alpha) > 0, H_{\alpha\alpha}(c, \alpha) < 0, \forall c \in (\underline{c}, \bar{c}).$$

[17] If R&D effort is observable, then it is specified by the contract.

[18] Note that since the family of CDFs $H(c, \alpha)$ has common support, $H(\underline{c}, \alpha) = 0$, $H(\bar{c}, \alpha) = 1$ for any $\alpha \geq 0$. Therefore, $H_\alpha(\underline{c}, \alpha) = H_\alpha(\bar{c}, \alpha) = 0$ for any $\alpha \geq 0$.

Positive $H_\alpha(c,\alpha)$ means that higher effort leads to a better cost distribution in the sense of first-order stochastic dominance. Negative $H_{\alpha\alpha}(c,\alpha)$ means that the marginal effect of α decreases. Since $H(c,\alpha) \in [0,1]$, these conditions mean that when α goes to infinity, the marginal effect $H_\alpha(c,\alpha)$ diminishes to zero.

Our formulation of $H(c,\alpha)$ covers the following widely adopted form as a special case:

$$H(c,\alpha) = 1 - (1 - F(c))^{\alpha+\beta_0},$$

where $F(\cdot)$ is a CDF with strictly positive density function everywhere over the support $[\underline{c}, \overline{c}]$ and $\beta_0 \geq 0$ is the initial technology endowment of the agent.[19] The case $\beta_0 = 0$ corresponds to the R&D technology used in Tan [28].

3 The First-Best: A Benchmark

We first analyze a full information benchmark case where the agent's action and the agent's types are public information. This is the first-best outcome that the principal can achieve. In this first-best environment, there is no moral hazard or adverse selection issues involved.

Suppose for the agent with type $\theta \in \{\theta_l, \theta_h\}$ and realized cost c, the contract specifies the payment to the agent $y^{FB}(\theta, c)$ and the acquisition probability $p^{FB}(\theta, c)$. In the first period, the contract prescribes the agent's effort level $\alpha^{FB}(\theta)$ and the payment to the agent $x^{FB}(\theta)$.[20] Therefore, the agent's second period payoff when his type is θ and the cost realization is c can be expressed as[21]

$$\tilde{\pi}^{FB}(\theta, c) = y^{FB}(\theta, c) - p^{FB}(\theta, c)c.$$

The first period expected payoff for the agent with type θ is

$$\pi^{FB}(\theta) = x^{FB}(\theta) - C(\alpha^{FB}(\theta), \theta) + \int_{\underline{c}}^{\overline{c}} \tilde{\pi}^{FB}(\theta, c) h(c, \alpha^{FB}(\theta)) dc.$$

[19] Thus, if the agent does not invest in R&D, then his delivery cost is randomly drawn according to CDF $H(c, 0)$.

[20] As we shall see later, the total procurement cost is convex in effort when the second stage mechanism is efficient (which is optimal given any effort level), so there is no loss of generality to assume that the required effort $\alpha^{FB}(\theta)$ is deterministic. Also note that the agent's payoffs are linear in transfers $x^{FB}(\theta)$ and $y^{FB}(\theta, c)$ so that we can focus on deterministic transfers.

[21] For the time being, we ignore the individual rationality constraints. We will verify that the agent's payoff is nonnegative for both two stages under the first-best contract.

As a result, the expected total procurement cost when the agent's type is θ is

$$x^{FB}(\theta) + \int_{\underline{c}}^{\overline{c}} (y^{FB}(\theta, c) + (1 - p^{FB}(\theta, c))c_0)h(c, \alpha^{FB}(\theta))dc$$

$$= C(\alpha^{FB}(\theta), \theta) + \int_{\underline{c}}^{\overline{c}} p^{FB}(\theta, c)(c - c_0)h(c, \alpha^{FB}(\theta))dc + \pi^{FB}(\theta) + c_0,$$

which is the sum of social cost and the agent's first period expected utility. Obviously, to minimize the above expression, we should set $\pi^{FB}(\theta) = 0$. And then notice that for any fixed α, to minimize the total cost, we should set $p^{FB}(\theta, c) = 1$ when $c \leq c_0$, and $p^{FB}(\theta, c) = 0$ when $c > c_0$. Thus the optimization problem amounts to minimizing

$$C(\alpha, \theta) + \int_{\underline{c}}^{c_0} (c - c_0)h(c, \alpha)dc = C(\alpha, \theta) - \int_{\underline{c}}^{c_0} H(c, \alpha)dc. \tag{1}$$

Note that this function has increasing difference in (α, θ) since $C_{\alpha\theta} > 0$. The first order derivative with respect to α is

$$C_\alpha(\alpha, \theta) - \int_{\underline{c}}^{c_0} H_\alpha(c, \alpha)dc.$$

Second order derivative with respect to α is

$$C_{\alpha\alpha}(\alpha, \theta) - \int_{\underline{c}}^{c_0} H_{\alpha\alpha}(c, \alpha)dc > 0.$$

Note that the cutoff is fixed at c_0 regardless of the effort level, and the objective function (1) is convex in effort. Thus, the principal would implement a deterministic effort.

The unique optimal solution $\alpha^{FB}(\theta)$ satisfies[22]

$$C_\alpha(\alpha^{FB}(\theta), \theta) - \int_{\underline{c}}^{c_0} H_\alpha(c, \alpha^{FB}(\theta))dc \geq 0, \text{ with equality when } \alpha^{FB}(\theta) > 0. \tag{2}$$

Note that since the function (1) has increasing difference and $\theta_l < \theta_h$, we have $\alpha^{FB}(\theta_h) \leq \alpha^{FB}(\theta_l)$, with equality only when $\alpha^{FB}(\theta_l) = 0$.

Payment $y^{FB}(\theta, c)$ is set at c_0 when $p^{FB}(\theta, c) = 1$; otherwise $y^{FB}(\theta, c) = 0$. Payment $x^{FB}(\theta)$ equals $C(\alpha^{FB}(\theta), \theta) - \int_{\underline{c}}^{\overline{c}} \tilde{\pi}^{FB}(\theta, c)h(c, \alpha^{FB}(\theta))dc$ to extract all the surplus. It is easy to verify that the agent's payoff is nonnegative for both two stages so that the individual rationality constraints are satisfied.

[22] Note that the optimal solution must be finite. This is because $C_\alpha(\alpha^{FB}(\theta), \theta)) \geq C_\alpha(0, \theta) > 0$ and $\lim_{\alpha \to +\infty} \int_{\underline{c}}^{c_0} H_\alpha(c, \alpha)dc = 0$.

To summarize, in the first-best environment, the second period mechanism is ex post efficient; both θ types agent has a zero expected payoff in the first period; and $\alpha^{FB}(\theta_h) \leq \alpha^{FB}(\theta_l)$, with strict inequality if and only if $\alpha^{FB}(\theta_l) > 0$. It is clear that the first-best outcome coincides with social efficiency.

To rule out the uninteresting case of $\alpha^{FB}(\theta_l) = \alpha^{FB}(\theta_h) = 0$, hereafter, we assume that $\alpha^{FB}(\theta_l) > 0$, which is satisfied if and only if $C_\alpha(0, \theta_l) < \int_{\underline{c}}^{c_0} H_\alpha(c, 0)dc$. This means that it is socially efficient to induce the efficient type agent to exert strictly positive effort. For notational simplicity, we use α_l^{FB} to denote $\alpha^{FB}(\theta_l)$, similarly for a_h^{FB}.

In the next section, we shall study how introducing adverse selection (but with observable R&D effort) distorts the R&D investment but still retains the ex post efficiency in the second stage. After that, we examine how further introducing moral hazard issue (unobservable R&D investment) affects the agent's R&D incentive and the second stage allocation efficiency.

4 Observable R&D Investment: Pure Adverse Selection

We now turn to the situation in which the agent's R&D efficiency θ and provision cost c are his private information, but his R&D investment (i.e., α) is observable. That is, there is no moral hazard issue. According to the revelation principle (Myerson [20]), there is no loss of generality to restrict to direct mechanisms, which is truthful on the equilibrium path. The mechanism specifies the first stage payment to the agent $\tilde{x}(\theta)$ and the effort required $\tilde{\alpha}(\theta)$ after receiving the agent's report θ. The mechanism also specifies the acquisition probability $\tilde{p}(\theta, c)$ and the payment to the agent $\tilde{y}(\theta, c)$ in the second stage, which depend on both the first stage report θ and the second stage report c.[23,24] To simplify the notations, let $\tilde{\alpha}_h = \tilde{\alpha}(\theta_h)$ and $\tilde{\alpha}_l = \tilde{\alpha}(\theta_l)$. Similar for the notations $\tilde{x}_h$ and $\tilde{x}_l$.

4.1 Stage Two

Assume that the agent truthfully reported his type $\theta \in \{\theta_l, \theta_h\}$ in stage one, and his realized cost is c. Let $\tilde{\pi}_p(\theta, \hat{c}, c)$ be his expected payoff in stage two if he reports $\hat{c}$. Then[25]

$$\tilde{\pi}_p(\theta, \hat{c}, c) = \tilde{y}(\theta, \hat{c}) - \tilde{p}(\theta, \hat{c})c. \tag{3}$$

[23] Since the agent's first stage and second stage payoffs are linear in payment, there is no loss of generality to focus on deterministic payment rules $\tilde{x}(\theta)$ and $\tilde{y}(\theta, c)$. In addition, since $C_{\alpha\theta} > 0$, we can apply the result in Strausz [27] (with some modifications) to show that there is no loss of generality to focus on deterministic mechanisms. We shall come to this point later.

[24] It is clear that allowing the mechanism depending on investment cannot improve the generality as investment is a function of the first stage report.

[25] We first ignore the second stage individual rationality constraint and then verify that this is not an issue.

Envelope Theorem yields

$$\frac{d\tilde{\pi}_p(\theta,c,c)}{dc} = \frac{\partial\tilde{\pi}_p(\theta,\hat{c},c)}{\partial c}|_{\hat{c}=c} = -\tilde{p}(\theta,c).$$

Thus

$$\tilde{\pi}_p(\theta,c,c) = \tilde{\pi}_p(\theta,\overline{c},\overline{c}) + \int_c^{\overline{c}} \tilde{p}(\theta,s)ds. \tag{4}$$

It is standard to establish that the second stage incentive compatibility (IC) is equivalent to that (4) holds and that $\tilde{p}(\theta,c)$ is decreasing in c for any fixed θ. Note that the agent will still truthfully report his second stage type c on the off-equilibrium path. That is, if the agent misreported his type in stage one as $\hat{\theta}$, then he will still truthfully report c in stage two. The reason is that the true type does not affect the realized value of c ex post. The reported first stage type solely changes the distribution of cost through the required effort level. The detailed arguments are as follows. Suppose the reported type is $\hat{\theta}$ in stage one, the realized cost is c, and he reports $\hat{c}$ instead in stage two. Then his payoff (3) becomes

$$\tilde{\pi}_p(\hat{\theta},\hat{c},c) = \tilde{y}(\hat{\theta},\hat{c}) - \tilde{p}(\hat{\theta},\hat{c})c.$$

Note if $\hat{\theta}$ is the true type, then $\hat{c}=c$ maximizes $\tilde{\pi}_p(\hat{\theta},\hat{c},c)$ by the optimality of truthful reporting. However, since $\tilde{\pi}_p(\hat{\theta},\hat{c},c)$ does not depend on the true type θ, the optimality of truthful reporting at the second stage holds regardless of the report of the first stage. By the same argument, (4) holds for any reported type $\hat{\theta}$.

Note that here we only consider IC in stage two. As will be shown later, at the optimum the agent's second stage individual rationality (IR) constraint is satisfied.

4.2 Stage One

We consider both IC and IR in the first stage. Recall that (4) holds for any reported type $\hat{\theta}$. If the efficient type θ_l agent misreports his type as θ_h, his expected payoff is

$$\pi_{hl} = \tilde{x}_h - C(\tilde{\alpha}_h,\theta_l) + \int_{\underline{c}}^{\overline{c}} [\tilde{\pi}_p(\theta_h,\overline{c},\overline{c}) + \int_c^{\overline{c}} \tilde{p}(\theta_h,s)ds]h(c,\tilde{\alpha}_h)dc.$$

Note that there is no loss of generality to assume that $\tilde{\pi}_p(\theta_h,\overline{c},\overline{c})=0$ since we can always define a new first stage payment $\tilde{x}'_h = \tilde{x}_h + \tilde{\pi}_p(\theta_h,\overline{c},\overline{c})$. Similarly, we can also assume that $\tilde{\pi}_p(\theta_l,\overline{c},\overline{c})=0$.

Then

$$\pi_{hl} = \tilde{x}_h - C(\tilde{\alpha}_h, \theta_l) + \int_{\underline{c}}^{\overline{c}} \int_{c}^{\overline{c}} \tilde{p}(\theta_h, s) h(c, \tilde{\alpha}_h) ds dc$$

$$= \tilde{x}_h - C(\tilde{\alpha}_h, \theta_h) + \int_{\underline{c}}^{\overline{c}} \tilde{p}(\theta_h, c) H(c, \tilde{\alpha}_h) dc + (C(\tilde{\alpha}_h, \theta_h) - C(\tilde{\alpha}_h, \theta_l))$$

$$= \pi_{hh} + C(\tilde{\alpha}_h, \theta_h) - C(\tilde{\alpha}_h, \theta_l),$$

where π_{hh} is the first stage expected payoff of the agent with type θ_h and he truthfully reports it. Similarly, when the type θ_h agent misreports his type as θ_l, his expected payoff is

$$\pi_{lh} = \tilde{x}_l - C(\tilde{\alpha}_l, \theta_h) + \int_{\underline{c}}^{\overline{c}} \tilde{p}(\theta_l, c) H(c, \tilde{\alpha}_l) dc$$

$$= \pi_{ll} - (C(\tilde{\alpha}_l, \theta_h) - C(\tilde{\alpha}_l, \theta_l)),$$

where π_{ll} is the first stage expected payoff of the agent with type θ_l and he truthfully reports it.

The IC requires that $\pi_{hl} \leq \pi_{ll}$ and $\pi_{lh} \leq \pi_{hh}$. That is,

$$\pi_{ll} - (C(\tilde{\alpha}_l, \theta_h) - C(\tilde{\alpha}_l, \theta_l)) \leq \pi_{hh} \leq \pi_{ll} - (C(\tilde{\alpha}_h, \theta_h) - C(\tilde{\alpha}_h, \theta_l)).$$

4.3 The Principal's Objective

The principal's objective is to minimize the expected procurement cost which is the sum of the expected social cost (denoted as SC) and the agent's first stage expected payoff. That is,

$$TC_p = SC + q\pi_{ll} + (1-q)\pi_{hh}$$

$$= q\{C(\tilde{\alpha}_l, \theta_l) + \int_{\underline{c}}^{\overline{c}} [\tilde{p}(\theta_l, c)c + (1 - \tilde{p}(\theta_l, c))c_0] h(c, \tilde{\alpha}_l) dc + \pi_{ll}\}$$

$$+ (1-q)\{C(\tilde{\alpha}_h, \theta_h) + \int_{\underline{c}}^{\overline{c}} [\tilde{p}(\theta_h, c)c + (1 - \tilde{p}(\theta_h, c))c_0] h(c, \tilde{\alpha}_h) dc + \pi_{hh}\}.$$

The constraints are:

$$\pi_{ll} - (C(\tilde{\alpha}_l, \theta_h) - C(\tilde{\alpha}_l, \theta_l)) \leq \pi_{hh} \leq \pi_{ll} - (C(\tilde{\alpha}_h, \theta_h) - C(\tilde{\alpha}_h, \theta_l)); \tag{5}$$

$$\pi_{ll} \geq 0,\ \pi_{hh} \geq 0; \tag{6}$$

$$\tilde{p}(\theta, c) \text{ is decreasing in } c, \forall\theta. \tag{7}$$

(5) and (6) are the IC and IR constraints for the first stage, respectively; (7) is the second stage IC constraint.[26]

As in standard discrete adverse selection model, at the optimum it must be the case that $\pi_{hh} = 0$ and $\pi_{ll} = C(\tilde{\alpha}_h, \theta_h) - C(\tilde{\alpha}_h, \theta_l)$, since $\theta_h > \theta_l$ leads to $C(\tilde{\alpha}_h, \theta_h) - C(\tilde{\alpha}_h, \theta_l) \geq 0$ as $C_\theta \geq 0$. Therefore, the θ_l type enjoys an information rent of $C(\tilde{\alpha}_h, \theta_h) - C(\tilde{\alpha}_h, \theta_l)$. And then IC is equivalent to $\tilde{\alpha}_h \leq \tilde{\alpha}_l$,[27] while IR can be dropped. Applying these results, we obtain the following equivalent problem of the principal:

$$\min_{\tilde{\alpha}_l, \tilde{\alpha}_h, \tilde{p}(\theta_l, c), \tilde{p}(\theta_h, c)} TC_p = c_0 + q[C(\tilde{\alpha}_l, \theta_l) + \int_{\underline{c}}^{\bar{c}} \tilde{p}(\theta_l, c)(c - c_0)h(c, \tilde{\alpha}_l)dc]$$

$$+ (1-q)[\frac{C(\tilde{\alpha}_h, \theta_h) - qC(\tilde{\alpha}_h, \theta_l)}{1-q} + \int_{\underline{c}}^{\bar{c}} \tilde{p}(\theta_h, c)(c - c_0)h(c, \tilde{\alpha}_h)dc]$$

subject to

$$\tilde{\alpha}_l \geq \tilde{\alpha}_h \geq 0; \tag{8}$$

$$\tilde{p}(\theta, c) \text{ is decreasing in } c, \forall\theta. \tag{9}$$

The procedure of solving the above minimization problem is standard: First drop the above two monotonicity constraints ((8) and (9)) and solve for the optimal mechanism in the relaxed problem, then check that the dropped two constraints are satisfied at the solution.

In fact, for the relaxed problem, we can minimize it pointwisely. It is clear that at the optimum, $\tilde{p}^*(\theta, c) = 1$ if $c \leq c_0$, and $\tilde{p}^*(\theta, c) = 0$ if $c > c_0$ for $\theta \in \{\theta_l, \theta_h\}$. It follows that for type θ_l, the minimization problem amounts to

$$\min_{\tilde{\alpha}_l \geq 0} C(\tilde{\alpha}_l, \theta_l) + \int_{\underline{c}}^{c_0} (c - c_0)h(c, \tilde{\alpha}_l)dc, \tag{10}$$

which is the same as the one in (1). Thus the optimal solution $\tilde{\alpha}_l^* = \alpha_l^{FB}$. While for type θ_h, the minimization problem boils down to

$$\min_{\tilde{\alpha}_h \geq 0} \frac{C(\tilde{\alpha}_h, \theta_h) - qC(\tilde{\alpha}_h, \theta_l)}{1-q} + \int_{\underline{c}}^{c_0} (c - c_0)h(c, \tilde{\alpha}_h)dc = \min_{\tilde{\alpha}_h \geq 0} \frac{C(\tilde{\alpha}_h, \theta_h) - qC(\tilde{\alpha}_h, \theta_l)}{1-q} - \int_{\underline{c}}^{c_0} H(c, \tilde{\alpha}_h)dc. \tag{11}$$

[26] Note that we have incorporated the envelope condition (4) into the objective function, thus the second stage IC reduces to (7).

[27] To see this, note that IC is equivalent to $C(\tilde{\alpha}_h, \theta_h) - C(\tilde{\alpha}_h, \theta_l) \leq C(\tilde{\alpha}_l, \theta_h) - C(\tilde{\alpha}_l, \theta_l)$, which holds if and only if $\tilde{\alpha}_h \leq \tilde{\alpha}_l$ since $C_{\alpha\theta} > 0$.

The first order derivative with respect to $\tilde{\alpha}_h$ is given by:

$$\frac{C_\alpha(\tilde{\alpha}_h,\theta_h)-qC_\alpha(\tilde{\alpha}_h,\theta_l)}{1-q}-\int_{\underline{c}}^{c_0}H_\alpha(c,\tilde{\alpha}_h)dc.$$

Note that the optimal $\tilde{\alpha}_h$ must be finite since the first term is always strictly positive, while the second term $\int_{\underline{c}}^{c_0}H_\alpha(c,\tilde{\alpha}_h)dc$ decreases to zero as $\tilde{\alpha}_h$ tends to infinity. The optimum, $\tilde{\alpha}_h^*$, must satisfy

$$\frac{C_\alpha(\tilde{\alpha}_h^*,\theta_h)-qC_\alpha(\tilde{\alpha}_h^*,\theta_l)}{1-q}-\int_{\underline{c}}^{c_0}H_\alpha(c,\tilde{\alpha}_h^*)dc\geq 0,\text{ with equality if }\tilde{\alpha}_h^*>0.$$

Note that

$$\begin{aligned}&\frac{C_\alpha(\tilde{\alpha}_h,\theta_h)-qC_\alpha(\tilde{\alpha}_h,\theta_l)}{1-q}-\int_{\underline{c}}^{c_0}H_\alpha(c,\tilde{\alpha}_h)dc\\ =\;& C_\alpha(\tilde{\alpha}_h,\theta_h)+\frac{q}{1-q}[C_\alpha(\tilde{\alpha}_h,\theta_h)-C_\alpha(\tilde{\alpha}_h,\theta_l)]-\int_{\underline{c}}^{c_0}H_\alpha(c,\tilde{\alpha}_h)dc.\end{aligned}$$

Recall that $C_{\alpha\alpha}\geq 0$, $C_{\alpha\alpha\theta}\geq 0$ and $H_{\alpha\alpha}(c,\alpha)<0$, which means $C_\alpha(\tilde{\alpha}_h,\theta_h)$ and $[C_\alpha(\tilde{\alpha}_h,\theta_h)-C_\alpha(\tilde{\alpha}_h,\theta_l)]$ (i.e., $\int_{\theta_l}^{\theta_h}C_{\alpha\theta}(\tilde{\alpha}_h,\theta)d\theta$) increase with $\tilde{\alpha}_h$, and $\int_{\underline{c}}^{c_0}H_\alpha(c,\tilde{\alpha}_h)dc$ decreases with $\tilde{\alpha}_h$. Thus the optimal $\tilde{\alpha}_h^*$ is pinned down by the unique solution of the first order condition, and the global second order condition must hold.

Recall that α_h^{FB} is the solution to (2) when $\theta=\theta_h$. We must have $\tilde{\alpha}_h^*\leq\alpha_h^{FB}$ with equality only when $\alpha_h^{FB}=0$.[28] Since $\alpha_h^{FB}<\alpha_l^{FB}$ and $\alpha_l^{FB}=\tilde{\alpha}_l^*$, we have $\tilde{\alpha}_h^*<\tilde{\alpha}_l^*$.

Notice that the specification of the second stage mechanism satisfies the monotonicity constraint (9), and (8) is also satisfied since $\tilde{\alpha}_h^*<\tilde{\alpha}_l^*$. Thus, as mentioned in footnote 23, the optimal (deterministic) mechanism involves no bunching so that the deterministic mechanism is indeed optimal (cf. Strausz [27]). Also note that the second stage mechanism is always efficient. We summarize the properties of the optimal pure adverse selection mechanism in the following proposition.

Proposition 1 *In the pure adverse selection setting with observable R&D investment, the second stage mechanism is always efficient at the optimum. The optimal mechanism requires the efficient type (θ_l) to exert more effort than the inefficient type (θ_h). Relative to the first-best outcome, there is no effort distortion for the efficient type, while there is a downward distortion for the inefficient type if and only if the first-best effort $\alpha_h^{FB}>0$.*

The downward effort distortion of the inefficient type reflects the usual rent extraction-efficiency trade-off in the adverse selection model. The ex post efficiency in the second stage is rather

[28] Note $C_\alpha(\tilde{\alpha}_h,\theta_h)-C_\alpha(\tilde{\alpha}_h,\theta_l)>0$.

surprising given the optimality of discriminatory mechanism that is typical in dynamic screening literature (e.g., Courty and Li [10] and Esős and Szentes [13]). The reason is that in our setting the observableness of the R&D effort can be utilized to cut off the linkage between the first stage signal (i.e., the R&D efficiency) and the second stage signal (i.e., the provision cost). Suppose at the optimum the second stage mechanism discriminates against the inefficient type θ_h such that the corresponding acquisition cutoff $\tilde{c}_h^*$ is lower than the efficient cutoff c_0. By raising the cutoff $\tilde{c}_h^*$ to the efficient level while maintaining the same R&D investment, the surplus generated in the second stage can be shared between the principal and the agent for the fixed (incentive compatible) second stage payment rule. Proper adjustments in the first stage payment rule can make sure that in the first stage the two types' expected payoffs remain unchanged as well as the expected payoffs from deviations. As a result, the first stage IC still holds and all additional surplus generated goes to the principal. The observableness of the R&D effort is crucial for the above argument, which guarantees that the proposed change in acquisition cutoff does not affect the R&D investment.

In Section 5, we will move on to the setting of our main interest, where the moral hazard issue arises. We shall discuss the efficiency loss and the effort distortion when R&D effort is unobservable.

4.4 Optimal Mechanism

By (4) and $\tilde{\pi}_p(\theta, \overline{c}, \overline{c}) = 0$ for both θ's,

$$\tilde{\pi}_p(\theta, c, c) = \int_c^{\overline{c}} \tilde{p}^*(\theta, s)ds = \begin{cases} 0, \text{ if } c > c_0; \\ c_0 - c, \text{ if } \underline{c} \leq c \leq c_0. \end{cases}$$

Thus by (3)

$$\tilde{y}^*(\theta, c) = \tilde{p}^*(\theta, c)c + \tilde{\pi}_p(\theta, c, c) = \begin{cases} 0, \text{ if } c > c_0; \\ c_0, \text{ if } \underline{c} \leq c \leq c_0. \end{cases} \tag{12}$$

Recall that

$$\begin{aligned} \pi_{hh} &= \tilde{x}_h - C(\tilde{\alpha}_h, \theta_h) + \int_{\underline{c}}^{\overline{c}} \tilde{p}(\theta_h, c)H(c, \tilde{\alpha}_h)dc, \\ \text{and } \pi_{ll} &= \tilde{x}_l - C(\tilde{\alpha}_l, \theta_l) + \int_{\underline{c}}^{\overline{c}} \tilde{p}(\theta_l, c)H(c, \tilde{\alpha}_l)dc, \end{aligned}$$

and at the optimum, $\pi_{hh} = 0$ and $\pi_{ll} = C(\tilde{\alpha}_h, \theta_h) - C(\tilde{\alpha}_h, \theta_l)$. Substituting the optimal solution into these expressions, we obtain

$$\tilde{x}_h^* = C(\tilde{\alpha}_h^*, \theta_h) - \int_{\underline{c}}^{c_0} H(c, \tilde{\alpha}_h^*)dc,$$

$$\text{and } \tilde{x}_l^* = C(\tilde{\alpha}_h^*, \theta_h) - C(\tilde{\alpha}_h^*, \theta_l) + C(\tilde{\alpha}_l^*, \theta_l) - \int_{\underline{c}}^{c_0} H(c, \tilde{\alpha}_l^*) dc.$$

Recall $C(0,\theta) = 0$, which means $\tilde{x}_h^* = -\int_{\underline{c}}^{c_0} H(c,0)dc \leq 0$ when $\tilde{\alpha}_h^* = 0$. When $\tilde{\alpha}_h^* > 0$, $\frac{C(\tilde{\alpha}_h^*,\theta_h) - qC(\tilde{\alpha}_h^*,\theta_l)}{1-q} - \int_{\underline{c}}^{c_0} H(c,\tilde{\alpha}_h^*)dc$ decreases on $[0, \tilde{\alpha}_h^*]$ by first order condition. We thus have

$$\begin{aligned}
\tilde{x}_h^* &= \frac{C(\tilde{\alpha}_h^*, \theta_h) - qC(\tilde{\alpha}_h^*, \theta_l)}{1-q} - \int_{\underline{c}}^{c_0} H(c, \tilde{\alpha}_h^*)dc + \frac{q(C(\tilde{\alpha}_h^*, \theta_l) - C(\tilde{\alpha}_h^*, \theta_h))}{1-q} \\
&\leq \frac{C(\tilde{\alpha}_h^*, \theta_h) - qC(\tilde{\alpha}_h^*, \theta_l)}{1-q} - \int_{\underline{c}}^{c_0} H(c, \tilde{\alpha}_h^*)dc \\
&\leq \frac{C(0, \theta_h) - qC(0, \theta_l)}{1-q} - \int_{\underline{c}}^{c_0} H(c, 0)dc \\
&= -\int_{\underline{c}}^{c_0} H(c, 0)dc \leq 0.
\end{aligned}$$

Similarly, since $\tilde{\alpha}_l^* > 0$ is the unique minimizer of the function $C(\alpha, \theta_l) - \int_{\underline{c}}^{c_0} H(c,\alpha)dc$ with respect to $\alpha \geq 0$ and $\tilde{\alpha}_l^* > \tilde{\alpha}_h^*$, we have

$$C(\tilde{\alpha}_l^*, \theta_l) - \int_{\underline{c}}^{c_0} H(c, \tilde{\alpha}_l^*)dc < C(\tilde{\alpha}_h^*, \theta_l) - \int_{\underline{c}}^{c_0} H(c, \tilde{\alpha}_h^*)dc.$$

Then

$$\tilde{x}_l^* = \tilde{x}_h^* + [C(\alpha_l^*, \theta_l) - \int_{\underline{c}}^{c_0} H(c, \tilde{\alpha}_l^*)dc] - [C(\alpha_h^*, \theta_l) - \int_{\underline{c}}^{c_0} H(c, \tilde{\alpha}_h^*)dc] < \tilde{x}_h^*.$$

Therefore, the agent pays to the principal in the first stage with the efficient type paying more. Note that in our setting, the agent's second stage *ex post* payoff must be nonnegative since the agent bears the production cost only when the principal acquires the product from him. As a result, the second stage IR is satisfied.

The optimal mechanism is summarized as follows.

Proposition 2 *In the pure adverse selection setting with observable R&D investment, the optimal mechanism is implemented by a menu of two contracts* $\{\tilde{x}_l^*, \tilde{\alpha}_l^*, c_0\}$ *and* $\{\tilde{x}_h^*, \tilde{\alpha}_h^*, c_0\}$ *where* $\tilde{x}_l^* < \tilde{x}_h^* \leq 0$ *and* $\tilde{\alpha}_l^* > \tilde{\alpha}_h^* \geq 0$. *In the first stage, the agent choosing the first contract (respectively the second contract) is required to pay* $\tilde{x}_l^*$ *(respectively* $\tilde{x}_h^*$*) and exert effort* $\tilde{\alpha}_l^*$ *(respectively* $\tilde{\alpha}_h^*$*). In the second stage, a nondiscriminatory take-it-or-leave-it acquisition price* c_0 *is in place.*

5 Unobservable R&D Investment: Mixed Problem

We now turn to a technically more challenging and economically more interesting environment where the agent's types of two stages and his R&D effort are all his private information. We thus have a mixed adverse selection and moral hazard problem in a dynamic setting.[29] As usual, we restrict attention to the truthful direct mechanisms according to Myerson [20]. In the first stage, the mechanism is a mapping $\rho : \{\theta_l, \theta_h\} \to \mathbb{R} \times \triangle\mathbb{R}_+$ such that when the agent reports θ, he receives from the principal a payment $x(\theta)$ and a stochastic R&D effort recommendation $\alpha \geq 0$ that has a density $\rho(\alpha|\theta)$. The agent decides on his R&D effort α after reporting θ and his delivery cost c is realized according to $H(\cdot, \alpha)$. In the second stage, the agent further reports his cost realization c and then the payment rule $y(\theta, c)$ and the acquisition probability $p(\theta, c)$ are executed.[30]

5.1 Stage Two

For the second stage, we ignore the IR for the time being as usual and consider IC. We will show later that at the optimum the agent's second stage IR is satisfied for the proposed optimal mechanism. Assuming truthfully reported $\theta \in \{\theta_l, \theta_h\}$ in stage one, suppose that the agent's true provision cost is c, but he reports $\hat{c}$. Let $\tilde{\pi}(\theta, \hat{c}, c)$ be his expected payoff in stage two. Then

$$\tilde{\pi}(\theta, \hat{c}, c) = y(\theta, \hat{c}) - p(\theta, \hat{c})c. \tag{13}$$

Envelope Theorem yields

$$\frac{d\tilde{\pi}(\theta, c, c)}{dc} = \frac{\partial \tilde{\pi}(\theta, \hat{c}, c)}{\partial c}|_{\hat{c}=c} = -p(\theta, c),$$

which leads to

$$\tilde{\pi}(\theta, c, c) = \tilde{\pi}(\theta, \overline{c}, \overline{c}) + \int_c^{\overline{c}} p(\theta, s) ds. \tag{14}$$

It is clear that the second stage IC is equivalent to that (14) holds and that $p(\theta, c)$ is decreasing in c for any fixed θ. For the same reason provided in Section 4.1, if the agent misreported his type in stage one as $\hat{\theta}$, then he will still truthfully report c in stage two. Moreover, as in Section 4.1, (14) holds for any reported type $\hat{\theta}$.

[29] We assume that the total R&D investment cost is also unobservable by the principal, so no information about R&D effort can be inferred.

[30] Since the agent has quasi-linear preference, there is no loss of generality to restrict attention to mechanisms in which the transfers x and y are deterministic.

5.2 Stage One

We consider both IC and IR in stage one. The IC requires that the agent will report his type truthfully and follow the principal's recommendation on R&D effort supply. This can be decomposed into two requirements: First, if the agent truthfully reports his type θ, then it is optimal for him to follow the principal's recommendation. Second, the agent will truthfully report his type given that he will accordingly choose the optimal effort level conditional on his report and the principal's recommendation. Note that though the recommendation can be stochastic, there is no loss of generality to focus on deterministic recommendation rules. The reason is that when the agent reports his type and then receives the recommendation (which depends on the report), he always chooses a unique optimal effort level regardless of the recommendation he receives.[31] Such effort level only depends on his true type and the type he reported to the principal. Therefore, we can analyze the IC in two steps: first we characterize the optimal $\alpha(\hat{\theta},\theta)$ if the agent with type θ reports $\hat{\theta}$ (so the recommendation must coincide with $\alpha(\hat{\theta},\hat{\theta})$ if the report is $\hat{\theta}$ because of the first requirement above), then we look at the second requirement.

Step One: the Optimal Response $\alpha(\hat{\theta},\theta)$

If the agent with type θ reports $\hat{\theta}$ and exerts effort α, his expected payoff is

$$\hat{\pi}(\alpha,\hat{\theta},\theta) = x(\hat{\theta}) - C(\alpha,\theta) + \int_{\underline{c}}^{\overline{c}} \tilde{\pi}(\hat{\theta},c,c)h(c,\alpha)dc.$$

The first term on the right hand side of the above equation is the payment, the second term is the agent's investment cost, and the last term is his expected profit from the second stage. There is no loss of generality to assume that $\tilde{\pi}(\hat{\theta},\overline{c},\overline{c}) = 0$ for both $\hat{\theta} \in \{\theta_l,\theta_h\}$. The reason is that we can always define

$$\bar{x}(\hat{\theta}) = x(\hat{\theta}) + \tilde{\pi}(\hat{\theta},\overline{c},\overline{c}),$$

and such change does not affect the first stage expected payoff $\hat{\pi}(\alpha,\hat{\theta},\theta)$ so that the agent's reporting and R&D incentive remain the same. Based on this observation and the fact that (14) holds for

[31] First, the belief of the agent is not affected by the recommendation. Second, as we shall show later, the agent's payoff is concave in effort so that there is no loss of generality to consider deterministic effort decisions.

any reported type $\hat{\theta}$, we can rewrite the expected payoff as

$$\begin{aligned}\hat{\pi}(\alpha,\hat{\theta},\theta) &= x(\hat{\theta}) - C(\alpha,\theta) + \int_{\underline{c}}^{\overline{c}} (\int_{c}^{\overline{c}} p(\hat{\theta},s)ds)h(c,\alpha)dc \\ &= x(\hat{\theta}) - C(\alpha,\theta) + \int_{\underline{c}}^{\overline{c}} p(\hat{\theta},c)(\int_{\underline{c}}^{c} h(s,\alpha)ds)dc \\ &= x(\hat{\theta}) - C(\alpha,\theta) + \int_{\underline{c}}^{\overline{c}} p(\hat{\theta},c)H(c,\alpha)dc.\end{aligned}$$

Taking derivative with respect to α yields

$$\frac{\partial\hat{\pi}(\alpha,\hat{\theta},\theta)}{\partial\alpha} = -C_\alpha(\alpha,\theta) + \int_{\underline{c}}^{\overline{c}} p(\hat{\theta},c)H_\alpha(c,\alpha)dc. \tag{15}$$

Second order derivative

$$\frac{\partial^2\hat{\pi}(\alpha,\hat{\theta},\theta)}{\partial\alpha^2} = -C_{\alpha\alpha}(\alpha,\theta) + \int_{\underline{c}}^{\overline{c}} p(\hat{\theta},c)H_{\alpha\alpha}(c,\alpha)dc < 0,$$

when $p(\hat{\theta},c) > 0$ on a positive measure set. Since the agent's expected payoff $\hat{\pi}(\alpha,\hat{\theta},\theta)$ is strictly concave in α, the optimal α is unique.[32] Since $\lim_{\alpha\to+\infty} H_\alpha(c,\alpha) = 0$ and $C_\alpha(\alpha,\theta) \geq C_\alpha(0,\theta) > 0$, we have that $\frac{\partial\hat{\pi}(\alpha,\hat{\theta},\theta)}{\partial\alpha} < 0$ when α is sufficiently large. If $\frac{\partial\hat{\pi}(\alpha,\hat{\theta},\theta)}{\partial\alpha} = 0$ has a (unique) solution $\alpha(\hat{\theta},\theta) \geq 0$, then it must be the global optimum. If not, then it must be the case that $\frac{\partial\hat{\pi}(\alpha,\hat{\theta},\theta)}{\partial\alpha} < 0$ for all $\alpha > 0$, which means the agent will choose $\alpha = 0$. In this case, we define $\alpha(\hat{\theta},\theta) = 0$. Thus, the agent with type θ who reports $\hat{\theta}$ will choose the optimal effort level $\alpha(\hat{\theta},\theta)$ as defined above.

Step Two: Truthful Report

Define

$$\pi(\hat{\theta},\theta) = \hat{\pi}(\alpha(\hat{\theta},\theta),\hat{\theta},\theta) = x(\hat{\theta}) - C(\alpha(\hat{\theta},\theta),\theta) + \int_{\underline{c}}^{\overline{c}} p(\hat{\theta},c)H(c,\alpha(\hat{\theta},\theta))dc. \tag{16}$$

This is the agent's expected utility when his true type is θ but he reports $\hat{\theta}$ given that he will respond optimally when receiving the recommendation. Now the first stage IC can be written as

$$\pi(\theta_l,\theta_l) \geq \pi(\theta_h,\theta_l), \text{ and } \pi(\theta_h,\theta_h) \geq \pi(\theta_l,\theta_h), \tag{17}$$

[32] $\frac{\partial^2\hat{\pi}(\alpha,\hat{\theta},\theta)}{\partial\alpha^2} = 0$ only when $p(\hat{\theta},c) = 0$ for almost all c except on a zero measure set. However, in this case, from the first order condition we know that the agent will optimally choose $\alpha = 0$. Therefore, in all cases the optimal α is unique.

where $\alpha(\hat{\theta}, \theta) \geq 0$ satisfies

$$-C_\alpha(\alpha(\hat{\theta}, \theta), \theta) + \int_{\underline{c}}^{\overline{c}} p(\hat{\theta}, c) H_\alpha(c, \alpha(\hat{\theta}, \theta)) dc \leq 0, \text{ with equality if } \alpha(\hat{\theta}, \theta) > 0.$$

Recall that the agent's utility function $\hat{\pi}(\alpha, \hat{\theta}, \theta)$ is strictly concave in α (> 0) (refer to footnote 32), thus the "first-order approach" is valid. We can replace the original incentive compatibility constraint for moral hazard with the above first order condition.

For notational simplicity, let $\alpha_{hl} = \alpha(\theta_h, \theta_l)$ be the optimal effort level that the agent with type θ_l and reports θ_h will choose. α_{ll}, α_{hh}, and α_{lh} are defined analogically. Now we have

$$\begin{aligned}
\pi(\theta_h, \theta_l) &= x(\theta_h) - C(\alpha_{hl}, \theta_l) + \int_{\underline{c}}^{\overline{c}} p(\theta_h, c) H(c, \alpha_{hl}) dc \\
&= [x(\theta_h) - C(\alpha_{hh}, \theta_h) + \int_{\underline{c}}^{\overline{c}} p(\theta_h, c) H(c, \alpha_{hh}) dc] + C(\alpha_{hh}, \theta_h) - \int_{\underline{c}}^{\overline{c}} p(\theta_h, c) H(c, \alpha_{hh}) dc \\
&- C(\alpha_{hl}, \theta_l) + \int_{\underline{c}}^{\overline{c}} p(\theta_h, c) H(c, \alpha_{hl}) dc \\
&= \pi(\theta_h, \theta_h) + C(\alpha_{hh}, \theta_h) - C(\alpha_{hl}, \theta_l) + \int_{\underline{c}}^{\overline{c}} p(\theta_h, c)[H(c, \alpha_{hl}) - H(c, \alpha_{hh})] dc.
\end{aligned}$$

Thus $\pi(\theta_l, \theta_l) \geq \pi(\theta_h, \theta_l)$ is equivalent to

$$\pi(\theta_h, \theta_h) \leq \pi(\theta_l, \theta_l) - C(\alpha_{hh}, \theta_h) + C(\alpha_{hl}, \theta_l) + \int_{\underline{c}}^{\overline{c}} p(\theta_h, c)(H(c, \alpha_{hh}) - H(c, \alpha_{hl})) dc. \tag{18}$$

Similarly,

$$\begin{aligned}
\pi(\theta_l, \theta_h) &= x(\theta_l) - C(\alpha_{lh}, \theta_h) + \int_{\underline{c}}^{\overline{c}} p(\theta_l, c) H(c, \alpha_{lh}) dc \\
&= \pi(\theta_l, \theta_l) - C(\alpha_{lh}, \theta_h) + C(\alpha_{ll}, \theta_l) + \int_{\underline{c}}^{\overline{c}} p(\theta_l, c)[H(c, \alpha_{lh}) - H(c, \alpha_{ll})] dc.
\end{aligned}$$

And $\pi(\theta_h, \theta_h) \geq \pi(\theta_l, \theta_h)$ is equivalent to

$$\pi(\theta_h, \theta_h) \geq \pi(\theta_l, \theta_l) - C(\alpha_{lh}, \theta_h) + C(\alpha_{ll}, \theta_l) + \int_{\underline{c}}^{\overline{c}} p(\theta_l, c)(H(c, \alpha_{lh}) - H(c, \alpha_{ll})) dc. \tag{19}$$

Therefore, the IC constraint (17) is equivalent to (18) and (19), where

$$-C_\alpha(\alpha_{ij},\theta_j)+\int_{\underline{c}}^{\overline{c}} p(\theta_i,c)H_\alpha(c,\alpha_{ij})dc \le 0, \text{ with equality if } \alpha_{ij}>0, \forall i,j\in\{l,h\}. \tag{20}$$

Note that $C_{\alpha\theta}>0$ implies the following result.

Lemma 1 *We have $\alpha_{ll}\ge\alpha_{lh}$ and $\alpha_{hl}\ge\alpha_{hh}$, with equality if and only if $\alpha_{ll}=0$ and $\alpha_{hl}=0$, respectively.*

5.3 The Principal's Objective

The expected social cost can be written as

$$SC = q\{C(\alpha_{ll},\theta_l)+\int_{\underline{c}}^{\overline{c}}[p(\theta_l,c)c+(1-p(\theta_l,c))c_0]h(c,\alpha_{ll})dc\}$$

$$+(1-q)\{C(\alpha_{hh},\theta_h)+\int_{\underline{c}}^{\overline{c}}[p(\theta_h,c)c+(1-p(\theta_h,c))c_0]h(c,\alpha_{hh})dc\}$$

$$=q[C(\alpha_{ll},\theta_l)+\int_{\underline{c}}^{\overline{c}} p(\theta_l,c)(c-c_0)h(c,\alpha_{ll})dc]$$

$$+(1-q)[C(\alpha_{hh},\theta_h)+\int_{\underline{c}}^{\overline{c}} p(\theta_h,c)(c-c_0)h(c,\alpha_{hh})dc]+c_0.$$

The principal's total cost is the sum of the expected social cost and the agent's expected profit:

$$TC = q\pi(\theta_l,\theta_l)+(1-q)\pi(\theta_h,\theta_h)+SC$$

The principal's problem is

$$\min_{\{\alpha_{ij}\ge 0, x(\theta), p(\theta,c), y(\theta,c)\}} TC$$

subject to

$$\pi(\theta_h,\theta_h)\ge\pi(\theta_l,\theta_l)-C(\alpha_{lh},\theta_h)+C(\alpha_{ll},\theta_l)+\int_{\underline{c}}^{\overline{c}} p(\theta_l,c)(H(c,\alpha_{lh})-H(c,\alpha_{ll}))dc; \tag{21}$$

$$\pi(\theta_h,\theta_h) \le \pi(\theta_l,\theta_l) - C(\alpha_{hh},\theta_h) + C(\alpha_{hl},\theta_l) + \int_{\underline{c}}^{\overline{c}} p(\theta_h,c)(H(c,\alpha_{hh}) - H(c,\alpha_{hl}))dc; \tag{22}$$

$$\text{constraints (20);} \tag{23}$$

$$\pi(\theta,\theta) \ge 0,\ \forall\theta; \tag{24}$$

$$\tilde{\pi}(\theta,c,c) = \int_{c}^{\overline{c}} p(\theta,s)ds, \forall c, \forall\theta; \tag{25}$$

$$p(\theta,c) \text{ is decreasing in } c,\ \forall\theta; \tag{26}$$

$$0 \le p(\theta,c) \le 1, \forall c, \forall\theta. \tag{27}$$

(21) and (22) are the first stage IC constraints; (23) is the moral hazard constraints; (24) is the first stage IR constraint; (25) and (26) together are the equivalent conditions for second stage IC; (27) is the constraint imposed on the acquisition probability. We call this original problem as Problem (O). Note that (25) has been used to simplify the expressions of $\pi(\theta_i,\theta_j), \forall i,j \in \{h,l\}$ in Problem (O), therefore if it is dropped the optimal solution remains the same as that in Problem (O). Now we drop constraints (21) and (25) to form a relaxed problem Problem (O-R) of Problem (O). If we can show that the optimal solution in Problem (O-R) satisfies constraint (21), then such solution must be the optimum of Problem (O).

5.4 Problem (O-R)

As is standard, at the optimum of Problem (O-R), (22) is binding and $\pi(\theta_h,\theta_h) = 0$. This is the following lemma.

Lemma 2 *In Problem (O-R), at the optimum, we must have*

$$\pi(\theta_h,\theta_h) = 0 \tag{28}$$

and (22) is binding, i.e.,

$$\pi(\theta_l,\theta_l) = C(\alpha_{hh},\theta_h) - C(\alpha_{hl},\theta_l) + \int_{\underline{c}}^{\overline{c}} p(\theta_h,c)(H(c,\alpha_{hl}) - H(c,\alpha_{hh}))dc \ge 0. \tag{29}$$

Proof: See Appendix. □

(29) is the rent that the principal must concede to the efficient type. Note that when $\alpha_{hl} = \alpha_{hh}$, the rent reduces to $C(\alpha_{hh},\theta_h) - C(\alpha_{hh},\theta_l)$, which takes the same form with the information rent established in Section 4.3 for the case where the R&D effort is observable (i.e., pure adverse selection). However, recall that $\alpha_{hl} \ge \alpha_{hh}$ with strict inequality whenever $\alpha_{hl} > 0$ (Lemma 1). In addition, no-

tice that by (20), α_{hl} is the (unique) maximizer of the function $-C(\alpha,\theta_l)+\int_{\underline{c}}^{\overline{c}} p(\theta_h,c)H(c,\alpha)dc$ with respect to $\alpha \geq 0$.[33] Thus, the rent of the efficient type is strictly larger than $C(\alpha_{hh},\theta_h)-C(\alpha_{hh},\theta_l)$ whenever $\alpha_{hl} > 0$. This means that whenever the efficient type exerts positive effort when he pretends to be the inefficient type, he also enjoys a moral hazard rent in addition to the information rent due to private R&D efficiency. Therefore, the principal has to concede to the agent more rent because of the moral hazard if she still wishes to induce the same level of effort (as in the pure adverse selection case) from the inefficient type. However, this does not imply that at the optimum, the efficient type definitely enjoys more rent than that in the pure adverse selection setting, since the rent for both cases depends on the effort levels of the inefficient type, which are potentially different.

5.5 Problem (O-R-E)

Substituting (28) and (29) into the objective function in Problem (O-R) gives

$$TC_E = q[C(\alpha_{hh},\theta_h) - C(\alpha_{hl},\theta_l) + \int_{\underline{c}}^{\overline{c}} p(\theta_h,c)(H(c,\alpha_{hl}) - H(c,\alpha_{hh}))dc + C(\alpha_{ll},\theta_l)$$

$$+ \int_{\underline{c}}^{\overline{c}} p(\theta_l,c)(c-c_0)h(c,\alpha_{ll})dc] + (1-q)[C(\alpha_{hh},\theta_h) + \int_{\underline{c}}^{\overline{c}} p(\theta_h,c)(c-c_0)h(c,\alpha_{hh})dc] + c_0.$$

We thus obtain the following Problem (O-R-E) which is equivalent to Problem (O-R).

$$\min_{\{\alpha_{ll},\alpha_{hl},\alpha_{hh},\alpha_{lh}\geq 0,p(\theta,c)\}} TC_E$$

subject to

$$\text{constraints (20);} \tag{30}$$

$$p(\theta,c) \text{ is decreasing in } c, \forall\theta; \tag{31}$$

$$0 \leq p(\theta,c) \leq 1, \forall c, \forall\theta. \tag{32}$$

Note that we have dropped the variables $x(\theta_l)$, $x(\theta_h)$ and $y(\theta,c)$ in the choice variable set. We also drop constraints (28) and (29), which have been substituted into the objective function of Problem (O-R). Payments $x(\theta_l)$, $x(\theta_h)$ and $y(\theta,c)$ appear in neither the objective function nor the constraints. Payments $x(\theta_l)$ and $x(\theta_h)$ are eventually determined by (28), (29) and the second stage allocation rule $p(\theta,c)$, and $y(\theta,c)$ is eventually determined by the second stage allocation rule.

Now we introduce the notion of cutoff mechanism.

[33] Note that this is a concave function of α.

Definition 1 ***(Cutoff Mechanism)****: A mechanism is called cutoff mechanism if the second stage allocation rule satisfy the following conditions: there exist c_l, $c_h \in [\underline{c}, \overline{c}]$ such that*

$$p(\theta_l, c) = \begin{cases} 1, \text{ if } c \leq c_l, \\ 0, \text{ if } c > c_l, \end{cases} \text{ and } p(\theta_h, c) = \begin{cases} 1, \text{ if } c \leq c_h, \\ 0, \text{ if } c > c_h. \end{cases}$$

For Problem (O-R-E), the following lemma shows that, at the optimum there is no distortion for the efficient type.

Lemma 3 *In Problem (O-R-E), at the optimum,*

$$p(\theta_l, c) = \begin{cases} 1, & \textit{if } c \leq c_0; \\ 0, & \textit{if } c > c_0. \end{cases}$$

Moreover, there is no effort distortion for θ_l type, i.e., the induced effort $\alpha_{ll} = \alpha_l^{FB}$.

Proof: See Appendix. □

Obviously, all cutoff mechanisms satisfy (31) and (32). Lemma 3 shows that for the efficient type, the optimal second stage mechanism is a deterministic one with the optimal cutoff $c_l^* = c_0$. In the following analysis, we restrict our attention to cutoff mechanisms to study the optimal mechanism.[34]

5.6 Problem (O-R-E-E)

Since we focus on cutoff mechanisms, Problem (O-R-E-E) below is equivalent to Problem (O-R-E):[35]

$$\min_{\{\alpha_{hh}, \alpha_{hl} \geq 0, \underline{c} \leq c_h \leq \overline{c}\}} TC_{E'}$$

subject to

$$-C_\alpha(\alpha_{ij}, \theta_j) + \int_{\underline{c}}^{c_h} H_\alpha(c, \alpha_{ij}) dc \leq 0, \text{ with equality if } \alpha_{ij} > 0, \text{ for } i = h, \text{ and } j \in \{l, h\}, \tag{33}$$

[34] It is an open question whether this restriction introduces loss of generality. It is not rare in the literature to focus on deterministic mechanisms, such as Krähmer and Strausz [15]. As pointed out by Laffont and Martimort [16], the commitment issue is more involved with stochastic mechanisms.

[35] Note that Lemma 3 shows that the optimal $c_l^* = c_0$, which further pins down $\alpha_{ll}, \alpha_{lh} \geq 0$ through (20). α_{hl} and α_{hh} are determined by c_h through (33).

where

$$TC_{E'} = \underbrace{(1-q)[c_0 + C(\alpha_{hh}, \theta_h) + \int_{\underline{c}}^{c_h} (c - c_0) h(c, \alpha_{hh}) dc]}_{\text{expected social cost from the inefficient type}}$$

$$+ \underbrace{q[C(\alpha_{hh}, \theta_h) - C(\alpha_{hl}, \theta_l) + \int_{\underline{c}}^{c_h} (H(c, \alpha_{hl}) - H(c, \alpha_{hh})) dc]}_{\text{expected rent to the efficient type}} + \underbrace{q[c_0 + C(\alpha_{ll}, \theta_l) + \int_{\underline{c}}^{c_0} (c - c_0) h(c, \alpha_{ll}) dc]}_{\text{expected social cost from the efficient type}}. \tag{34}$$

Intuitively, when c_h is higher, the efficient type is more tempted to mimic the inefficient type. As a result, the rent to the efficient type would increase. The following lemma confirms this intuition. Note that by Lemma 2, $\pi(\theta_l, \theta_l) = C(\alpha_{hh}, \theta_h) - C(\alpha_{hl}, \theta_l) + \int_{\underline{c}}^{c_h} (H(c, \alpha_{hl}) - H(c, \alpha_{hh})) dc$ for cutoff mechanisms.

Lemma 4 *With cutoff mechanisms, the rent $\pi(\theta_l, \theta_l)$ to the efficient type, i.e., $[C(\alpha_{hh}, \theta_h) - C(\alpha_{hl}, \theta_l) + \int_{\underline{c}}^{c_h} (H(c, \alpha_{hl}) - H(c, \alpha_{hh})) dc]$, is increasing in c_h. Moreover, it is strictly increasing whenever the induced $\alpha_{hl} > 0$.*

Proof: See Appendix. □

Regarding the optimal c_h^*, we have the following result.

Lemma 5 *In Problem (O-R-E-E), the optimal cutoff $c_h^* < c_0$.*

Proof: See Appendix. □

The intuition of Lemma 5 is as follows. If the moral hazard does not cause additional agency cost to the principal, then it implies that 1) the second stage mechanism must be ex post efficient; 2) the efficient type does not enjoy moral hazard rent. However, when the second stage is efficient, i.e., $c_h = c_0$, the efficient type exerts the same level of effort as that in the first-best situation when he pretends to be the inefficient type. That is, $\alpha_{hl} = \alpha_l^{FB}$. In addition, the second requirement is fulfilled only when $\alpha_{hl} = \alpha_{hh}$, which holds only when $\alpha_{hl} = 0$. Thus, unless it is socially inefficient to induce the efficient type to exert strictly positive effort, the principal always needs to concede moral hazard rent to the efficient type agent. As a result, moral hazard always causes additional agency cost to the principal.

As shown in Lemma 3, there is no distortion for the efficient type. The result is quite intuitive. Fix the other choice variables in Problem (O-R-E), then minimizing the objective function by jointly choosing α_{ll} and $p(\theta_l, c)$ is equivalent to maximizing total surplus. Clearly, this leads to a cutoff of $c_l^* = c_0$, which corresponds to an ex post efficient second stage mechanism for the efficient type. Given the second stage mechanism is efficient, it is incentive compatible for the efficient type to choose the efficient R&D effort since the second stage VCG mechanism perfectly aligns the incentive of the agent with the social surplus.

For the inefficient type, the principal faces the efficiency-rent extraction trade-off. By Lemma 4, the second under-bracketed expression of (34)–the rent to the efficient type is increasing in the cutoff c_h in $[\underline{c}, \overline{c}]$. Note that α_{hh} and α_{hl} are determined by c_h. It is clear that the expected social cost from the inefficient type–the first under-bracketed expression in (34) is decreasing in c_h in $[\underline{c}, c_0]$ and increasing in $[c_0, \overline{c}]$. Note that minimizing this term is equivalent to maximizing the total surplus from the inefficient type. Setting an ex post efficient cutoff c_0 would induce an efficient R&D effort, thus the concerned term is minimized. When c_h increases from $\underline{c}$ to c_0, the R&D effort increases to the efficient level and the second stage allocation gets more efficient, which means the total surplus keeps improving. When c_h increases from c_0 to $\overline{c}$, the R&D effort continues diverging from the efficient level and the second stage allocation gets less efficient, which means the total surplus is deteriorating. Therefore, to minimize the total expected cost $TC_{E'}$, c_h cannot be strictly larger than c_0.

The optimal cutoff c_h^* is obtained by equating the gain in social surplus from the inefficient type (the absolute value of the first order derivative of the first under-bracketed expression) to the increase in rent to the efficient type (the first order derivative of the second under-bracketed expression)[36,37]:

$$-(1-q)(c_h-c_0)[h(c_h, \alpha_{hh}(c_h))+\alpha'_{hh}(c_h)H_\alpha(c_h, \alpha_{hh}(c_h))] = q[H(c_h, \alpha_{hl}(c_h))-H(c_h, \alpha_{hh}(c_h))]. \quad (35)$$

[36] In this footnote, we derive the first order derivative of the first under-bracketed expression in (34). Take derivative with respect to c_h and bear in mind that α_{hh} is determined by c_h through (33) (denote it as $\alpha_{hh}(c_h)$). There exists a cutoff $\check{c} \in [\underline{c}, \overline{c}]$, such that $\alpha_{hh}(c_h) = 0$ when $c_h \leq \check{c}$ and $\alpha_{hh}(c_h) > 0$ when $c_h > \check{c}$. (It is possible that such $\check{c}$ does not exist, which means that $\alpha_{hh}(c_h) = 0$ for all $c_h \in [\underline{c}, \overline{c}]$.) Thus, $\alpha'_{hh}(c_h) = 0$ when $c_h \leq \check{c}$; and (33) is binding when $c_h > \check{c}$ (setting $j = h$). That is, $\alpha'_{hh}(c_h)(-C_\alpha(\alpha_{hh}, \theta_h) + \int_{\underline{c}}^{c_h} H_\alpha(c, \alpha_{hh})dc) = 0$ for all $c_h \in [\underline{c}, \overline{c}]$. It is easy to see that $\alpha'_{hh}(c_h) > 0$ when $c_h > \check{c}$. Now take derivative with respect to c_h for the concerned first expression in (34) (neglecting the factor $(1-q)$), we obtain

$$\begin{aligned}
&\alpha'_{hh}(c_h)C_\alpha(\alpha_{hh}, \theta_h) + (c_h - c_0)h(c_h, \alpha_{hh}(c_h)) + \alpha'_{hh}(c_h)\int_{\underline{c}}^{c_h}(c - c_0)dH_\alpha(c, \alpha_{hh}(c_h)) \\
&= \alpha'_{hh}(c_h)(C_\alpha(\alpha_{hh}, \theta_h) - \int_{\underline{c}}^{c_h} H_\alpha(c, \alpha_{hh}(c_h))dc) + (c_h - c_0)(h(c_h, \alpha_{hh}(c_h)) + \alpha'_{hh}(c_h)H_\alpha(c_h, \alpha_{hh}(c_h))) \\
&= (c_h - c_0)(h(c_h, \alpha_{hh}(c_h)) + \alpha'_{hh}(c_h)H_\alpha(c_h, \alpha_{hh}(c_h))).
\end{aligned}$$

[37] By the proof of Lemma 4, the first order derivative of the second under-bracketed expression in (34) is $q[H(c_h, \alpha_{hl}(c_h)) - H(c_h, \alpha_{hh}(c_h))]$.

Note that when $c_h = c_0$, the marginal surplus gain from the procurement is zero while the marginal rent is strictly positive.[38] As a result, it is never optimal to set the cutoff greater than or equal to c_0. When c_h is set below c_0, such discrimination against the inefficient type will make it less profitable for the efficient type to mimic the inefficient type. Therefore, the rent to the efficient type is reduced. The lower the cutoff is, the less the rent will be. On the other hand, lower cutoff incurs more social cost. The principal has to find the optimal way to balance between these two conflicting effects, which is determined by (35).

5.7 Going back to Problem (O)

Let the optimal solution to Problem (O-R-E-E) be $\{\alpha_{ll}^*, \alpha_{hl}^*, \alpha_{hh}^*, \alpha_{lh}^*, c_l^*, c_h^*\}$ in which $c_l^* = c_0$ and $c_h^* < c_0$. Note that Problem (O-R-E-E) is equivalent to Problem (O-R-E) with deterministic second stage allocation rules for the inefficient type. Also note that Problem (O-R-E) is equivalent to Problem (O-R), which is a relaxed problem of the original problem Problem (O) in the sense that (21) is dropped. Thus, if we can show that the optimal solution of Problem (O-R-E-E) satisfies (21), then such solution must be the optimal solution to Problem (O), if we restrict to deterministic second stage allocation rules for the inefficient type. In fact, the following lemma shows that (21) is indeed satisfied.[39]

Lemma 6 *The optimal solution of Problem (O-R-E-E) is feasible in Problem (O). Therefore, it is the optimal solution to Problem (O) within the class of mechanism with deterministic second stage allocation rules for the inefficient type.*

Proof: See Appendix. □

Combining with Lemmas 5 and 6, we have the following characterization of the second stage allocation rule.[40]

Corollary 1 *Given $\alpha_l^{FB} > 0$ (i.e., it is socially efficient to induce the efficient type to exert a strictly positive R&D effort), we have $c_h^* < c_0$, i.e., the optimal acquisition cutoff c_h^* for the inefficient type is smaller than that (i.e., c_0) of the efficient type.*

This means that moral hazard renders the second stage inefficient for the inefficient type and that there is an additional agency cost comparing to the observable effort scenario.

[38] When $c_h = c_0$, the induced $\alpha_{hl} = \alpha_l^{FB} > 0$. Also note that $\alpha_{hl} > \alpha_{hh}$ whenever $\alpha_{hl} > 0$ (Lemma 1).

[39] In the proof of Lemma 6 we show that $c_h \leq c_l$ is sufficient for first stage IC for cutoff mechanisms. In fact, this weak monotonicity in cutoff is also necessary for first stage IC. This result can be extended to the continuous first stage type setting. More details will be provided in the conclusion section.

[40] Please refer to footnote 38.

5.8 Optimal Mechanism and Implementation

We first describe the second stage optimal allocation rule and payments. By Lemmas 3, 5 and 6, the second stage optimal allocation rule is described as in the following proposition. Moreover, by (13) and (25), we also have the following characterizations of the second stage optimal payment rule.

Proposition 3 *At the optimum, (i) the second stage allocation rule is:*
$p^*(\theta_l, c) = \begin{cases} 1, \text{ if } c \leq c_0, \\ 0, \text{ if } c > c_0, \end{cases}$ *and* $p^*(\theta_h, c) = \begin{cases} 1, \text{ if } c \leq c_h^*, \\ 0, \text{ if } c > c_h^*, \end{cases}$ *where* c_h^* *is the optimal solution of Problem (O-R-E-E); and (ii) the second stage payment rule is:*
$y^*(\theta_l, c) = \begin{cases} c_0, \text{ if } c \leq c_0, \\ 0, \text{ if } c > c_0, \end{cases}$ *and* $y^*(\theta_h, c) = \begin{cases} c_h^*, \text{ if } c \leq c_h^*, \\ 0, \text{ if } c > c_h^*. \end{cases}$

Note that the agent's second stage *ex post* payoff must be nonnegative since the agent bears the production cost only when the principal buys the product from him. As a result, we confirm that the second stage IR is satisfied.

We next pin down the first stage payment rule. Since $\pi(\theta_h, \theta_h) = 0$ and $\pi(\theta_h, \theta_h) = x^*(\theta_h) - C(\alpha_{hh}^*, \theta_h) + \int_{\underline{c}}^{c_h^*} H(c, \alpha_{hh}^*)dc$ by (16), we have

$$x^*(\theta_h) = C(\alpha_{hh}^*, \theta_h) - \int_{\underline{c}}^{c_h^*} H(c, \alpha_{hh}^*)dc \leq - \int_{\underline{c}}^{c_h^*} H(c, 0) \leq 0,$$

with the first inequality being strict if $\alpha_{hh}^* > 0$. This is because α_{hh}^* is the (unique) minimizer of

$$C(\alpha, \theta_h) - \int_{\underline{c}}^{c_h^*} H(c, \alpha)dc, \alpha \geq 0,$$

so that setting $\alpha = 0$ always leads to a weakly higher function value than that when $\alpha = \alpha_{hh}^*$. $x^*(\theta_h) \leq 0$ means that the type θ_h agent pays to the principal.

By Lemma 2

$$\pi(\theta_l, \theta_l) = C(\alpha_{hh}^*, \theta_h) - C(\alpha_{hl}^*, \theta_l) + \int_{\underline{c}}^{c_h^*} (H(c, \alpha_{hl}^*) - H(c, \alpha_{hh}^*))dc,$$

and by (16)

$$\pi(\theta_l, \theta_l) = x^*(\theta_l) - C(\alpha_{ll}^*, \theta_l) + \int_{\underline{c}}^{c_0} H(c, \alpha_{ll}^*)dc.$$

We thus have

$$x^*(\theta_l) = C(\alpha^*_{hh}, \theta_h) - C(\alpha^*_{hl}, \theta_l) + \int_{\underline{c}}^{c^*_h} (H(c, \alpha^*_{hl}) - H(c, \alpha^*_{hh}))dc + C(\alpha^*_{ll}, \theta_l) - \int_{\underline{c}}^{c_0} H(c, \alpha^*_{ll})dc$$

$$\begin{aligned}
&= x^*(\theta_h) + [-C(\alpha^*_{hl}, \theta_l) + \int_{\underline{c}}^{c^*_h} H(c, \alpha^*_{hl})dc] - [-C(\alpha^*_{ll}, \theta_l) + \int_{\underline{c}}^{c_0} H(c, \alpha^*_{ll})dc] \\
&\leq x^*(\theta_h) + [-C(\alpha^*_{hl}, \theta_l) + \int_{\underline{c}}^{c_0} H(c, \alpha^*_{hl})dc] - [(-C(\alpha^*_{ll}, \theta_l) + \int_{\underline{c}}^{c_0} H(c, \alpha^*_{ll})dc)] \\
&< x^*(\theta_h),
\end{aligned}$$

where the first inequality follows from $c^*_h < c_0$ and the second inequality follows from α^*_{ll} $(= \alpha_l^{FB} > 0)$ is the unique maximizer of

$$-C(\alpha, \theta_l) + \int_{\underline{c}}^{c_0} H(c, \alpha)dc, \alpha \geq 0.$$

Thus $x^*(\theta_l) < x^*(\theta_h) \leq -\int_{\underline{c}}^{c^*_h} H(c, 0) \leq 0$. The above results are formally stated in the following proposition.

Proposition 4 *At the optimum, both types pay to the principal in the first stage and the efficient type pays more. Specifically,*

$$\begin{aligned}
x^*(\theta_h) &= C(\alpha^*_{hh}, \theta_h) - \int_{\underline{c}}^{c^*_h} H(c, \alpha^*_{hh})dc, \\
x^*(\theta_l) &= C(\alpha^*_{hh}, \theta_h) - C(\alpha^*_{hl}, \theta_l) + \int_{\underline{c}}^{c^*_h} (H(c, \alpha^*_{hl}) - H(c, \alpha^*_{hh}))dc + C(\alpha^*_{ll}, \theta_l) - \int_{\underline{c}}^{c_0} H(c, \alpha^*_{ll})dc.
\end{aligned}$$

Implementation

The optimal two-stage mechanism can be implemented by a menu of two put contracts. Let $x^*_l = -x^*(\theta_l)$ and $x^*_h = -x^*(\theta_h)$. In the first stage, the agent is offered a menu of two put contracts with strike prices $\{c^*_l, c^*_h\}$. The premium for the put with strike price c^*_i is x^*_i, $i \in \{l, h\}$. The agent can choose to pay x^*_i to the principal for buying the put with strike price c^*_i. In the second stage, if he selects the put with strike price c^*_i, he is entitled to charge the price c^*_i if he decides to supply the good. If the agent wishes to have a higher strike price in the second stage, he has to pay a higher premium upfront in the first stage.

5.9 Discussions

Now we go on to discuss the effects of introducing moral hazard (i.e., unobservable R&D effort). Since the second stage acquisition cutoff for the efficient type is set at c_0, there is no efficiency loss in the second stage for the efficient type, i.e., the second stage mechanism is ex post efficient for the efficient type. However, the second stage mechanism discriminates against the inefficient type by setting an acquisition cutoff c_h^* strictly lower than c_0. This leads to a second stage efficiency loss for the inefficient type.

Since the second stage acquisition cutoff for the efficient type is set at the efficient level, there is no effort distortion for the efficient type relative to the first-best level. Since the inefficient type's effort α_{hh}^* is determined by (33) and $c_h^* < c_0$, we have $\alpha_{hh}^* \leq \alpha_h^{FB}$ by comparing (33) with (2). The inequality is strict whenever $\alpha_h^{FB} > 0$. This means that in general there is a downward effort distortion for the inefficient type.

Recall that in the pure adverse selection (observable R&D effort) setting, the R&D effort level for the efficient type also equals the first-best level. Hence, introducing moral hazard in addition to adverse selection does not affect the effort induced for the efficient type. On the other hand, introducing moral hazard requires a lower acquisition cutoff for the inefficient type in order to reduce the information rent to the efficient type. This leads to a downward distortion of this type's R&D effort relative to the first-best. Although the acquisition cutoff is set at the efficient level for the inefficient type when the R&D effort is observable, the R&D effort level of the inefficient type is also set at a level lower than the first-best in order to lower the information rent to the efficient type.

When R&D effort is not observable, the principal has to rely on the second stage acquisition prices to incentivize the agent to invest and screen different types at the same time. Although a lower acquisition cutoff for the inefficient type damages the inefficient type's investment incentive, it reduces the information rent to the efficient type. When there is only adverse selection, the principal can rather directly rely on specifying the R&D effort levels to screen different types without sacrificing the second stage allocation efficiency. With observable R&D effort, the second stage acquisition cutoff is fixed at the efficient level–the principal does not discriminate different types of agent. The payment scheme in the first stage is sufficient to induce the agent to invest at the specified levels because the investment levels are observable.

6 Concluding Remarks

This paper studies the optimal procurement design in a two-stage setting where the R&D efficiency of the supplier is his private information, and the supplier can make R&D effort to improve his chance of discovering a more cost-efficient way of providing the good. The first stage mechanism specifies the contingent effort level and transfer when agent's R&D effort is observable; otherwise, it can only specify the transfer rule. In the second stage (after the R&D effort is incurred by the agent in the first stage), the good provision and payment rules are carried out, which are contingent on reports of both two stages. To our best knowledge, this is the first time in the procurement design literature that the contractor's private R&D ability and endogenized R&D effort (observable or unobservable) are jointly integrated into an analytical framework of dynamic mechanism design.

We find that observable effort cuts off the impact of the agent's first stage private information on the second stage allocation rule. Regardless of the R&D efficiency, the second stage allocation rule is always ex post efficient. The principal solely relies on the first stage mechanism (effort provision and transfer rules) to elicit the agent's private information on R&D efficiency. To reduce information rent to the efficient type, the inefficient type's effort provision is distorted downward relative to the first-best level.

When the agent's R&D effort is unobservable, the principal must also rely on discriminatory second stage allocation rule to optimally elicit first stage private information and induce the desired R&D effort level. At the optimum, the second stage allocation rule discriminates against the inefficient type by setting a lower acquisition threshold in provision cost. The second stage allocation rule for the efficient type remains ex post efficient regardless of the observability of R&D effort. The optimal two-stage mechanism thus takes a form of a menu of two put contracts with different strike prices. Such favoritism incentivizes the efficient type to exert efficient effort and dampens the effort of the inefficient type. While the inefficient type's R&D effort is distorted downward, together with the discriminatory second stage allocation rule, it reduces the information rent to the efficient type, and thus strikes the optimal efficiency-information rent trade-off.

Our analysis illustrates the subtle impacts of the agent's private information of R&D efficiency on optimal procurement designs for both observable and unobservable R&D effort. Moreover, we find that the optimal mechanism crucially depends on the observability of R&D effort. These observations provide useful guidelines for the designer to appropriately take these factors into account when considering the optimal procurement design that targets on acquisition cost effectiveness.

The insights obtained in our paper extend to other environments. Suppose the principal can observe the agent's total R&D cost though his R&D effort is not directly observed. The principal's problem with observable total R&D cost is equivalent to that with observable R&D effort.[41] When total R&D cost is contractable, once the agent reports his type, the principal can impose an R&D cost that equals the total effort cost induced by the optimal mechanism with observable R&D effort. Clearly, the resulting mechanism duplicates the optimal mechanism with observable R&D effort.

[41] In our paper, the R&D cost is completely determined by the agent's type and effort level. If an additional noise kicks in, then we have a mixed problem (cf. Laffont and Tirole [17]).

While our analysis is carried out assuming that the first stage types are discrete, our main results are still valid when the first stage types are continuous. When R&D effort is observable, the principal's problem can be fully solved in a similar way, with the non-discrimination result (ex post efficiency) holding in the second stage. When R&D effort is not observable, we find that for deterministic second stage mechanisms, the first stage IC is equivalent to decreasing second stage acquisition thresholds in the first stage types together with a usual envelope condition.[42] This equivalence characterization implies that in general the discrimination against the inefficient type in the second stage is robust in the continuous type setting.

We have focused on an environment with single agent. While we expect that the main insights can be extended to a multi-agent setting, new issues of information disclosure and belief-updating would arise and create additional challenges in the analysis. We leave these interesting issues to future works.

[42] Krähmer and Strausz [15] also provide an equivalence result for deterministic mechanisms in a different setting.

7 Appendix

Proof of Lemma 2: We first show that

$$-C(\alpha_{hh},\theta_h)+C(\alpha_{hl},\theta_l)+\int_{\underline{c}}^{\overline{c}} p(\theta_h,c)(H(c,\alpha_{hh})-H(c,\alpha_{hl}))dc \leq 0, \tag{36}$$

that is,

$$-C(\alpha_{hh},\theta_h)+\int_{\underline{c}}^{\overline{c}} p(\theta_h,c)H(c,\alpha_{hh})dc \leq -C(\alpha_{hl},\theta_l)+\int_{\underline{c}}^{\overline{c}} p(\theta_h,c)H(c,\alpha_{hl})dc.$$

As a result, $\pi(\theta_h,\theta_h) \leq \pi(\theta_l,\theta_l)$.

In fact,

$$\begin{aligned}
& -C(\alpha_{hh},\theta_h)+\int_{\underline{c}}^{\overline{c}} p(\theta_h,c)H(c,\alpha_{hh})dc \\
&= -C(\alpha_{hh},\theta_l)+\int_{\underline{c}}^{\overline{c}} p(\theta_h,c)H(c,\alpha_{hh})dc + C(\alpha_{hh},\theta_l)-C(\alpha_{hh},\theta_h) \\
&\leq -C(\alpha_{hl},\theta_l)+\int_{\underline{c}}^{\overline{c}} p(\theta_h,c)H(c,\alpha_{hl})dc + C(\alpha_{hh},\theta_l)-C(\alpha_{hh},\theta_h) \\
&\leq -C(\alpha_{hl},\theta_l)+\int_{\underline{c}}^{\overline{c}} p(\theta_h,c)H(c,\alpha_{hl})dc.
\end{aligned}$$

Here, the first inequality follows from the fact that α_{hl} is the maximizer of the function

$$-C(\alpha,\theta_l)+\int_{\underline{c}}^{\overline{c}} p(\theta_h,c)H(c,\alpha)dc, \alpha \geq 0.$$

And the second inequality follows from $\theta_l < \theta_h$ and $C_\theta \geq 0$.

At the optimum, $\pi(\theta_h,\theta_h)=0$. Otherwise, the principal can reduce $\pi(\theta_l,\theta_l)$ and $\pi(\theta_h,\theta_h)$ by the same amount without violating (22) and (24) since we have shown that $\pi(\theta_h,\theta_h) \leq \pi(\theta_l,\theta_l)$. By doing so, the total cost decreases.

(22) must be binding as well at the optimum. Suppose this is not the case, then the designer can reduce $\pi(\theta_l,\theta_l)$ to decrease the total cost without violating (24) since we have shown that (36) holds.

Therefore, at the optimum, we have $\pi(\theta_h,\theta_h)=0$ and $\pi(\theta_l,\theta_l)=C(\alpha_{hh},\theta_h)-C(\alpha_{hl},\theta_l)+\int_{\underline{c}}^{\overline{c}} p(\theta_h,c)(H(c,\alpha_{hl})-H(c,\alpha_{hh}))dc$. □

Proof of Lemma 3: It suffices to show that for any fixed α_{hl}, α_{hh}, and $p(\theta_h, c)$ satisfying (20), setting $p(\theta_l, c)$ as the one in Lemma 3 is optimal. To show this, first ignore constraint (20) for $i = j = l$, which is the only relevant constraint about both $p(\theta_l, c)$ and α_{ll}.[43] For any α_{ll}, to minimize the objective function TC_E, it is easy to see that it is optimal to set $p(\theta_l, c) = 1$ when $c \leq c_0$, and $p(\theta_l, c) = 0$ when $c > c_0$. The next step is to choose the optimal α_{ll} to minimize TC_E. If the optimal α_{ll} satisfies (20) for $i = j = l$, then we actually have found the above optimal $p(\theta_l, c)$ and α_{ll}. Note that we have fixed all the other variables, i.e., α_{hl}, α_{hh}, and $p(\theta_h, c)$. The only relevant component in TC_E is

$$q[C(\alpha_{ll}, \theta_l) + \int_{\underline{c}}^{c_0} (c - c_0) h(c, \alpha_{ll}) dc]. \tag{37}$$

Drop the factor q since it is irrelevant for minimization. First order derivative with respect to α_{ll}:

$$\begin{aligned}
&C_\alpha(\alpha_{ll}, \theta_l) + \int_{\underline{c}}^{c_0} (c - c_0) H_{c\alpha}(c, \alpha_{ll}) dc \\
&= C_\alpha(\alpha_{ll}, \theta_l) + \int_{\underline{c}}^{c_0} (c - c_0) dH_\alpha(c, \alpha_{ll}) \\
&= C_\alpha(\alpha_{ll}, \theta_l) + (c - c_0) H_\alpha(c, \alpha_{ll})|_{c=\underline{c}}^{c_0} - \int_{\underline{c}}^{c_0} H_\alpha(c, \alpha_{ll}) dc \\
&= C_\alpha(\alpha_{ll}, \theta_l) - \int_{\underline{c}}^{c_0} H_\alpha(c, \alpha_{ll}) dc.
\end{aligned}$$

Second order derivative with respect to α_{ll}:

$$C_{\alpha\alpha}(\alpha_{ll}, \theta_l) - \int_{\underline{c}}^{c_0} H_{\alpha\alpha}(c, \alpha_{ll}) dc > 0.$$

Therefore, to minimize (37), the (unique) optimal α_{ll}^m satisfies

$$C_\alpha(\alpha_{ll}^m, \theta_l) - \int_{\underline{c}}^{c_0} H_\alpha(c, \alpha_{ll}^m) dc \geq 0 \text{ with equality when } \alpha_{ll}^m > 0,$$

which is precisely the moral hazard constraint (20) we omitted in the beginning. In addition, comparing with (2), it is obvious that the solution is exactly the same as α_l^{FB}. $\square$

[43] Though (20) is relevant to $p(\theta_l, c)$ for $i = l$, $j = h$, however, the variable α_{lh} does not appear in the objective function TC_E. In fact, once $p(\theta_l, c)$ is determined, α_{lh} is also determined by (20).

Proof of Lemma 4: Consider the function

$$g(t) = C(\beta^*(t), \theta_h) - C(\gamma^*(t), \theta_l) + \int_{\underline{c}}^{t} (H(c, \gamma^*(t)) - H(c, \beta^*(t)))dc, t \in [\underline{c}, \overline{c}],$$

where $\beta^*(t)$ is the (unique) maximizer of

$$\tau_1(\beta) = -C(\beta, \theta_h) + \int_{\underline{c}}^{t} H(c, \beta)dc, \beta \geq 0,$$

and $\gamma^*(t)$ is the (unique) maximizer of

$$\tau_2(\gamma) = -C(\gamma, \theta_l) + \int_{\underline{c}}^{t} H(c, \gamma)dc, \gamma \geq 0.$$

That is,

$$-C_\alpha(\beta^*(t), \theta_h) + \int_{\underline{c}}^{t} H_\alpha(c, \beta^*(t))dc \leq 0, \text{ with equality if } \beta^*(t) > 0, \tag{38}$$

and

$$-C_\alpha(\gamma^*(t), \theta_l) + \int_{\underline{c}}^{t} H_\alpha(c, \gamma^*(t))dc \leq 0, \text{ with equality if } \gamma^*(t) > 0. \tag{39}$$

We need to show that $g(t)$ is increasing in $[\underline{c}, \overline{c}]$ and strictly increasing whenever $\gamma^*(t) > 0$. Note that $g(t)$ is a continuous function. There are two cases to consider.

Case 1: $-C_\alpha(0, \theta_l) + \int_{\underline{c}}^{\overline{c}} H_\alpha(c, 0)dc > 0$ and $-C_\alpha(0, \theta_h) + \int_{\underline{c}}^{\overline{c}} H_\alpha(c, 0)dc \leq 0$.[44] In this case, $\beta^*(t) = 0$ for all $t \in [\underline{c}, \overline{c}]$. And there exists a $\tilde{t}_1 \in (\underline{c}, \overline{c})$ such that (39) holds with equality for all $t \in [\tilde{t}_1, \overline{c}]$ and $\gamma^*(t) = 0$ for all $t \in [\underline{c}, \tilde{t}_1]$. Then, when $t \leq \tilde{t}_1$, $g(t) = 0$; when $t > \tilde{t}_1$, $g(t) > 0$ since choosing $\gamma = 0$ is not optimal. Therefore, to show that $g(t)$ is increasing in $[\underline{c}, \overline{c}]$ and strictly increasing in $[\tilde{t}_1, \overline{c}]$, it suffices to show that it is strictly increasing in $(\tilde{t}_1, \overline{c}]$. In this interval,

$$g(t) = -C(\gamma^*(t), \theta_l) + \int_{\underline{c}}^{t} (H(c, \gamma^*(t)) - H(c, 0))dc,$$

and

$$-C_\alpha(\gamma^*(t), \theta_l) + \int_{\underline{c}}^{t} H_\alpha(c, \gamma^*(t))dc = 0.$$

Note that $g(t)$ is differentiable in this range (by implicit function theorem), and by envelope theorem

[44] $-C_\alpha(0, \theta_l) + \int_{\underline{c}}^{\overline{c}} H_\alpha(c, 0)dc \leq 0$ is impossible since we have assumed that $C_\alpha(0, \theta_l) < \int_{\underline{c}}^{c_0} H_\alpha(c, 0)dc$ to ensure that $\alpha_l^{FB} > 0$.

(or taking derivative with respect to t),

$$g'(t) = H(t, \gamma^*(t)) - H(t, 0) > 0.$$

Therefore, $g(t)$ is strictly increasing in $(\tilde{t}_1, \overline{c}]$.

Case 2: $-C_\alpha(0, \theta_h) + \int_{\underline{c}}^{\overline{c}} H_\alpha(c, 0)dc > 0$. In this case, there exists $\hat{t}_1$ and $\hat{t}_2$ with $\underline{c} < \hat{t}_1 < \hat{t}_2 < \overline{c}$, such that: $\beta^*(t) = 0$ when $t \leq \hat{t}_2$, $\beta^*(t) > 0$ when $t > \hat{t}_2$; $\gamma^*(t) = 0$ when $t \leq \hat{t}_1$, $\gamma^*(t) > 0$ when $t > \hat{t}_1$. Therefore, $g(t) = 0$ when $t \leq \hat{t}_1$. When $t \in (\hat{t}_1, \hat{t}_2]$,

$$g(t) = -C(\gamma^*(t), \theta_l) + \int_{\underline{c}}^{t} (H(c, \gamma^*(t)) - H(c, 0))dc > 0,$$

where

$$-C_\alpha(\gamma^*(t), \theta_l) + \int_{\underline{c}}^{t} H_\alpha(c, \gamma^*(t))dc = 0.$$

When $t \in (\hat{t}_2, \overline{c}]$,

$$g(t) = C(\beta^*(t), \theta_h) - C(\gamma^*(t), \theta_l) + \int_{\underline{c}}^{t} (H(c, \gamma^*(t)) - H(c, \beta^*(t)))dc > 0,$$

where

$$-C_\alpha(\beta^*(t), \theta_h) + \int_{\underline{c}}^{t} H_\alpha(c, \beta^*(t))dc = 0,$$

and

$$-C_\alpha(\gamma^*(t), \theta_l) + \int_{\underline{c}}^{t} H_\alpha(c, \gamma^*(t))dc = 0.$$

Therefore, it suffices to show that $g(t)$ is strictly increasing in $(\hat{t}_1, \overline{c}]$. To see this, note that $g(t)$ is continuous in this interval and it is differentiable in $(\hat{t}_1, \hat{t}_2) \cup (\hat{t}_2, \overline{c})$.[45] Taking derivative with respect to t: When $t \in (\hat{t}_1, \hat{t}_2)$,

$$g'(t) = H(t, \gamma^*(t)) - H(t, 0) > 0.$$

When $t \in (\hat{t}_2, \overline{c})$,

$$g'(t) = H(t, \gamma^*(t)) - H(t, \beta^*(t)) > 0$$

since $\gamma^*(t) > \beta^*(t)$ when $\gamma^*(t) > 0$.

In sum, we have shown that $g(t)$ is increasing in $[\underline{c}, \overline{c}]$ and strictly increasing whenever $\gamma^*(t) > 0$.

[45] $\beta^*(t) = 0$ when $t \in (\hat{t}_1, \hat{t}_2)$, which is trivially differentiable. When $t \in (\hat{t}_2, \overline{c})$, it is differentiable by implicit function theorem. $\gamma^*(t)$ is differentiable when $t \in (\hat{t}_1, \overline{c})$ by implicit function theorem.

$\square$

Proof of Lemma 5: Notice that since α_{ll}^* and c_l^* have been solved, the only relevant components in the minimization problem are

$$\min_{\{\alpha_{hh},\alpha_{hl}\geq 0,\underline{c}\leq c_h\leq \overline{c}\}} \widehat{TC}$$

subject to

$$-C_\alpha(\alpha_{hl},\theta_l)+\int_{\underline{c}}^{c_h} H_\alpha(c,\alpha_{hl})dc \leq 0, \text{ with equality if } \alpha_{hl}>0; \tag{40}$$

$$-C_\alpha(\alpha_{hh},\theta_h)+\int_{\underline{c}}^{c_h} H_\alpha(c,\alpha_{hh})dc \leq 0, \text{ with equality if } \alpha_{hh}>0, \tag{41}$$

where

$$\widehat{TC} = C(\alpha_{hh},\theta_h) - qC(\alpha_{hl},\theta_l) + q\int_{\underline{c}}^{c_h}(H(c,\alpha_{hl})-H(c,\alpha_{hh}))dc$$
$$+ (1-q)\int_{\underline{c}}^{c_h}(c-c_0)h(c,\alpha_{hh})dc.$$

For convenience, call this Problem (R). Recall by Lemma 1 that $\alpha_{hl} \geq \alpha_{hh}$ with strict inequality when $\alpha_{hl} > 0$.

Also recall that $C_\alpha(0,\theta_l) < \int_{\underline{c}}^{c_0} H_\alpha(c,0)dc$ by assumption. Define $\hat{c}_1$ as the cutoff such that $\int_{\underline{c}}^{\hat{c}_1} H_\alpha(c,0)dc = C_\alpha(0,\theta_l)$. So we have $\hat{c}_1 < c_0$. Then when $c_h \in (\hat{c}_1, \overline{c}]$, (40) is binding and $\alpha_{hl} > 0$. Similarly, define $\hat{c}_2$ as the cutoff such that $\int_{\underline{c}}^{\hat{c}_2} H_\alpha(c,0)dc = C_\alpha(0,\theta_h)$. If such cutoff does not exist (which means that $\int_{\underline{c}}^{\overline{c}} H_\alpha(c,0)dc < C_\alpha(0,\theta_h)$), define $\hat{c}_2 = \overline{c}$. It is clear that $\hat{c}_2 \geq \hat{c}_1$.

Our goal is to show that $c_h \geq c_0$ is not optimal. There are two cases to consider: Case 1: $\hat{c}_2 > c_0$ and Case 2: $\hat{c}_2 \leq c_0$.

Case 1 ($\hat{c}_2 > c_0$) consists of two subcases: Case 1.1: $c_h \in [\hat{c}_1, \hat{c}_2]$ and Case 1.2: $c_h \in (\hat{c}_2, \overline{c}]$.

Case 1.1: $c_h \in [\hat{c}_1, \hat{c}_2]$. In this case, (40) is binding while (41) is non-binding, i.e., $\alpha_{hl} > 0 = \alpha_{hh}$. Then

$$\widehat{TC} = -qC(\alpha_{hl},\theta_l) + q\int_{\underline{c}}^{c_h}(H(c,\alpha_{hl})-H(c,0))dc + (1-q)\int_{\underline{c}}^{c_h}(c-c_0)h(c,0)dc, \;\; c_h \in [\hat{c}_1,\hat{c}_2].$$

Using envelope theorem,

$$\frac{\partial \widehat{TC}}{\partial c_h} = q[H(c_h,\alpha_{hl}(c_h)) - H(c_h,0)] + (1-q)(c_h-c_0)h(c_h,0),$$

where $\alpha_{hl}(c_h)$ is the effort determined by (40) with equality. Note that $c_0 \in [\hat{c}_1, \hat{c}_2]$ for Case 1.1.

Take any $c_h \in [c_0, \hat{c}_2]$, since $\alpha_{hl}(c_h) > 0$, $\frac{\partial \widehat{TC}}{\partial c_h} > 0$. Therefore, it is never optimal to set $c_h \in [c_0, \hat{c}_2]$.

Case 1.2: $c_h \in (\hat{c}_2, \overline{c}]$. In this case, (40) and (41) are both binding. Then Problem (R) becomes a minimization problem with two equality constraints. Construct the Lagrangian by multiplying λ to (40) and μ to (41), That is,

$$L = \widehat{TC} + \lambda(-C_\alpha(\alpha_{hl}, \theta_l) + \int_{\underline{c}}^{c_h} H_\alpha(c, \alpha_{hl})dc) + \mu(-C_\alpha(\alpha_{hh}, \theta_h) + \int_{\underline{c}}^{c_h} H_\alpha(c, \alpha_{hh})dc)$$

The first order conditions are[46]

$$\begin{aligned} \frac{\partial L}{\partial c_h} &= q(H(c_h, \alpha_{hl}) - H(c_h, \alpha_{hh})) + (1-q)(c_h - c_0)h(c_h, \alpha_{hh}) \\ &\quad + \lambda H_\alpha(c_h, \alpha_{hl}) + \mu H_\alpha(c_h, \alpha_{hh}) = 0; \end{aligned} \tag{42}$$

$$\begin{aligned} \frac{\partial L}{\partial \alpha_{hh}} &= C_\alpha(\alpha_{hh}, \theta_h) - q\int_{\underline{c}}^{c_h} H_\alpha(c, \alpha_{hh})dc + (1-q)\int_{\underline{c}}^{c_h} (c - c_0)H_{c\alpha}(c, \alpha_{hh})dc \\ &+ \mu(-C_{\alpha\alpha}(\alpha_{hh}, \theta_h) + \int_{\underline{c}}^{c_h} H_{\alpha\alpha}(c, \alpha_{hh})dc) \\ &= C_\alpha(\alpha_{hh}, \theta_h) - qC_\alpha(\alpha_{hh}, \theta_h) + (1-q)\int_{\underline{c}}^{c_h} (c - c_0)dH_\alpha(c, \alpha_{hh}) \\ &+ \mu(-C_{\alpha\alpha}(\alpha_{hh}, \theta_h) + \int_{\underline{c}}^{c_h} H_{\alpha\alpha}(c, \alpha_{hh})dc) \\ &= (1-q)C_\alpha(\alpha_{hh}, \theta_h) + (1-q)[(c_h - c_0)H_\alpha(c_h, \alpha_{hh}) - \int_{\underline{c}}^{c_h} H_\alpha(c, \alpha_{hh})dc] \\ &+ \mu(-C_{\alpha\alpha}(\alpha_{hh}, \theta_h) + \int_{\underline{c}}^{c_h} H_{\alpha\alpha}(c, \alpha_{hh})dc) \\ &= (1-q)(c_h - c_0)H_\alpha(c_h, \alpha_{hh}) + \mu(-C_{\alpha\alpha}(\alpha_{hh}, \theta_h) + \int_{\underline{c}}^{c_h} H_{\alpha\alpha}(c, \alpha_{hh})dc) = 0; \end{aligned} \tag{43}$$

$$\begin{aligned} \frac{\partial L}{\partial \alpha_{hl}} &= -qC_\alpha(\alpha_{hl}, \theta_l) + q\int_{\underline{c}}^{c_h} H_\alpha(c, \alpha_{hl})dc + \lambda(-C_{\alpha\alpha}(\alpha_{hl}, \theta_l) + \int_{\underline{c}}^{c_h} H_{\alpha\alpha}(c, \alpha_{hl})dc) \\ &= \lambda(-C_{\alpha\alpha}(\alpha_{hl}, \theta_l) + \int_{\underline{c}}^{c_h} H_{\alpha\alpha}(c, \alpha_{hl})dc) = 0. \end{aligned} \tag{44}$$

[46] $\frac{\partial L}{\partial c_h}$ is allowed to be negative at boundary $\overline{c}$.

Note that $-C_{\alpha\alpha}(\alpha_{hh},\theta_h)+\int_{\underline{c}}^{c_h} H_{\alpha\alpha}(c,\alpha_{hh})dc<0$ and $-C_{\alpha\alpha}(\alpha_{hl},\theta_l)+\int_{\underline{c}}^{c_h} H_{\alpha\alpha}(c,\alpha_{hl})dc<0$ when $c_h>\underline{c}$. If $c_h\geq\hat{c}_2(>c_0)$, from (44), we obtain $\lambda=0$. And from (43), $\mu>0$. Since $\alpha_{hl}>\alpha_{hh}>0$ in Case 1.2, $H(c_h,\alpha_{hl})-H(c_h,\alpha_{hh})>0$. However, substituting $c_h\geq c_0$, $\lambda=0$, and $\mu>0$ into (42) leads to

$$q(H(c_h,\alpha_{hl})-H(c_h,\alpha_{hh}))<0,$$

a contradiction.

Therefore, under Case 1 it is never optimal to set $c_h^*\geq c_0$.

Case 2: $\hat{c}_2\leq c_0$. In this case, both (40) and (41) are binding when $c_h\geq c_0$. Problem (R) becomes a minimization problem with two equality constraints when $c_h\geq c_0$. Then we can set up the Lagrangian. Using similar arguments as those in Case 1.2, it can be shown that setting $c_h^*\geq c_0$ is never optimal.

Summing up, we have shown that $c_h^*<c_0$ when $C_\alpha(0,\theta_l)<\int_{\underline{c}}^{c_0} H_\alpha(c,0)dc$. $\square$

Proof of Lemma 6: Note that $\pi(\theta_h,\theta_h)=0$ and

$$\pi(\theta_l,\theta_l)=C(\alpha_{hh}^*,\theta_h)-C(\alpha_{hl}^*,\theta_l)+\int_{\underline{c}}^{c_h^*}(H(c,\alpha_{hl}^*)-H(c,\alpha_{hh}^*))dc,$$

therefore, to check (21), it suffices to check

$$C(\alpha_{hh}^*,\theta_h)-C(\alpha_{hl}^*,\theta_l)+\int_{\underline{c}}^{c_h^*}(H(c,\alpha_{hl}^*)-H(c,\alpha_{hh}^*))dc$$

$$\leq C(\alpha_{lh}^*,\theta_h)-C(\alpha_{ll}^*,\theta_l)+\int_{\underline{c}}^{c_0}(H(c,\alpha_{ll}^*)-H(c,\alpha_{lh}^*))dc.$$

Notice that in terms of the $g(\cdot)$ function defined in the proof of Lemma 4, the LHS is $g(c_h^*)$ while the RHS is $g(c_0)$. Since we have shown that $g(t)$ is increasing by Lemma 4 and $c_h^*<c_0$ holds by Lemma 5, (21) holds. $\square$

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