On Equilibrium Player Ordering in Dynamic Team Contests*

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Abstract

We study equilibrium player ordering in a dynamic all-pay contest between two teams. The contest lasts two periods, and each team consists of two players who perform in different periods on behalf of their teams. The team with the higher aggregate output wins the prize, which is a public good to its players. Each team has one stronger player and one weaker player, and the two teams can differ in their values of the prize. The teams maximize their winning odds by strategically assigning their players to different periods. We find that when the intra-team heterogeneity in player ability is not excessive, the teams would allocate their stronger players to the late positions as the "anchormen." When both the intra-team ability gap and inter-team heterogeneity in teams' values become excessively large, the team with high value always places its stronger player in the early position, who will place a large bid to preempt late competition.

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1 Introduction

Many contest-like competitions demand contenders' continued input, rather than a single stroke of effort, and involve dynamic interactions (see Konrad, 2009; Fu and Wu, 2019). Contentions often take place between teams or alliances instead of individual parties; players on a team join forces to compete collectively for prizes. Consider, for instance, a running or swimming relay race. Each player is assigned to a leg, and the trophy is awarded based on

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teams' overall performance. In an R&D race between two research teams, the project is split among scientists on a team, and each is tasked with one phase of the development. Consider, alternatively, an electoral competition. As a common feature of U.S. politics, candidates rely on key surrogates' efforts to campaign on their behalf.

This paper combines the two aforementioned observations and analyzes a dynamic contest between two teams. The contest lasts two periods. Each team consists of two players, and each player performs in one period. In each period, two players from rival teams simultaneously exert effort, and early performance is revealed before late players move.[1] The two players' efforts are summed for each team, and the team with the higher aggregate effort wins the prize, which is a public good to its players.

We analyze a particular strategic problem. Assume that players on a team are heterogeneous, with one stronger and the other weaker. Suppose that each team is governed by a "planner," who decides on the placement of her players, i.e., in the early position or the late position. How should a planner place her players in order to maximize her team's winning probability? Which player should be the "anchorman" to perform in the late period, the stronger or the weaker? Consider a coach who allocates her athletes' positions in a relay race to ensure her team's win. Alternatively, imagine a campaign manager who has to plan and schedule key surrogates' appearances at major campaign events to maximize their impact on votes.[2] In real-life sports competitions, the most competent player on a team is usually assigned to the last leg of a relay as the anchorman. Do such observations reflect game theoretic regularity as a result of the teams' conscious and rational choices, or are they simply common and habitual routines without strategic substance?

In our model, two planners simultaneously commit to the ordering of their players, and the placement profile is revealed publicly. The planners' choices involve interesting strategic concerns. Suppose that one planner places her stronger player in the early position to secure an early lead, while the other does the opposite. It is not straightforward how the competition would unfold. First, it is far from transparent how players in the late period would respond to the early outcome. On the one hand, an early lead allows the late player on the leading team to shirk, which tends to offset the leading team's advantage, while on the other hand, the conventional wisdom in the literature on dynamic contests (see Harris

[1] For instance, the regulatory regimes in most of EU countries require mandatory disclosure of firms' R&D activities (La Rosa and Giovanni Liberatore, 2014). Disclosure allows firms to observe competitors' progress, but they make their current R&D investment decisions without observing those of the others. Alternatively, consider an electoral competition. Candidates' relative standing can largely be inferred from their status in public opinion polls, which, in turn, affects the momentum in the dynamics (Denter and Sisak, 2015).

[2] In the United States, a political campaign management team usually includes a dedicated scheduling and advance department. The department makes sure that the candidate and campaign surrogates are effectively scheduled so as to maximize their influence on voters, which plays an important role in campaign strategy.

and Vickers, 1987; Klumpp and Polborn, 2006; Konrad and Kovenock, 2006) holds that a lead tends to discourage the laggard in future competition. Furthermore, a mismatch is formed in the second period. The team's early lead comes at the cost of obliging its weaker player to fight with a stronger competitor. The latter team's late player has a good prospect of catching up in the presence of a weak opponent, which could offset the discouragement caused by the lag and incentivize his effort. As a result, it is unclear whether a stronger player, when being placed in an early position, indeed has an incentive to secure a substantial lead for his team in the first place, when anticipating the rival team's catching up in the late period. Second, the dynamic competition naturally encourages early players to freeride intertemporally, as shirking might still be made up for by late teammates.

Following the tradition of group/team contests, the prize is a public good to the players on each team, and they value it equally. However, the players on each team could differ in the capability, in that one bears a lower cost for effort; further, one team's players could value the prize more than those on the other. The model thus allows for both intra- and inter-team heterogeneities. We show that with moderate intra-team heterogeneity, a unique equilibrium, with early players bidding in pure strategies, exists in the contest regardless of the prevailing placement profile and level of inter-team heterogeneity: In the equilibrium, early players freerides on their late teammates, so the winner is determined purely by the bidding outcome in the late period. As a result, a planner would find it to be a dominant strategy to place her stronger player in the late period. We thus provide a strategic rationale for the commonly observed anchorman choice.

Further, we show that with large intra-team heterogeneity, inter-team heterogeneity no longer plays an entirely passive role. When the two teams differ substantially in valuations, the more motivated team must place its stronger player in the early position, because he will place a large bid to preempt future competition and secure a sure win for the team, regardless of the rival team's placement arrangement.

This paper belongs to the extensive literature on contests between groups or teams, including Skaperdas (1998); Nitzan (1991); Baik (1993, 2008); Esteban and Ray (2001, 2008); Nitzan and Ueda (2009, 2011); Münster (2007, 2009); Konrad and Leininger (2007); Konrad and Kovenock (2009b); Barbieri, Malueg, and Topolyan (2014); Chowdhury, Lee, and Topolyan (2016); Chowdhury and Topolyan (2016); and a recent study by Eliaz and Wu (2016). All of these studies assume that group/team members simultaneously contribute their efforts, which are translated into a single aggregate bid through a production function. They focus on static competitions, while we consider a dynamic contest in which the performance of early players is observed by late players.[3]

[3]Our model bears a link to Varian (1994), who studies a two-player public contribution game. He shows that when they move sequentially, the early contributor may freeride on the later one. In contrast to Varian,

Our paper is therefore linked to the literature on dynamic contests, which includes Harris and Vickers (1987); Klumpp and Polborn (2006); Yildirim (2005); Konrad and Kovenock (2009a, 2010); Hirata (2014); Joffrion and Parreiras (2014); and Gelder (2014). These studies, however, all consider competitions between individual contenders. In particular, Harris and Vickers (1987), Klumpp and Polborn (2006), and Gelder (2014) all study settings in which players participate in a series of discrete battles, and the winner is chosen by counting the number of battles won by each player. Yildirim (2005) and Hirata (2014) both consider contests in which players are allowed to add to their own bids in late periods, and the player with a higher total bid wins with a higher probability.[4] Fu, Lu, and Pan (2015) and Häfner (2015) are the two exceptions to this strand of literature, as they focus on team contests. While they study environments in which each player participates in one distinct component battle of a whole contest, we assume that players on a team contribute perfectly substitutable efforts. In addition, both studies assume that players' ordering and matching are fixed, while we focus on the ex ante choice of teams' plans in assigning and ordering players in the dynamics. Our study thus adds to the literature in this regard.[5]

2 A Model of A Two-Period Team Contest

Two teams, indexed by $i = A, B$, compete for a prize in a contest. Each team consists of two players. The prize is a public good to players on the same team.

The contest proceeds in two periods. In each period, a pair of players from opposing teams are matched and simultaneously exert their efforts to perform. Each player performs only once. Players' performance in the early period is immediately observed by late players before they place their bids. A player is indexed by $i(t)$, where $i \in \{A, B\}$ denotes his affiliation, and $t \in \{1, 2\}$ denotes the temporal order of his performance in the series.

The model allows for both intra- and inter-team heterogeneity. On each team, there is one strong player and one weak player. A player's output $x_{i(t)}$ incurs a linear effort cost $k_{i(t)}x_{i(t)}$, with $k_{i(t)} > 0$. If $i(t)$ is a strong player, then $k_{i(t)} = 1$; and if $i(t)$ is a weak player,

our study considers a competitive setting: Players on a team join forces to compete against another team, which thus leads to the important trade-off between freeriding and preemption. As a result, by Theorem 4, the early player on the stronger team may preempt the competition by placing a large bid, which contrasts with the prediction of Varian (1994) in the noncompetitive setting.

[4]Yildirim (2005) considers Tullock contests, while Hirata (2014) considers all-pay auctions. Hirata shows that there exists a continuum of equilibria in which players play pure strategies in the first period and randomize their bids in the second period.

[5]Hamilton and Romano (1998) also study a game in which two teams' planners strategically order their players. In contrast to our setting, they assume that players participate in pairwise battles, and the winner is chosen by teams' discrete counts of victories in battles. In addition, they abstract away players' effort decisions, assuming that players have fixed winning probabilities in a battle under a fixed matching.

then $k_{i(t)} = k > 1$. The level of k measures the *intra-team heterogeneity* and the difference between players in terms of their abilities. We impose the following requirement on the size of k.

Assumption 1 *(moderate intra-heterogeneity) $k < 2$.*

For analytical tractability, our analysis primarily focuses on the most plausible case of moderate intra-team heterogeneity, which requires that contenders not differ drastically in their abilities within a team. We will discuss the case of $k > 2$ after we present our main results.

Following the tradition in the literature on group contests with group-specific public-good prizes (e.g., Esteban and Ray, 1999; Baik, 2008; Barbieri, Malueg, and Topolyan, 2014; and Eliaz and Wu, 2018), we assume that players on a team equally value the prize. The two teams, however, may differ in their players' valuations. Let

$$\varphi_i = \begin{cases} \varphi \ (\geq 1) & \text{if } i = A, \\ 1 & \text{if } i = B. \end{cases}$$

The prize has a value $\varphi_i v$, with $v > 0$, to players on team $i = A, B$. The parameter φ measures the degree of *inter-team heterogeneity*.

The winning rule of the contest resembles that of a standard all-pay auction. A team i wins the contest if its aggregate performance exceeds that of the rival team, i.e., $x_{i(1)} + x_{i(2)} > x_{j(1)} + x_{j(2)}$, $i, j \in \{1, 2\}$, $i \neq j$. A tie is broken randomly and fairly with the following caveat: Whenever a tie occurs at a level where there is a mass point for one and only one player's bid, the tie-breaking rule favors the other player. Players on the winning team collect the prize, while a player's costly effort is nonrefundable regardless of the outcome. The payoff structure is commonly known.

We assume that each team has a planner, who strategically decides the placement of her players in the series to maximize the winning odds of the team. In our setup, she simply decides whether to place her stronger player in the late position. We assume that planners simultaneously commit to their players' order prior to the contest. The placement profile is immediately revealed to players before the contest begins and is common knowledge.

3 Analysis

We solve the game by backward induction. Assuming a fixed placement profile, we first characterize the bidding equilibrium in the second period, which is no different from a standard all-pay auction with a headstart. The solution allows us to obtain the payoff functions of players who perform in period 1 and characterize their bidding strategies. Finally, we explore planners' placement decisions based on the equilibrium results obtained under each placement profile.

3.1 Bidding Equilibrium in Period 2

For ease of exposition, we introduce the following definition of effective prizes for the players competing in the second period.

Definition 1 $v_{i(2)} = \frac{\varphi_i}{k_{i(2)}} v$, $i = A, B$.

Note that $v_{i(2)}$ is the maximum bid player $i(2)$ is willing to exert in the second-period competition, as he would end up with negative payoff regardless of win or loss if he bids more than that. Let $v_{i(2)} \geq v_{j(2)}, i, j \in \{A, B\}, i \neq j$. In other words, let i denote the team with higher effective value in the second period. Define $\Delta = x_{j(1)} - x_{i(1)}$, which is player $j(2)$'s headstart in the second-period competition. The bidding competition remains active in the second period if and only if $\Delta \in (-v_{j(2)}, v_{i(2)})$: Player $j(2)$ would give up if he has to make up for a gap of a size more than $v_{j(2)}$—which his maximum bid—and team i secures a sure win; the same is the case of $\Delta \geq v_{i(2)}$, which would discourages $i(2)$ entirely and gives team i a sure win.

The bidding equilibrium depends on the particular size of Δ, and the competition is simply an all-pay auction with headstart. Such all-pay auction games have been thoroughly studied in the literature, such as Konrad (2002), Li and Yu (2012), Pastine and Pastine (2012), and Kawamura and Moreno de Barreda (2014).[6]

There are altogether three possibilities.[7]

Lemma 1 *Suppose* $\Delta \in (-v_{j(2)}, 0)$. *In the unique mixed-strategy equilibrium, player* $i(2)$ *bids zero with a probability* $-\Delta/v_{j(2)}$. *His effort* $x_{i(2)}$ *is continuously distributed over the support* $(0, v_{j(2)} + \Delta]$, *with cumulative distribution function*

$$F_{x_{i(2)}}(x_{i(2)}) = \frac{x_{i(2)} - \Delta}{v_{j(2)}}.$$

Player $j(2)$ *bids zero with a probability of* $1 - \frac{v_{j(2)}+\Delta}{v_{i(2)}}$, *and* $x_{j(2)}$ *is continuously distributed over the support* $(-\Delta, v_{j(2)}]$, *with cumulative distribution function*

$$F_{x_{j(2)}}(x_{j(2)}) = 1 - \frac{v_{j(2)} - x_{j(2)}}{v_{i(2)}}.$$

[6]Kawamura and Moreno de Barreda (2014) allow for convex bidding costs. Siegel (2014) considers an all-pay contest in which a contestant can produce a certain range of scores with zero cost. In addition, Siegel (2014) allows for multiple prizes.

[7]The detailed proofs for Lemmata 1-3 are omitted to save space, but is available from the authors upon request.

Player $j(2)$ is weaker than his matched opponent $i(2)$, as his effective prize is lower. In Case (i), he is further afflicted with a negative headstart $\Delta = x_{j(1)} - x_{i(1)} \leq 0$ because of his teammate's early underperformance $(\Delta < 0)$, which magnifies his innate disadvantage. He is discouraged by bidding zero with a positive probability. The magnified lead for $i(2)$ allows him to slacken off and also bid zero with a positive probability. Further, we consider a case of $\Delta \in [0, v_{i(2)} - v_{j(2)}]$, i.e., player $j(2)$ is endowed with a positive lead.

Lemma 2 *Suppose $\Delta \in [0, v_{i(2)} - v_{j(2)}]$. In the unique mixed-strategy bidding equilibrium, player $i(2)$'s effort $x_{i(2)}$ is continuously distributed over the support $[\Delta, v_{j(2)} + \Delta]$, with cumulative distribution function*

$$F_{x_{i(2)}}(x_{i(2)}) = \frac{x_{i(2)} - \Delta}{v_{j(2)}}.$$

Player $j(2)$ bids zero with a probability $1 - \frac{v_{j(2)}}{v_{i(2)}}$, and his effort $x_{j(2)}$ is continuously distributed over the support $(0,\ v_{j(2)}]$, with cumulative distribution function

$$F_{x_{j(2)}}(x_{j(2)}) = 1 - \frac{v_{j(2)} - x_{j(2)}}{v_{i(2)}}.$$

In this case, player $j(2)$ has a positive headstart, but the headstart is insufficient to offset his innate disadvantage, as measured by the difference between $i(2)$'s and $j(2)$'s maximum bids, i.e., $v_{i(2)} - v_{j(2)}$. He still bids zero with a positive probability. Player $i(2)$, however, must counteract the initial disadvantage, so his bid must exceed Δ.

As will be shown later, Lemma 2 depicts the most relevant scenario in the game, so we provide more detail about the equilibrium. Player $j(2)$ wins with a probability

$$P_{j(2)} = \Pr(x_{j(2)} + \Delta > x_{i(2)}) = \int_0^{v_{j(2)}} \left[\frac{x_{j(2)}}{v_{j(2)}} \frac{1}{v_{i(2)}}\right] dx_{i(2)} = \frac{v_{j(2)}}{2v_{i(2)}}, \tag{1}$$

and $i(2)$ wins with the complementary probability $1 - \frac{v_{j(2)}}{2v_{i(2)}}$. It is noteworthy that in this case equilibrium winning probabilities are independent of Δ: The distribution of player $i(2)$'s bid is moved up by exactly Δ, which perfectly offsets team j's early lead. In this case, player $i(2)$'s expected bid amounts to

$$E(x_{i(2)}) = \int_\Delta^{\Delta + v_{j(2)}} \frac{x_{i(2)}}{v_{j(2)}} dx_{i(2)} = \frac{v_{j(2)}}{2} + \Delta. \tag{2}$$

Player $j(2)$ ends up with zero surplus, so his expected bid is simply $\frac{v_{j(2)}}{2v_{i(2)}} v$, where $\frac{v_{j(2)}}{2v_{i(2)}}$ is his winning probability.

We next consider the case in which a large headstart reverses the innate imbalance between $i(2)$ and $j(2)$ in their effective prizes.

Lemma 3 *Suppose $\Delta \in (v_{i(2)} - v_{j(2)}, v_{i(2)})$. In the unique mixed-strategy bidding equilibrium, player $i(2)$ bids zero with a probability $\frac{[v_{j(2)}-v_{i(2)}]+\Delta}{v_{j(2)}}$ and his effort $x_{i(2)}$ is continuously distributed over the support $[\Delta, v_{i(2)}]$, with cumulative distribution function*

$$F_{x_{i(2)}}(x_{i(2)}) = \frac{[v_{j(2)} - v_{i(2)}] + x_{i(2)}}{v_{j(2)}}.$$

Player $j(2)$ bids zero with a probability $\Delta/v_{i(2)}$ and his effort $x_{j(2)}$ is continuously distributed over the support $(0,\, v_{i(2)} - \Delta]$, with cumulative distribution function

$$F_{x_{j(2)}}(x_{j(2)}) = \frac{x_{j(2)} + \Delta}{v_{i(2)}}.$$

In this case, both players bid zero with positive probabilities. Player $i(2)$ does so because of the excessive discouragement; in contrast, the large lead allows $j(2)$ to slacken off, as he can win even without an input.

3.2 Bidding in the Early Period and Equilibrium Ordering

The bidding equilibrium in the subgame allows us to solve for the equilibrium in the first period. Early players' bids determine $\Delta = x_{j(1)} - x_{i(1)}$, which subsequently determines their teams' ultimate win likelihoods and their own payoffs. We first write their payoffs as functions of early bidding profiles.

If $\Delta \geq v_{i(2)}$, i.e., $x_{i(1)} \leq x_{j(1)} - v_{i(2)}$, then both players $i(2)$ and $j(2)$ exert zero effort at period 2. As a result, team j would definitely win the contest, which gives $\pi_{i(1)}(x_{i(1)}, x_{j(1)}) = -k_{i(1)}x_{i(1)}$ and $\pi_{j(1)}(x_{i(1)}, x_{j(1)}) = \varphi_j v - k_{j(1)}x_{j(1)}$. If $\Delta \leq -v_{j(2)}$, i.e., $x_{i(1)} \geq x_{j(1)} + v_{j(2)}$, then both players $i(2)$ and $j(2)$ exert zero effort at period 2. As a result, team i would definitely win the contest, which gives $\pi_{i(1)}(x_{i(1)}, x_{j(1)}) = \varphi_i v - k_{i(1)}x_{i(1)}$ and $\pi_{j(1)}(x_{i(1)}, x_{j(1)}) = -k_{j(1)}x_{j(1)}$.

We next consider the case in which $\Delta \in (-v_{j(2)}, v_{i(2)})$. By Lemmata 1-3, we have the following three subcases for player $i(1)$'s payoff function, which correspond to the three ranges of Δ.[8] Denote by p_i team i's winning probability.

$$\begin{aligned} &\pi_{i(1)}(x_{i(1)}, x_{j(1)}) = p_i\varphi_i v - k_{i(1)}x_{i(1)} \\ = &\begin{cases} \left[1 - \frac{1}{2v_{i(2)}v_{j(2)}}\left(v_{j(2)}^2 - \Delta^2\right)\right]\varphi_i v - k_{i(1)}x_{i(1)} & \text{if } x_{i(1)} \in (x_{j(1)}, x_{j(1)} + v_{j(2)}); \\ \left(1 - \frac{v_{j(2)}}{2v_{i(2)}}\right)\varphi_i v - k_{i(1)}x_{i(1)} & \text{if } x_{i(1)} \in [x_{j(1)} - v_{i(2)} + v_{j(2)}, x_{j(1)}]; \\ [\frac{1}{2v_{i(2)}v_{j(2)}}\left(v_{i(2)}^2 - \Delta^2\right)]\varphi_i v - k_{i(1)}x_{i(1)} & \text{if } x_{i(1)} \in (x_{j(1)} - v_{i(2)}, x_{j(1)} - v_{i(2)} + v_{j(2)}). \end{cases} \end{aligned} \quad (3)$$

[8]Note that the following ranges for $x_{i(1)}$ and $x_{j(1)}$ might cover negative effort, which is not eligible. For notational conveniency, we ignore this issue for the time being. This issue will be well taken care of, however, when we derive the equilibrium.

Since one and only one team wins the contest, player $j(1)$'s payoff is simply $\pi_{j(1)}(x_{i(1)}, x_{j(1)}) = (1-p_i)\varphi_j v - k_{j(1)}x_{j(1)}$. It is straightforward to observe that the payoff functions are continuous in their own choice variables, but have kinked points when $x_{i(1)} = x_{j(1)}$ and when $x_{i(1)} = x_{j(1)} - v_{i(2)} + v_{j(2)}$.

We are ready to characterize players' bidding equilibrium in this period. Analogous to Hirata (2014), we focus on a pure-strategy bidding equilibrium in period 1. Altogether, there are four possible scenarios:

1. Both teams place their stronger players in the 2nd position (Scenario 1).
2. Both teams place their weaker players in the 2nd position (Scenario 2).
3. Team A places its stronger player in the 2nd position and team B does the opposite (Scenario 3).
4. Team A places its weaker player in the 2nd position and team B does the opposite (Scenario 4).

Despite these variations, our analysis concludes the following unequivocally.

Theorem 1 *With moderate intra-team heterogeneity, i.e., $k \in (1, 2)$, under any given placement profile, a contest has a unique subgame perfect equilibrium with pure-strategy bidding in the first period, in which players $A(1)$ and $B(1)$ exert zero effort, and no nonzero headstart is carried over to period 2, i.e., $\Delta^* = 0$.*

Proof. See Appendix. ■

Theorem 1 states that for moderate intra-team heterogeneity, players in the early period simply have no incentive to deliver effort, regardless of the level of inter-team heterogeneity. As a result, the outcome of the contest is purely determined by the performance in the late period, which—in our setup—is a standard all-pay auction with no headstart in the equilibrium. The prediction of freeriding in group echoes Baik (1993, 2008). He studies group contests in which players simultaneously commit to their efforts. With players within a group to differ in their valuations, he shows that only the one with the highest valuation exerts positive effort. In contrast, our study considers a sequential-move game, in which the early player on each team shirks and freerides intertemporally, irrespective of his strength to bid. Our result thus complements those of Baik (1993, 2008).

The logic of Theorem 1 will be discussed later in this paper, as the result on equilibrium ordering can be obtained immediately. Recall that team planners simply maximize their teams' winning odds. Because early players bid zero, $\Delta = 0$ results. By Lemma 2 and (1), team i's winning probability is given by $1 - \frac{v_{j(2)}}{v_{i(2)}}$, and team j wins with the complementary probability. The payoff matrix can be summarized as follows:

		Team B	
		weaker first	stronger first
Team A	weaker first	$1-\frac{1}{2\varphi}, \frac{1}{2\varphi}$	$1-\frac{1}{2\varphi k}, \frac{1}{2\varphi k}$
	stronger first	$1-\frac{k}{2\varphi}, \frac{k}{2\varphi}$ if $k \leq \varphi$; $\frac{\varphi}{2k}, 1-\frac{\varphi}{2k}$ if $k > \varphi$	$1-\frac{1}{2\varphi}, \frac{1}{2\varphi}$

Since $\varphi \geq 1$ and $k \in (1,2)$, the planners' ordering game has a unique equilibrium, as in the following theorem.

Theorem 2 *With moderate intra-team heterogeneity, i.e., $k \in (1,2)$, both team planners place their stronger players in the late period. That is, Scenario 1—which is defined above—emerges in the equilibrium.*

The claim is straightforward. Given that only the outcome in the late period would make a difference, it is a dominant strategy for a planner to reserve her most competent player for that competition.[9] Our results thus provide a rationale for the popular practice in various relay races, i.e., giving the anchor leg to the most capable athlete on a team.

It should be noted that the size of k—i.e., the weaker players' marginal effort cost—does not play a role in these equilibrium outcomes because the competition essentially takes place between the two stronger players whose marginal effort cost is one.

Theorem 1, together with Lemmata 1-3, also allows us to explore this competitive setting from a contest designer's perspective. Suppose that a designer—who sets the rules for the contest—is allowed to enforce players' placement in the competition, and that she knows players' cost profiles, e.g., based on athletes' track records in professional sports. As commonly assumed in contest literature, the designer maximizes the expected total effort of the contest, or equivalently, the aggregate performance of the two teams measured by $\sum_{i=A}^{B}\sum_{t=1}^{2} x_{i(t)}$. The following is obtained.

Corollary 1 *With moderate intra-team heterogeneity—i.e., $k \in (1,2)$, a contest designer—who maximizes the total effort or aggregate performance of the contest—would require that*

(i) each team's stronger player be placed in the late position if $\varphi \leq k$; and

(ii) team A place its weaker player in the late position, and team B do the opposite if $\varphi \geq k$.

By Theorem 1, maximizing the total effort/performance of the contest is reduced to maximizing the total effort/performance in period 2. The prediction subtly differs from that of Theorem 2. When the inter-team heterogeneity remains mild, i.e., $\varphi \leq k$, the optimum requires Scenario 1, and stronger players should be assigned to the late-period competition:

[9] It should be noted that this prediction would remain in place when planners are allowed to randomize their placement plan.

The optimum for the design problem is consistent with the equilibrium outcome predicted by Theorem 2. However, when the heterogeneity is large, i.e., $\varphi \geq k$, the designer should let the weaker player on the stronger team perform in the late period and compete head-to-head to the stronger player on the rival team, which leads to Scenario 4. The intuition is straightforward. By the conventional wisdom in contest literature, a standard contest elicits more effort when players' efforts are less costly, and when the competitions are more balanced. Placing stronger players in late positions achieves both when the inter-team heterogeneity remains mild. A subtler trade-off arises in the presence of large heterogeneity.

We provide numerican examples to illustrate the two cases in Corollary, which demonstrates how the contest behave in each possible scenario, and how the equilibrium and effort-maximizing optimum are formed. We normalize the common factor in prize valuations, v, to one. Contestants' equilibrium efforts are linear in it; equilibrium winning probabilities are homogenous of degree zero with respect to v: They are solely determined by contestants' relative competitiveness, i.e., $P_{j(2)} = v_{j(2)}/2v_{i(2)}$, which entirely cancels out the role played by v. We first consider a setting of $\varphi = 1.2, k = 1.5$, which corresponds to case (i) of Corollary 1. We provide the outcomes in each of the four scenarios in the following table.

	winning odds	A(2)'s effort	B(2)'s effort	total efforts
Scenario 1 (equibrium/optimum)	$\frac{7}{12}, \frac{5}{12}$	$\frac{1}{2}$	$\frac{5}{12}$	$\frac{11}{12}$
Scenario 2	$\frac{7}{12}, \frac{5}{12}$	$\frac{1}{3}$	$\frac{5}{18}$	$\frac{11}{18}$
Scenario 3	$\frac{13}{18}, \frac{5}{18}$	$\frac{1}{3}$	$\frac{5}{27}$	$\frac{14}{27}$
Scenario 4	$\frac{2}{5}, \frac{3}{5}$	$\frac{8}{25}$	$\frac{2}{5}$	$\frac{18}{25}$

Table 1: The setting of $\varphi = 1.2, k = 1.5$

In this case, the equilibrium ordering outcome—i.e., Scenario 1— coincides with the optimum that maximizes total effort. We then consider an alternative setting of $\varphi = 1.8, k = 1.5$, which corresponds to case (ii) of Corollary 1: The inter-team heterogeneity is enlarged, and the result also varies.

	winning odds	A(2)'s effort	B(2)'s effort	total efforts
Scenario 1 (equibrium/optimum)	$\frac{13}{18}, \frac{5}{18}$	$\frac{1}{2}$	$\frac{5}{18}$	$\frac{7}{9}$
Scenario 2	$\frac{13}{18}, \frac{5}{18}$	$\frac{1}{3}$	$\frac{5}{27}$	$\frac{14}{27}$
Scenario 3	$\frac{22}{27}, \frac{5}{27}$	$\frac{1}{3}$	$\frac{10}{81}$	$\frac{37}{81}$
Scenario 4	$\frac{7}{12}, \frac{5}{12}$	$\frac{1}{2}$	$\frac{5}{12}$	$\frac{11}{12}$

Table 2: The setting of $\varphi = 1.8, k = 1.5$

The setting of ($\varphi = 1.8, k = 1.5$) exemplifies Corollary 1(ii). As observed from Table 2, Scenario 4 generates the maximal total effort, which departs from the equilibrium: The excessive inter-team heterogeneity is corrected by placing team A's weaker player in the late position to level the playing field, which leads to a higher total effort than that in the equilibrium.

3.3 Discussion

This part begins with a discussion of the game's strategic nature. This allows us to interpret the logic underlying our main result, i.e., Theorem 1. We then explore the roles played by players' heterogeneity.

Let us consider the role played by the headstart Δ on the bidding competition in the late period. An early lead, on the surface, would increase a team's winning odds, while it changes late players' bidding behaviors.

Imagine that the innately weaker player in the 2nd period—i.e., $j(2)$—has a headstart $\Delta > 0$. Its overall effect is ex ante unclear and depends on its particular size. As demonstrated by Lemma 2, when Δ falls into the range $[0, v_{i(2)} - v_{j(2)}]$, it uniformly shifts player $i(2)$'s bid upward, while player $j(2)$'s strategy does not respond to Δ. Team j's lead—established by player $j(1)$—compels the rival player to step up his effort—which entirely offsets the early lead—and does not help team j to win. When Δ exceeds the cutoff $v_{i(2)} - v_{j(2)}$, the early lead reverses the balance of late competition and discourages the innately stronger player $i(2)$. The lead is counterproductive, as it tempts player $j(2)$'s to shirk: As shown by Lemma 3, the upper support of player $j(2)$'s bid drops to $v_{i(2)} - \Delta$. In short, a team's early lead would never further incentivizes its own late player, which limits its effect on the team's win, while it could discourage the rival player only when it is sufficiently large.

Having understood the role played by the headstart Δ in late competition, we are ready to unveil the strategic nature of the bidding game and explore early players' bidding incentives. For an early player, creating a lead allows their teammates to shirk. As revealed by the marginal payoff functions laid out in Appendix and the proof of Theorem 1, whenever he has an incentive to generate a lead, he would prefer to further raise his bid until he preempts the rival team—i.e., creating a lead that exceeds the maximum bid of the late player on the rival team, such that the late player on the rival team is completely discouraged. Alternatively, he would lower his bid below that of his matched opponent, placing a small bid simply to ensure that his opponent and his late teammate would not be excessively discouraged.

In our main setting, the former is always too costly for first-period players. We use the following example to illustrate the point. Intuitively, one team can preempt the other only when its stronger player is placed in the early slot. Imagine a subgame in which both teams place their stronger players in early positions. Suppose that player $B(1)$ bids zero. Player $A(1)$ has no incentive to preempt late competition by bidding $\frac{v}{k}$: By doing this, he receives a payoff $\varphi v - \frac{v}{k}$, while by bidding zero, his teammate wins the contest for him with a probability $1 - \frac{1}{2\varphi}$, with the latter rendering a payoff $\varphi v - \frac{v}{2} > \varphi v - \frac{v}{k}$ because $k < 2$. First, with a moderate ability differential, i.e., $k < 2$, preemption is difficult and costly: A lead of $\frac{v}{k}$ becomes larger because of a smaller k. Second, it deserves to be noted that a large inter-team heterogeneity, i.e., a larger φ, would not make a difference: A larger φ entices

$A(1)$ to preempt $B(2)$, but it also implies a larger advantage for his teammate $A(2)$ in his race against $B(2)$, which increases player $A(1)$'s payoff when he freerides on his teammate vis-à-vis preemption. These forces lead to a downward spiral to complete freeriding and zero-bid in the early stage of the contest.

3.4 Planners' Alternative Objective Function

A team planner may optimize toward other objectives. So far we have assumed that the planner is "selfish," in the sense that she does not internalize the costs incurred to the players on her team. For instance, a campaign manager may only seek to maximize the electoral impact of his candidate, but not the interests of surrogates in his camp. A coach of a relay team is primarily concerned with the team's achievement and his own career record, but not athletes' effort input. However, a planner can be subject to other objectives. For example, a firm, in an R&D race, can be concerned about the input its internal research teams commit to the project. Our analysis also provides predictions for such a scenario.

We first consider a variation in which a team planner aims to maximize the joint surplus of the two players on her team. The planner may have to take into account of the well-being of the players to maintain her leadership. Lemmata 1-3 and Theorem 1 allow us to write down the following payoff matrix for the two planners, which provides the joint surplus of the two players on each team under every possible circumstance. The detailed derivation of the matrix is provided in Appendix.

		Team B	
		weaker first	stronger first
Team A	weaker first	$2\varphi v - \frac{3}{2}v, \frac{1}{2\varphi}v$	$2\varphi v - \frac{3}{2k}v, \frac{1}{2\varphi k}v$
	stronger first	$2\varphi v - \frac{3k}{2}v, \frac{k}{2\varphi}v$ if $k \leq \varphi$; $\frac{\varphi^2}{2k}v, 2v - \frac{3\varphi}{2k}v$ if $k > \varphi$	$2\varphi v - \frac{3}{2}v, \frac{1}{2\varphi}v$

Theorem 3 *Suppose that each planner aims to maximize the joint surplus of the players on her team. There exists a unique equilibrium in which both planners place their stronger players in late positions, as in the baseline model.*

The prediction does not differ from our baseline setup in which planners maximize the winning probabilities of their teams. We now consider an alternative scenario. Assume that each team, as a stand-alone entity, has a valuation of the prize $v_i \geq \varphi_i v, i = \{A, B\}$ independent of that of the players, while the players' input incurs cost to the team. Consider, for instance, a technological breakthrough accrues to the profit of a firm, while it also marks scientists' career success: The values could differ between the firm and its scientists. Let each planner maximize the profit of the team. Then the payoff matrix is given by the following, and its detailed derivation is relegated to Appendix.

Team B

		weaker first	stronger first
Team A	weaker first	$(1-\frac{1}{2\varphi})v_A - \frac{v}{2}, \frac{1}{2\varphi}(v_B - v)$	$(1-\frac{1}{2\varphi k})v_A - \frac{v}{2k}, \frac{1}{2\varphi k}(v_B - v)$
	stronger first	$(1-\frac{k}{2\varphi})v_A - \frac{k}{2}v, \frac{k}{2\varphi}(v_B - v)$ if $k \leq \varphi$; $\frac{\varphi}{2k}(v_A - \varphi v), (1-\frac{\varphi}{2k})v_B - \frac{\varphi}{2k}v$ if $k > \varphi$	$(1-\frac{1}{2\varphi})v_A - \frac{v}{2}, \frac{1}{2\varphi}(v_B - v)$

Theorem 4 *Suppose that each team is a stand-alone entity, has a valuation of the prize $\tilde{v} > v$ and the players' input incurs cost to the team. When each planner maximizes the profit of her team. There exists a unique equilibrium in which both planners place their stronger players in late positions, as in the baseline model.*

Again, the game ends up with a unique equilibrium in which both planners place their stronger players in late positions, which preserves the prediction obtained in the baseline setting.

4 Extensions and Robustness

In this part, we consider a few extensions to explore the boundary of our main results, which also allow us to understand the strategic nature of the game in greater depth. We first discuss the nuance caused by large intra-team heterogeneity, i.e., $k \geq 2$. We also consider an extension in which players are not subject to capacity constraints—i.e., a player is allowed to show up in both periods—and a planner can change her placement decision after observing first-period results. We demonstrate that multiple equilibria exist, while our prediction continues to be one of the equilibrium outcomes. We then explore a case in which a team's performance in the two periods counts different in the scoring rule. Finally, we discuss briefly the robustness of our results to a few other possible variations in modelling.

4.1 Large Heterogeneity

Our main analysis has focused on the case of moderate heterogeneity, i.e., $k < 2$. We now discuss the situation with large heterogeneity. The rationale presented in Section 3.3 explains why preemption is excessively costly, which leads to zero bidding in the early stage. The same argument, however, implies that such an equilibrium would not necessarily hold if $k > 2$. Imagine that both teams place their stronger players in the first period. Obviously, a lead $\frac{v}{k}$ is less costly for player $A(1)$ when k increases, which temps him to bid aggressively to preempt the opponents.

With substantial intra-team heterogeneity ($k > 2$), inter-team heterogeneity ($\varphi > 1$) no longer plays a passive role. We first consider a case of large inter-team heterogeneity, with $\varphi > k$. The result is formally stated as follows.

Lemma 4 *When $k > 2$ and $\varphi > k$, the two-stage team contest has a unique subgame perfect bidding equilibrium in pure strategy:*

(i) When team A, the stronger team, places its stronger player in the late position, early players both bid zero, regardless of team B's placement;

(ii) when team A places its stronger player in the early position, the player makes a preemptive bid $\frac{v}{k_{B(2)}}$, regardless of team B's placement, and other players all bid zero.

By Lemma 4, the stronger player of team A, if placed in the early position, would preempt regardless of the rival team's placement. The players on team B simply gives up and responds by zero bid. The logic is straightforward: A large φ gives the stronger player on team A an excessive advantage and prevents rival players from thwarting his preemptive attempt.

Lemma 4 thus allows us to conclude the ordering outcome of the game when $k > 2$ and $\varphi > k$. Suppose that team A places its stronger player in the late position. Team B would win the contest with positive probabilities, because early players make zero efforts, and the outcome of the contest depends only on the bidding competition in the second stage. Obviously, placing its stronger player in the early position is a dominant strategy for team A, which is formally stated as follows.

Theorem 5 *When both intra- and inter-team heterogeneities are sufficiently large, i.e., $k > 2$ and $\varphi > k$, in a subgame perfect equilibrium of the ordering game, team A, the stronger team, must place its stronger player in the early position. Team A wins the contest with probability one, regardless of the rival team's placement decision.*

We can readily reconsider the hypothetical design question laid out in Section 3.2. Let a designer enforce players' ordering. The following is straightforward.

Corollary 2 *When both intra- and inter-team heterogeneities are sufficiently large, i.e., $k > 2$ and $\varphi > k$, a contest designer—who maximizes the total effort or aggregate performance of the contest—would require that team A place its stronger player in the early position, and team B do the opposite. That is, Scenario 4 generates the highest total effort.*

No formal proof is needed. When team A places its stronger player in the early slot, he will bid as much as the maximum player B(2) is willing to bid, and he is the sole supplier of effort in the contest. When team B places the weaker player in the late position, the equilibrium bid would be v/k; when team B places the stronger player in the late position, the equilibrium bid would be v. In the latter scenario, the rent v is fully dissipated, which is unlikely otherwise. We illustrate this result in the following numerical example, which sets $\varphi = 4, k = 3$.

	winning odds	A(2)'s effort	B(2)'s effort	total efforts
Scenario 1	$\frac{7}{8}, \frac{1}{8}$	$\frac{1}{2}$	$\frac{1}{8}$	$\frac{5}{8}$
Scenario 2 (equilibrium)	1,0	$\frac{1}{3}$	0	$\frac{1}{3}$
Scenario 3	$\frac{23}{24}, \frac{1}{24}$	$\frac{1}{6}$	$\frac{1}{72}$	$\frac{13}{72}$
Scenario 4 (equibrium/optimum)	$1, 0$	1	0	1

Table 3: The setting of $\varphi = 4, k = 3$

Both Scenarios 2 and 4 can be equilibra. If team A places its stronger player in the early position, ordering decision is no longer relevant to team B, as A(1) must preempt. However, Scenario 4 generates a higher amount of effort, as it costs more for A(1) to preempt.

The case of moderate inter-team heterogeneity ($\varphi < k$), however, involves substantially more complications. Imagine, again, a situation in which both teams place their stronger players in the first period. A large k makes it less costly for the two strong players—i.e., both $A(1)$ and $B(1)$—to preempt late competition in the early period. A small φ implies a close match between teams. Hence, it would be difficult for player $A(1)$ to force his competent opponent $B(1)$ to concede—i.e., bidding zero—in response to his preemptive bid. Player $B(1)$, with similar incentive to player $A(1)$ in terms of both winning value ($\varphi < k$) and bidding cost, can choose either to "retaliate" with a preemptive bid or simply raise his bid modestly from zero to maintain a nonexcessive lag, which keeps the contest unsettled. We obtain the following.

Claim 1 *Consider a contest in which both teams place their stronger players in early positions. For large intra-team heterogeneity and moderate inter-team heterogeneity, i.e., $k > 2$ and $\varphi < k$, there exists no equilibrium in which early players place their bids in pure strategies. However, there exist Nash equilibria in which the two early players bid in mixed strategies.*

Proof. See Appendix. ■

Suppose that teams place their stronger player in early positions. Claim 1 states that with substantial intra-team heterogeneity ($k > 2$) and moderate inter-team heterogeneity ($\varphi < k$), a pure-strategy bidding equilibrium would not persist. A Nash equilibrium must exist because early players' payoff function (1) are obviously continuous, although there are kinked points. The existence is directly implied by Glicksberg (1952) and Fudenberg and Tirole (1991; Theorem 1.3, p. 35). Randomized bidding would emerge in the early period given the nonexistence of pure-strategy equilibrium.

In this case, the bidding competition in the early period involves a more complex interaction than standard contests. A higher bidder does not necessarily win, which renders the competition an imperfectly discriminatory contest. This resembles Tullock contests and tournaments with additive noises (Lazear and Rosen, 1981), but the winner is selected through a noncanonical contest success function. When $k < 2$, the possibility of preemption is ruled out. This ensures that players' best responses are well-behaved, which substantially simplifies the search for equilibrium. The same, however, no longer holds when $k > 2$. A player is tempted to preempt late competition; this causes jump in their best responses, and the regularity of the game fades away.

In an all-pay auction, a player wins (loses) with certainty when he outbids (underbids) his opponent, which allows one to pin down players' equilibrium payoffs. This feature is missing in our setting, and it is thus difficult to pin down early players' bidding supports and equilibrium payoffs for the case of $k > 2$ and $\varphi < k$. In this sense, the bidding game is closely related to the all-pay auction games studied by Gelder, Kovenock, and Roberson (2015) and Gelder, Kovenock, and Sheremeta (2015) that allow for a tie. In the two studies, a bidder has to overbid his opponent above a margin to secure a win. However, they assume that when the difference between two players' bids remains below a cutoff, the rent is divided between them by a fixed sharing rule regardless of the actual bidding differential. In contrast, an early player's payoff in our setting is continuous in bidding differential, while the kinked points is determined endogenously by the bid placed by the other. Players' mutual best responses are then elusive.[10] Such a bidding game has not been systematically studied in the contest literature, but warrants future research.

For the same reason, the approach of Alcalde and Dahm (2010) would not help with the current game. They show that players randomize between zero bid and an all-pay-auction bidding strategy in Tullock contests with large discriminatory power. As stated above, placing zero bid to exit is not necessarily a best response to a large bid placed by the rival. As discussed above, one may place a small bid to ensure that the contest remains unsettled, and the optimal bid depends on the size of the bid placed by the opponent. By the same token, a bid up to the rival team's late player's effective prize does not guarantee a win, because of the rival player's ability to place a small bid to keep his teammate in the competition. As a result, we are unable to reduce players' strategy space, which also prevents us from pinning down players' equilibrium payoffs and bidding supports, as stated above.

4.2 When Players Do Not Have Capacity Constraints

In our baseline setting, we implicitly assume that (1) players have capacity constraints and each can perform in only one period of the contest; and (2) a team planner is able to commit to her placement decision, in that she cannot change her choice of late player after the first-period competition. Our model can be viewed as a simple but reasonable abstraction of a

[10] In a standard all-pay auction, one can secure a sure win by bidding as much as the opponent's valuation.

situation in which a player on a team is unable to spread himself over multiple assignments, and a placement plan cannot easily be changed.[11] We now relax these restrictions: A player is allowed to show up in both periods; furthermore, a planner can change her player placement decision at any time.

We focus on the case of moderate intra-team heterogeneity, i.e., $k < 2$. In this part, we examine whether the equilibrium outcome in our baseline setting will persist in the alternative setting. Obviously, the stronger player on each team will show up in the late period of this contest. It is a dominant strategy for a planner to utilize her stronger player in late stage, regardless of the initial placement and interim outcome. We only need to focus on planners' placement plan for the early period. There are altogether four possible scenarios in the early period:

1. Both teams place their weaker players in the early period;

2. Team A (the stronger team) places its weaker player in the early period, while team B (the weaker team) assigns its stronger player to this slot;

3. Team A (the stronger team) places its stronger player in the early period, while team B (the weaker team) assigns its weaker player to this slot;

4. Both teams place their stronger players in early slots.

In scenario 1, the game's outcome coincides with the equilibrium of the main setting. Both of the early players will bid zero, and the contest is settled solely by the late competition. As a result, team A wins with a probability $1 - \frac{1}{2\varphi}$, while team B wins with the complementary probability $\frac{1}{2\varphi}$. The same can be expected in scenario 2: The early player on team B (its strong player) will not attempt to preempt, which requires a bid of at least φv. We show in the Appendix that he will not place a positive bid in the first period to give himself a headstart in the second period. The contest will, again, be settled solely by the second-period competition between the two stronger players.

In scenario 4, the two strong players perform in both periods. The contest boils down to a head-to-head competition between two individual players. Hirata (2014) studies this situation. He shows that there exists a continuum of equilibria: In the early period, player $B(1)$ bids zero with probability one, while player $A(1)$ places a constant bid $b \in [0, v]$. These equilibria differ in their outcomes. When player $A(1)$ bids $b \geq 0$, he gives himself a headstart in the second-period competition, which allows him to win the contest with

[11] For instance, a celebrity—who serves as a surrogate for a politician—can presumably be subject to tight schedule, which must be planned ahead a long while ago and cannot easily be changed. For instance, there is a large demand from Republican candidates for Donald Trump's support. But not all these demands can be entertained, and many Senate Republicans had to resort to the president's son to rally for voters' support.

a probability $1 - \frac{1}{2\varphi v^2}(v^2 - b^2)$. Obviously, the winning probability strictly increases with b. In the equilibrium with $b = 0$, however, the outcome of the contest coincides with the equilibrium in the baseline setting.

The game play in scenario 3 is no different from that in scenario 4. The weaker player on team B will bid zero with probability one in the early period. The strong player on team A—who will show up in both periods—may or may not place positive bid in the early period to give himself a headstart for the late-period competition against the strong player on the opposing team. Again, a continuum of equilibria emerge; in each of them, the stronger player on team A bids $b \in [0, v]$ in the early period.

It is obvious that the placement of early player is no longer a relevant decision for the planner of the weaker team, as the early player on the team always bids zero. The outcome of the contest depends on not only the placement of team A but also which equilibrium will be played out in subgames with multiple bidding equilibria: When the stronger player of team A is placed in the early slot, there exist a continuum of equilibria; in each of them, he bids $b \in [0, v]$ in the early period of the contest. We obtain the following.

Theorem 6 *Multiple subgame perfect equilibria exist in the game. Each of the above four scenarios can be a part of an equilibrium for player ordering. In particular, there exists a subgame perfect equilibrium, in which both teams place their weak players in the early slot (scenario 1).*

An equilibrium exists that features scenario 1. The equilibrium outcome coincides with that of the game in our baseline setting: Both teams place their weak players in early slots, and the outcome of the contest is determined solely by the head-to-head competition between the two strong players in the second period. The emergence of this equilibrium requires that in subgames in which team A places its strong player in the early period, the player, in the early period of the contest, would bid zero. In such a scenario, the planner of team A is indifferent between placing the stronger player in early slot and placing the weak.

Hence, we conclude that the equilibrium player assignment profile obtained in the baseline setting remains a subgame perfect equilibrium even if players have no capacity constraints and planners cannot commit to player assignment plans. Of course, multiple equilibria may arise. The subgame equilibrium of the player assignment game depends on which bidding equilibrium will be played when the stronger team places its strong player in the early period.

4.3 When Early and Late Output Counts Differently

Our baseline setup assumes that a team's output is the sum of its players' output $x_{i(1)}$ and $x_{i(2)}$. The scoring rule implicitly assumes that for each team, early and late performance equally contribute to its overall output. We now consider a variation in which this assumption is relaxed. We allow for a more flexible and general aggregating rule. Specifically, we assume that the effective output of an early player is given by $hx_{i(1)}$ for a team i, $i \in \{A, B\}$, with $h > 0$: When $h > (<)1$, early (late) performance weighs more for the team's success. So a team i's overall performance is given by the weighted sum $hx_{i(1)} + x_{i(2)}$.

The analysis would not be substantially different from that in our baseline setting. Lemmata 1-3 continue to hold. However, the headstart Δ now is redefined accordingly, as $\Delta = h(x_{j(1)} - x_{i(1)})$. The payoff functions of early player do not fundamentally differ from (1), although one's output is scaled up or down by the factor h:

$$\pi_{i(1)}(x_{i(1)}, x_{j(1)}) = p_i \varphi_i v - k_{i(1)} x_{i(1)}$$
$$= \begin{cases} \left[1 - \frac{1}{2v_{i(2)}v_{j(2)}}\left(v_{j(2)}^2 - \Delta^2\right)\right]\varphi_i v - k_{i(1)}x_{i(1)} & \text{if } x_{i(1)} \in (x_{j(1)}, x_{j(1)} + \frac{v_{j(2)}}{h}); \\ \left(1 - \frac{v_{j(2)}}{2v_{i(2)}}\right)\varphi_i v - k_{i(1)}x_{i(1)} & \text{if } x_{i(1)} \in [x_{j(1)} - \frac{v_{i(2)}}{h} + \frac{v_{j(2)}}{h}, x_{j(1)}]; \\ [\frac{1}{2v_{i(2)}v_{j(2)}}\left(v_{i(2)}^2 - \Delta^2\right)]\varphi_i v - k_{i(1)}x_{i(1)} & \text{if } x_{i(1)} \in (x_{j(1)} - \frac{v_{i(2)}}{h}, x_{j(1)} - \frac{v_{i(2)}}{h} + \frac{v_{j(2)}}{h}). \end{cases} \quad (4)$$

Despite the presence of the factor h, the trade-off for an early player is not altered: He either bids aggressively to preempt future competition, or lowers his bid to merely ensure the continuation of the competition. We obtain the following.

Theorem 7 *When $hk < 2$, under any given placement profile, a contest has a unique subgame perfect equilibrium with pure-strategy bidding in the first period, in which players $A(1)$ and $B(1)$ exert zero effort, and no nonzero headstart is carried over to period 2, i.e., $\Delta^* = 0$. The game, at the placement stage, has a unique subgame perfect equilibrium in which both teams place their stronger players in the late period.*

The result shows that the prediction of our baseline setting would remain if $hk < 2$. As a result, the prediction is more likely to be preserved when h is small. When $h > 1$, i.e., early output counts more for a team's overall output, which encourages preemption: player $i(1)$ only needs to create a lead of $(-\Delta) = \frac{v_{j(2)}}{h}$ to preempt, which decreases with h.

4.4 Other Extensions

Our analysis can be extended in a number of additional directions. First, we may model the competition as an imperfectly discriminatory contest, in which a higher effort cannot guarantee a win. Our prediction would persist in a Tullock contest, which is frequently adopted to model noisy contests. In such setting, an early player's effort would be offset by his late teammate's shirking and does not affect the team's winning odds. As a result, early players have no incentive to place positive bids, which preserves the prediction we obtain in the all-pay auction setting.

Second, we have assumed that players on a team sequentially commit to their efforts and the performance in period 1 is observable to the late players. It would be interesting to explore the equilibrium bidding behavior when early players' performance is unobservable when late performers decide on their efforts. The contest thus reduces to a simultaneous bidding game. The literature, however, has yet to provide a general analysis of complete-information simultaneous-move all-pay auctions with perfectly substitutable efforts, due to the difficulty of characterizing mixed-bidding strategies in groups. Multiple equilibria may emerge, but a simple equilibrium exists in which only the strong player on each team makes positive bid. The outcome of the contest in this equilibrium thus coincides with that in our baseline model.

Third, we assume a linear effort cost function. Our result would also be robust when players' effort cost functions are strictly convex. In fact, our results would be strengthened under this variation, in the sense that the equilibrium with zero bidding in the early period would remain in place even if a large intra-team heterogeneity is present. As a result, the outcome of the contest, again, be determined solely by the competition in the second period. Both planners will place their stronger players in the early slots in the unique equilibrium.

Fourth, our results will qualitatively hold when we relax the assumption on the structure imposed on players' heterogeneity. In our main setting, players across team differ in their valuations, which is measured by the parameter φ, while players within a team differ in their marginal effort costs, which is measured by the parameter k. A similar result can be obtained in an alternative setting. Let the four players all bear a unitary marginal effort. Players' heterogeneity, however, is encapsulate in their valuations. Let their valuations of the prize be $v_1 > v_2 > v_3 > v_4$: the two players on team A have valuations v_1 and v_3, and the two on team B have valuations v_2 and v_4; so team A remains stronger. The prediction of Theorem 1 is well preserved: Both teams place their stronger players in the late positions, as long as intra-team heterogeneity remains mild—i.e., $v_1 < 2v_3$ and $v_2 < 2v_4$.

The analysis of these extensions is collected in an online appendix. In addition, our setting has assumed that each team consists of two players. A more general model would allow for more than two players on a team, and the ultimate outcome could depend more sensitively on the particular distribution of players' abilities within each team. A comprehensive study requires a full analysis of all possible subgames under the various possible placement profiles. The addition of another player causes substantial complexity in the strategic calculation: Preemption is less costly because additional players can join forces to create a large lead. This fact would substantially compound the analysis. However, it should be noted that the main logic of the game remains in place in the broader setting. First, the main prediction remains robust for moderate heterogeneity, because preemption remains costly when players are not excessively heterogeneous, although the exact cutoff would definitely vary. Second, consider a symmetric contests, assuming that symmetric equilibria prevail under symmetric placement, a team still gains an edge by placing its strong player in the last spot: By doing so, a team thwarts the rival team's attempt to preempt the contest prematurely, so the competition would definitely last until the late period, and the late competition decides the winner. The presence of a strong player in that period ensures a more favorable outcome. When inter-team heterogeneity is present, the favorite team is expected to be more likely to place its strongest player in the earliest period to preempt. The logic is consistent with that in our two-player-team model.

5 Concluding Remark

Our paper studies the endogenous ordering of players within teams in dynamic contests. We show that with moderate intra-team heterogeneity, a team always prefers to place its stronger player in the late position as the anchorman. In contrast, when intra-team heterogeneity is large and teams differ substantially in terms of their valuations—or, equivalently, their abilities—the stronger team places its stronger player in early position, which allows him to preempt late competition.

Our study leaves extensive room for future research. When intra-team heterogeneity is large and inter-team heterogeneity remains moderate, a noncanonical contest game arises in early-period bidding competitions under certain placement arrangements. Only mixed-strategy equilibrium exists, while previous analytical approaches in contest literature lose their bite. The setting poses a challenge for future research to explore equilibrium bidding behaviors in such settings. Furthermore, our analysis is limited to a two-period setting. Including additional periods and players would substantially expand team planners' strategy spaces and complicate the dynamics among players. The analysis is challenging, but warrants serious research.

Our study assumes that team members' efforts are perfectly substitutable. This is similar to the majority of studies of group contests based on imperfectly discriminatory contest models (Tullock contests), e.g., Esteban and Ray (2001, 2008) and Nitzan and Ueda (2009, 2011). In contrast, Barbieri, Malueg, and Topolyan (2014); Chowdhury, Lee, and Topolyan (2016); and Chowdhury and Topolyan (2016) consider complete-information perfectly discriminatory contests (all-pay auctions), assuming that a team's overall output is determined by either the "best shot"—i.e., the highest bid placed by its team member—or the "weakest link"—i.e., the lowest bid placed within the team. It is interesting to explore dynamic team contests with other team production functions.

We assume that a planner's placement decision is deterministic. It would be interesting to explore a situation in which a planner can randomize on her placement, and the placement profile is not publicly announced before the contest begins. This setting leads to subtle strategic interactions. It deserves to be explored in future research.

Appendix

A Generic Property of the Bidding Game in Period 1

The following result depicts a generic property of the equilibrium plays in period 1. The lemma, as well as its proof, lays a foundation for all subsequent analyses. Note that the claim does not depend on the assumption of $k < 2$.

Lemma A0 *Under any given placement profile, there exists no pure-strategy equilibrium in the early period in which both players make strictly positive bids, for all* $\varphi, k \geq 1$.

Proof. Suppose otherwise that there exists an equilibrium in which players $A(1)$ and $B(1)$ make strictly positive bids $x^*_{A(1)}$ and $x^*_{B(1)}$ with probability one. Note that for $x^*_{A(1)}$ and $x^*_{B(1)}$ to be a part of an equilibrium, we must have $\Delta^* = x^*_{j(1)} - x^*_{i(1)} \in (-v_{j(2)}, v_{i(2)})$ where i and j are defined as in the main text. Suppose $\Delta^* \notin (-v_{j(2)}, v_{i(2)})$, we have both the two players in the second period exert zero effort. Note that $\Delta^* \notin (-v_{j(2)}, v_{i(2)})$ means $x^*_{A(1)} \neq x^*_{B(1)}$. Clearly, the lower bidder has incentive to deviate to a zero bid. We thus focus on the $x^*_{j(1)}$ and $x^*_{i(1)}$ such that $\Delta^* = x^*_{j(1)} - x^*_{i(1)} \in (-v_{j(2)}, v_{i(2)})$ for candidate equilibrium, and verify Lemma A0 by contradiction.

Recall that in the second period $v_{i(2)} \geq v_{j(2)}$ by construction. Imagine a hypothetical equilibrium with $x^*_{i(1)}, x^*_{j(1)} > 0$. We first consider player $i(1)$'s best response to $x^*_{j(1)}$, and establish that his optimal response might be strictly lower than $x^*_{j(1)}$.

Suppose $x^*_{i(1)} \geq x^*_{j(1)}$. For $x_{i(1)} \geq x^*_{j(1)}$, from (2), $i(1)$'s marginal payoff is positive only if $\frac{k_{i(2)}}{v_{j(2)}}(-\Delta) - k_{i(1)} > 0 \Leftrightarrow (x_{i(1)} - x^*_{j(1)}) > \frac{k_{i(1)}v_{j(2)}}{k_{i(2)}}$. Player $i(1)$'s payoff strictly increases when his bid $x_{i(1)}$ is above $x^*_{j(1)} + \frac{k_{i(1)}v_{j(2)}}{k_{i(2)}}$ until it reaches the bound $x^*_{j(1)} + v_{j(2)}$, if $x^*_{j(1)} + v_{j(2)} \geq x^*_{j(1)} + \frac{k_{i(1)}v_{j(2)}}{k_{i(2)}}$, i.e. $k_{i(2)} \geq k_{i(1)}$. Hence, if $x^*_{j(1)} + v_{j(2)} \geq x^*_{j(1)} + \frac{k_{i(1)}v_{j(2)}}{k_{i(2)}}$, i.e. $k_{i(2)} \geq k_{i(1)}$, then $x^*_{j(1)} + v_{j(2)}$ must be the best response of $i(1)$, i.e. $x^*_{i(1)} = x^*_{j(1)} + v_{j(2)}$. Note bidding $x^*_{j(1)} + v_{j(2)}$ allows $i(1)$ to preempt the second period competition and lets the team win with probability 1. This means $x^*_{j(1)} > 0$ cannot be the optimal response to $x^*_{i(1)}$, which conflicts with the hypothetical equilibrium $(x^*_{i(1)}, x^*_{j(1)})$.

If $x^*_{j(1)} + v_{j(2)} < x^*_{j(1)} + \frac{k_{i(1)}v_{j(2)}}{k_{i(2)}}$, i.e. $k_{i(2)} < k_{i(1)}$, $i(1)$'s best response must be less than $x^*_{j(1)}$, because his marginal payoff is strictly negative for $x_{i(1)} \in [x^*_{j(1)}, x^*_{j(1)} + v_{j(2)}]$. This immediately conflicts with assumption $x^*_{i(1)} \geq x^*_{j(1)}$.

We thus conclude that for positive $x^*_{i(1)}$ and $x^*_{j(1)}$ to be a part of an equilibrium, we must have $x^*_{i(1)} < x^*_{j(1)}$. We next show we also must have $x^*_{j(1)} < x^*_{i(1)}$ by considering the optimal response of player $j(1)$.

Suppose $x^*_{j(1)} \geq x^*_{i(1)}$. For $x_{j(1)} \geq x^*_{i(1)}$, by (3), player $j(1)$'s marginal payoff is positive only if $\Delta = x_{j(1)} - x^*_{i(1)} > \frac{k_{j(1)}v_{i(2)}}{k_{j(2)}}$, i.e. $x_{j(1)} > x^*_{i(1)} + \frac{k_{j(1)}v_{i(2)}}{k_{j(2)}}$. Note that player $j(1)$ has no incentive to bid more than $x^*_{i(1)} + v_{i(2)}$, which is sufficient to preempt the second period competition. If $x^*_{i(1)} + \frac{k_{j(1)}v_{i(2)}}{k_{j(2)}} \leq x^*_{i(1)} + v_{i(2)}$, i.e. $k_{j(1)} \leq k_{j(2)}$, then $x^*_{i(1)} + v_{i(2)}$ must be the best response, i.e. $x^*_{j(1)} = x^*_{i(1)} + v_{i(2)}$, which allows $j(1)$ to preempt late competition and lets the team win with probability 1. This means $x^*_{i(1)} > 0$ cannot be the optimal response to $x^*_{j(1)}$, which conflicts to the hypothetical equilibrium $(x^*_{i(1)}, x^*_{j(1)})$.

If $x^*_{i(1)} + \frac{k_{j(1)}v_{i(2)}}{k_{j(2)}} > x^*_{i(1)} + v_{i(2)}$, i.e. $k_{j(1)} > k_{j(2)}$, the player $j(1)$'s best response must be less than $x^*_{i(1)}$, because his marginal payoff is strictly negative for $x_{j(1)} \in [x^*_{i(1)}, x^*_{i(1)} + v_{i(2)} - v_{j(2)}]$. This immediately conflicts with assumption $x^*_{j(1)} \geq x^*_{i(1)}$.

We thus conclude that for positive $x^*_{i(1)}$ and $x^*_{j(1)}$ to be a part of an equilibrium, we must have $x^*_{j(1)} < x^*_{i(1)}$.

In summary, we observe that such a hypothetical equilibrium requires both $x^*_{i(1)} < x^*_{j(1)}$ and $x^*_{j(1)} < x^*_{i(1)}$ hold at the same time. Contradiction thus arises. This completes the proof. ■

Early Players' Marginal Payoffs of Efforts

By the payoff function (1), we can obtain early players' marginal payoff functions for $\Delta \in (-v_{j(2)}, v_{i(2)})$:

$$\begin{aligned}
\frac{d\pi_{i(1)}}{dx_{i(1)}} &= \frac{dp_i}{dx_{i(1)}}\varphi_i v - k_{i(1)} \\
&= \begin{cases} -\frac{k_{i(2)}}{v_{j(2)}}\Delta - k_{i(1)} & \text{if } x_{i(1)} \in (x_{j(1)}, x_{j(1)} + v_{j(2)}); \\ -k_{i(1)} & \text{if } x_{i(1)} \in [x_{j(1)} - v_{i(2)} + v_{j(2)}, x_{j(1)}]; \\ \frac{k_{i(2)}}{v_{j(2)}}\Delta - k_{i(1)} & \text{if } x_{i(1)} \in (x_{j(1)} - v_{i(2)}, x_{j(1)} - v_{i(2)} + v_{j(2)}); \end{cases}
\end{aligned} \tag{5}$$

and

$$\frac{d\pi_{j(1)}}{dx_{j(1)}} = \begin{cases} -\frac{k_{j(2)}}{v_{i(2)}}\Delta - k_{j(1)} & \text{if } x_{j(1)} \in (x_{i(1)} - v_{j(2)}, x_{i(1)}); \\ -k_{j(1)} & \text{if } x_{j(1)} \in [x_{i(1)}, x_{i(1)} + v_{i(2)} - v_{j(2)}]; \\ \frac{k_{j(2)}}{v_{i(2)}}\Delta - k_{j(1)} & \text{if } x_{j(1)} \in (x_{i(1)} + v_{i(2)} - v_{j(2)}, x_{i(1)} + v_{i(2)}). \end{cases} \tag{6}$$

Proof of Theorem 1

Proof. In total, there are four scenarios, as stated in the paper. Lemma A0 reveals that there is no equilibrium such that both players in the first period bid positively. This implies at least one player must bid at zero at equilibrium. We first show that the optimal response of the other player must be zero or the minimum level to preempt the other team regardless of k.

Assume $x^*_{j(1)} = 0$. For $x_{i(1)} \geq 0$, from (2), $i(1)$'s marginal payoff is positive only if $\frac{k_{i(2)}}{v_{j(2)}}(-\Delta) - k_{i(1)} > 0 \Leftrightarrow x_{i(1)} > \frac{k_{i(1)}v_{j(2)}}{k_{i(2)}}$. Player $i(1)$'s payoff strictly increases when his bid $x_{i(1)}$ is above $\frac{k_{i(1)}v_{j(2)}}{k_{i(2)}}$ until it reaches the bound $v_{j(2)}$, if $v_{j(2)} \geq \frac{k_{i(1)}v_{j(2)}}{k_{i(2)}}$, i.e. $k_{i(2)} \geq k_{i(1)}$. Hence, if $v_{j(2)} \geq \frac{k_{i(1)}v_{j(2)}}{k_{i(2)}}$, i.e. $k_{i(2)} \geq k_{i(1)}$, then $v_{j(2)}$ must be the best response of $i(1)$ for the range $x_{i(1)} > \frac{k_{i(1)}v_{j(2)}}{k_{i(2)}}$. For the range $x_{i(1)} \leq \frac{k_{i(1)}v_{j(2)}}{k_{i(2)}}$, $i(1)$'s payoff strictly decreases when his bid $x_{i(1)}$, which means that zero must be the best response of $i(1)$ for the range $x_{i(1)} \leq \frac{k_{i(1)}v_{j(2)}}{k_{i(2)}}$. This means the best response of $i(1)$ must be either $v_{j(2)}$ or 0.

If $v_{j(2)} < \frac{k_{i(1)}v_{j(2)}}{k_{i(2)}}$, i.e. $k_{i(2)} < k_{i(1)}$, $i(1)$'s best response must be 0, because his marginal payoff is strictly negative for $x_{i(1)} \in [x^*_{j(1)}, x^*_{j(1)} + v_{j(2)}] = [0, v_{j(2)}]$.

Similarly, we can show that when $x^*_{i(1)} = 0$, then $x^*_{j(1)} = v_{i(2)}$ or 0 if $k_{j(1)} \leq k_{j(2)}$; and $x^*_{j(1)} = 0$ if $k_{j(1)} > k_{j(2)}$.

We next show that for $k < 2$, we can eliminate the possibility of $x^*_{i(1)} = v_{j(2)}$ when $k_{i(2)} \geq k_{i(1)}$, and the possibility of $x^*_{j(1)} = v_{i(2)}$ when $k_{j(2)} \geq k_{j(1)}$.

There are three relevant scenarios. In the first scenario, both teams place their weaker players in late positions. Assume $x_{B(1)} = 0$. For player $A(1)$, preemption requires $\frac{v}{k}$, which gives a payoff $\varphi v - \frac{v}{k}$. If he simply bids 0, his payoff is $(1 - \frac{1}{2\varphi})\varphi v = \varphi v - \frac{v}{2} > \varphi v - \frac{v}{k}$, because $k < 2$. Assume $x_{A(1)} = 0$. For player $B(1)$, preemption requires $\frac{\varphi v}{k}$. This does not make sense if $\varphi > k$ as it leads to negative payoff to $B(1)$. Suppose $\varphi \leq k$. His payoff from preemption would be $v - \frac{\varphi v}{k}$. If he bids 0, his payoff would be $\frac{1}{2\varphi}v$, which is strictly greater than $v - \frac{\varphi v}{k}$, because $1 - \frac{\varphi}{k} < \frac{1}{2\varphi}$. To see that, $1 - \frac{\varphi}{k} - \frac{1}{2\varphi} = \frac{2\varphi k - 2\varphi^2 - k}{2\varphi k}$. Note $\frac{d(2\varphi k - 2\varphi^2 - k)}{d\varphi} = 2k - 4\varphi < 0$ as $k < 2$ and $\varphi > 1$. Therefore, $2\varphi k - 2\varphi^2 - k < (2\varphi k - 2\varphi^2 - k)|_{\varphi=1} = k - 2 < 0$.

In the second scenario, team A places its stronger player in late position, while team B does the opposite. We only need to consider player $B(1)$. Assume $x_{A(1)} = 0$. For player $B(1)$, preemption requires φv, which leads to negative payoff.

In the third scenario, team A places its weaker player in late position, while team B does the opposite. We only need to consider player $A(1)$. Assume $x_{B(1)} = 0$. We need to discuss two possible cases. First, assume $\varphi > k$. For player $A(1)$ to preempt, he has to bid v, which gives a payoff $\varphi v - v$. If he bids 0, the payoff is $(1 - \frac{k}{2\varphi})\varphi v = \varphi v - \frac{k}{2}v > \varphi v - v$, because $k < 2$. Second, assume $\varphi \leq k$. For player $A(1)$ to preempt, he has to bid v, which gives a payoff $\varphi v - v$. If he bids 0, the payoff is $\frac{\varphi}{2k} \cdot \varphi v$. We claim the latter generates a better payoff. To see that, $\frac{\varphi}{2k} \cdot \varphi v - (\varphi v - v) = \frac{2k - 2k\varphi + \varphi^2}{2k}v = \frac{(2k - k^2) + (k^2 - 2k\varphi + \varphi^2)}{2k}v > 0$, because $k < 2$.

We thus conclude that in the first period, if one player exerts zero effort, then the other must exert zero effort. As a result, exerting zero effort is the unique pure strategy equilibrium regardless the placement of players. ■

5.1 Proof of Corollary 1

Proof. We only need to compare two possible cases.

First, suppose that both teams place their strong players in late slots. By Lemma 2, player $A(2)$ exerts an expected effort $\frac{v}{2}$, while player $B(2)$ does $\frac{1}{2\varphi}v$. The total effort is $\frac{v}{2}(1+\frac{1}{\varphi})$.

Second, suppose that team A places its weak players in the late slot, while team B does the strong one.

Suppose $\varphi \geq k$. Then Team A still has an upper hand although its weaker player faces the stronger on the rival team. Player $A(2)$ exerts an expected effort $\frac{v}{2}$. Team B wins with a probability $\frac{k}{2\varphi}$, and player $B(2)$ earns zero payoff. So player $B(2)$'s expected bid is $\frac{k}{2\varphi}v$. The expected total effort is $\frac{v}{2}+\frac{k}{2\varphi}v=\frac{v}{2}(1+\frac{k}{\varphi})$.

Suppose $\varphi < k$. Team A's advantage caused by larger valuation is more than offset by the disadvantage in effort cost. In this case, team A wins with a probability $\frac{\varphi v/k}{2v}=\frac{\varphi}{2k}<\frac{1}{2}$. Player A(2) earns zero payoff. So his expected effort is $\frac{\varphi}{2k^2}v$. Player B(2) exerts an expected effort $\frac{\varphi v/k}{2}=\frac{\varphi v}{2k}$. So the expected total effort is $\frac{\varphi}{2k^2}v+\frac{\varphi v}{2k}=\frac{v}{2}\cdot\frac{\varphi}{k}(\frac{1}{k}+1)$.

We then compare the two cases.

Suppose $\varphi < k$. Then $\frac{v}{2}(1+\frac{1}{\varphi}) > \frac{v}{2}\cdot\frac{\varphi}{k}(\frac{1}{k}+1)$, because $(1+\frac{1}{\varphi})-\frac{\varphi}{k}(\frac{1}{k}+1)=(1-\frac{\varphi}{k})+(\frac{1}{\varphi}-\frac{\varphi}{k}\frac{1}{k})>0$: $\frac{\varphi}{k}<1$ and $\frac{1}{\varphi}>\frac{1}{k}$.

Suppose $\varphi \geq k$. Then $\frac{v}{2}(1+\frac{k}{\varphi}) > \frac{v}{2}(1+\frac{1}{\varphi})$. ■

5.2 Proof of Theorems 3-4

We now derive the payoff matrices that are critical for the theorems.

5.2.1 Scenario for Theorem 3

Case 1: Both teams place weaker players in early slots.

The competition takes place only in the second period. By Lemma 2, team A wins with a probability $1-\frac{1}{2\varphi}$, and player $A(2)$ exerts an expected effort $\frac{v}{2}$. Hence, Team A's total surplus is

$$\underbrace{(1-\frac{1}{2\varphi})\varphi v}_{\text{player } A(1)\text{'s expected payoff}} + \underbrace{\left[(1-\frac{1}{2\varphi})\varphi v-\frac{v}{2}\right]}_{\text{player } A(2)\text{'s expected payoff}} = 2\varphi v-\frac{3}{2}v.$$

Team B wins with a probability $\frac{1}{2\varphi}$, and player $B(2)$ ends up with zero payoff. So the team's total surplus is $\frac{1}{2\varphi}v$, which exclusively accrues to the benefit of $B(2)$.

Case 2: Team A places the weaker player in the early slot, while Team B does the opposite.

The strong player on team A is matched to the weaker player on team B in the second-period competition. By Lemma 2, team A wins with a probability $1-\frac{1}{2\varphi k}$, and player $A(2)$'s expected bid is $\frac{v}{2k}$. So Team A's total surplus is $2(1-\frac{1}{2\varphi k})\varphi v - \frac{v}{2k} = 2\varphi v - \frac{3}{2k}v$.

Similar to Case 1, for Team B, only the early player receives positive payoff, which is $\frac{1}{2\varphi k}v$.

Case 3: Team A places its stronger player in the earlier slot, while Team B does the opposite.

Recall that in deriving Lemmata 1-3, we let i denote team whose late player being stronger in the competition. In this case, we need to compare φ and k to determine which player has an upper hand.

Suppose $\varphi \geq k$. Then Team A still has an upper hand although its weaker player faces the stronger on the rival team. Then team A wins with a probability $(1-\frac{k}{2\varphi})$. Player $A(2)$ exerts an expected effort $\frac{v}{2}$, so his expected effort cost is $\frac{kv}{2}$. The team's surplus is thus $2(1-\frac{k}{2\varphi})\varphi v - \frac{kv}{2} = 2\varphi v - \frac{3kv}{2}$. Team B wins with a probability $\frac{k}{2\varphi}$, and player $B(2)$ earns zero payoff. So the total surplus is $\frac{k}{2\varphi}v$.

Suppose $\varphi < k$. Team A's advantage caused by larger valuation is more than offset by the disadvantage in effort cost. In this case, team A wins with a probability $\frac{\varphi v/k}{2v} = \frac{\varphi}{2k} < \frac{1}{2}$. Player A(2) earns zero payoff. So the total surplus is $\frac{\varphi}{2k}\varphi v = \frac{\varphi^2}{2k}v$. Team B wins with a probability $1-\frac{\varphi}{2k}$. Player B(2) exerts an expected effort $\frac{\varphi v/k}{2} = \frac{\varphi v}{2k}$. So the total surplus is $2(1-\frac{\varphi}{2k})v - \frac{\varphi v}{2k} = 2v - \frac{3\varphi v}{2k}$.

Case 4: Both teams place their weaker stronger players in the early slots.

The case is similar to Case 1. The winning probabilities in the equilibrium are the same: Player B(2) wins with a probability $\frac{v/k}{2\varphi v/k} = \frac{1}{2\varphi}$. Player B(2) earns zero payoff. Player A(1) exerts an effort $\frac{v/k}{2} = \frac{v}{2k}$. As his marginal effort cost is k, the expected effort cost is $\frac{v}{2}$. Hence, team A's total surplus is $2(1-\frac{1}{2\varphi})\varphi v - \frac{v}{2} = 2\varphi v - \frac{3v}{2}$.

5.2.2 Scenario for Theorem 4

Case 1: Both teams place weaker players in early slots.

By Lemma 2, team A wins with a probability $1-\frac{1}{2\varphi}$, and player $A(2)$ exerts an expected effort $\frac{v}{2}$. So team A's payoff is $(1-\frac{1}{2\varphi})v_A - \frac{v}{2}$. Team B wins with a probability $\frac{1}{2\varphi}$, and player $B(2)$ exerts an expected effort $\frac{1}{2\varphi}v$. So team's payoff is $\frac{1}{2\varphi}v_B - \frac{1}{2\varphi}v = \frac{1}{2\varphi}(v_B - v)$.

Case 2: Team A places the weaker player in the early slot, while Team B does the opposite.

The strong player on team A is matched to the weaker player on team B in the second-period competition. By Lemma 2, team A wins with a probability $1-\frac{1}{2\varphi k}$, and player $A(2)$'s expected bid is $\frac{v}{2k}$. So Team A's payoff is $(1-\frac{1}{2\varphi k})v_A - \frac{v}{2k}$.

For Team B, it wins with a probability $\frac{1}{2\varphi k}$ and player B(2) exerts an expected effort $\frac{1}{2\varphi k^2}v$. The total effort cost $\frac{1}{2\varphi k}v$, which gives the team's payoff $\frac{1}{2\varphi k}(v_B - v)$.

Case 3: Team A places its stronger player in the earlier slot, while Team B does the opposite.

We let i denote team whose late player being stronger in the competition.

Suppose $\varphi \geq k$. Then Team A still has an upper hand although its weaker player faces the stronger on the rival team. Then team A wins with a probability $(1 - \frac{k}{2\varphi})$. Player $A(2)$ exerts an expected effort $\frac{v}{2}$, so his expected effort cost is $\frac{kv}{2}$. The team's payoff is thus $2(1 - \frac{k}{2\varphi})v_A - \frac{kv}{2}$. Team B wins with a probability $\frac{k}{2\varphi}$, and player $B(2)$ earns zero payoff. So the team's payoff is $\frac{k}{2\varphi}(v_B - v)$.

Suppose $\varphi < k$. Team A's advantage caused by larger valuation is more than offset by the disadvantage in effort cost. In this case, team A wins with a probability $\frac{\varphi v/k}{2v} = \frac{\varphi}{2k} < \frac{1}{2}$. Player A(2) earns zero payoff, so his effort cost must be $\frac{\varphi}{2k}\varphi v = \frac{\varphi^2}{2k}v$. So the team's payoff must be $\frac{\varphi}{2k}\varphi v = \frac{\varphi^2}{2k}v$.

Team B wins with a probability $1 - \frac{\varphi}{2k}$. Player B(2) exerts an expected effort $\frac{\varphi v/k}{2} = \frac{\varphi v}{2k}$. So the team's payoff is total surplus is $(1 - \frac{\varphi}{2k})v_B - \frac{\varphi}{2k}v$.

Case 4: Both teams place their weaker stronger players in the early slots.

The case is similar to Case 1. The winning probabilities in the equilibrium are the same: Player B(2) wins with a probability $\frac{1}{2\varphi}$. Player B(2) earns zero payoff. So team B's payoff is $\frac{1}{2\varphi}(v_B - v)$. Player A(1) exerts an effort $\frac{v/k}{2} = \frac{v}{2k}$. As his marginal effort cost is k, the expected effort cost is $\frac{v}{2}$. Hence, team A's payoff is $(1 - \frac{1}{2\varphi})v_A - \frac{v}{2}$.

Proof of Claim 1

Proof. By Lemma A0, if there is an equilibrium with pure strategies in early period, then at least one of the players must bid zero.

Suppose that player $B(1)$ bids zero with probability one. Based on the proof of Theorem 1, player $A(1)$ must choose between 0 and $\frac{v}{k}$. If he responds by bidding zero, he gets a payoff $(1 - \frac{1}{2\varphi})v$. If he instead bids $\frac{v}{k}$, he receives a payoff $(\varphi - \frac{1}{k})v$, which is strictly more than $(1 - \frac{1}{2\varphi})v$, because $\varphi - \frac{1}{k} - (1 - \frac{1}{2\varphi}) > \varphi - \frac{1}{k} - \varphi(1 - \frac{1}{2\varphi}) = \frac{1}{2} - \frac{1}{k} > 0$ for $k > 2$.

We next show that for player $B(1)$, bidding 0 is not optimal given that player $A(1)$ bids $\frac{v}{k}$. Suppose that $B(1)$ bids $\frac{v}{k} - \frac{\varphi v}{k^2}$, which is strictly less than $\frac{v}{k}$, which is $x_{A(1)}$. Because $\varphi < k$, $\frac{v}{k} - \frac{\varphi v}{k^2} > 0$, which is $x_{A(1)} - v_{B(2)}$ as $v_{B(2)} = \frac{v}{k}$. By placing the bid $\frac{v}{k} - \frac{\varphi v}{k^2}$, his payoff is given by

$$\begin{aligned}
\pi_{B(1)} &= \frac{k^2}{2\varphi v^2}\left[\frac{v^2}{k^2}-\left(\frac{\varphi v}{k^2}\right)^2\right]v-\left(\frac{v}{k}-\frac{\varphi v}{k^2}\right) \\
&= \left\{\frac{k^2}{2\varphi}\left[\frac{1}{k^2}-\left(\frac{\varphi}{k^2}\right)^2\right]-\left(\frac{1}{k}-\frac{\varphi}{k^2}\right)\right\}v \\
&= \left[\left(k^2-\varphi^2\right)-2\varphi\left(k-\varphi\right)\right]\frac{v}{2k^2\varphi} \\
&= \left(k^2-2k\varphi+\varphi^2\right)\frac{v}{2k^2\varphi} \\
&= (k-\varphi)^2\frac{v}{2k^2\varphi}>0.
\end{aligned}$$

This is sufficient to verify that zero is not a best response to $\frac{v}{k}$. Then there exists no pure-strategy equilibrium in which player $B(1)$ bids zero with probability one.

Suppose that player $A(1)$ bids zero with probability one. Based on the proof of Theorem 1, player $B(1)$ would respond by bidding either $\frac{\varphi v}{k}$ or zero. Suppose that his best response is zero. Then player $A(1)$ would bid $\frac{v}{k}$, as shown above. Then a pure-strategy equilibrium breaks down. If player $B(1)$ bids $\frac{\varphi v}{k}$, his payoff is $v-\frac{\varphi v}{k}$.

Then we consider the best response of $A(1)$ to $\frac{\varphi v}{k}$. By bidding zero, his payoff is zero. If he bids $\frac{v}{k}+\frac{\varphi v}{k}$, he preempts late competition, which gives him a payoff $\varphi v-(\frac{v}{k}+\frac{\varphi v}{k})=\varphi v(1-\frac{1}{k})-\frac{v}{k}$, which is strictly positive because, by $k>2$, $\varphi v(1-\frac{1}{k})-\frac{v}{k}>\frac{\varphi v}{k}-\frac{v}{k}\geq 0$. Hence, zero is never a best response.

In conclusion, no pure-strategy equilibrium exists in the contest. ■

Proof of Lemma 4

Proof. Note $\varphi>k>2$. There are four scenarios to consider.

Scenario I: Both two teams place their strong players in the first period.

According to Lemma A0, there are altogether three possibilities: Both two players $A(1)$ and $B(1)$ bid zero; $A(1)$ bids zero and $B(1)$ bids $\frac{\varphi v}{k}$; and $A(1)$ bids $\frac{v}{k}$ and $B(1)$ bids 0. Note that the proof of Theorem 1 has shown that when one player bids zero at period 1, the other must bid zero or preempt.

We first show that it cannot be an equilibrium that $A(1)$ bids zero and $B(1)$ bids $\frac{\varphi v}{k}$. This is clearly, otherwise, $B(1)$ gets negative payoff since $\varphi>k$.

We next show that it cannot be an equilibrium that both $A(1)$ and $B(1)$ bid zero. If $B(1)$ bids zero, then $A(1)$'s best response must be $\frac{v}{k}$. Preemption gives him a payoff $\varphi v-\frac{v}{k}$, strictly larger than the payoff $(1-\frac{1}{2\varphi})\varphi v$ from bidding zero, because $\frac{1}{2}>\frac{1}{k}$.

Finally, we show that it is an equilibrium that $A(1)$ bids $\frac{v}{k}$ and $B(1)$ bids 0. Based on the above discussion, we only need to show that bidding zero is $B(1)$'s best response when $A(1)$ bids $\frac{v}{k}$. Consider $x_{B(1)}\in(0,x_{A(1)}=\frac{v}{k})$. Player $B(1)$'s marginal payoff is positive if and only if $x_{B(1)} < \frac{v}{k} - \frac{\varphi v}{k^2} < 0$ since $\varphi > k$. Thus player $B(1)$'s payoff decreases with his bid until $x_{A(1)} = \frac{v}{k}$. Consider $x_{B(1)} \geq x_{A(1)} = \frac{v}{k}$. Player $B(1)$'s payoff decreases with his bid until $x_{A(1)} + v_{A(2)} - v_{B(2)} = \frac{v}{k} + \frac{\varphi v}{k} - \frac{v}{k} = \frac{\varphi v}{k}$. When $x_{B(1)} \geq x_{A(1)} + v_{A(2)} - v_{B(2)} = \frac{\varphi v}{k}$, $B(1)$'s marginal payoff is positive if and only if $x_{B(1)} > \frac{v}{k} + \frac{\varphi v}{k^2}$. Thus a candidate best response is ${}_{A(1)} + v_{A(2)} = \frac{v}{k} + \frac{\varphi v}{k} > \frac{v}{k} + \frac{\varphi v}{k^2}$. However, this preemptive bid leads to a payoff of $v - (\frac{v}{k} + \frac{\varphi v}{k}) < 0$. We thus conclude zero is $B(1)$'s best response when $A(1)$ bids $\frac{v}{k}$.

Scenario II: Team A places its stronger player in the first period, and team B places its strong player in the second period.

According to Lemma A0, there are altogether three possibilities: Both two players $A(1)$ and $B(1)$ bid zero; $A(1)$ bids zero and $B(1)$ bids $\frac{\varphi v}{k}$; and $A(1)$ bids v and $B(1)$ bids 0. Note that the proof of Theorem 1 has shown that when one player bids zero at period 1, the other must bid zero or preempt.

We first show that it cannot be an equilibrium that $A(1)$ bids zero and $B(1)$ bids $\frac{\varphi v}{k}$. This is clearly, otherwise, $B(1)$ gets negative payoff since $\varphi > 1$.

We next show that it is an equilibrium that $A(1)$ bids v and $B(1)$ bids 0. If $B(1)$ bids zero, then $A(1)$'s best response must be v rather than 0. Preemption gives him a payoff $\varphi v - v = (\varphi - 1)v$, strictly higher than the payoff $(1 - \frac{k}{2\varphi})\varphi v = \varphi v - \frac{k}{2}v$ from bidding zero, because $k > 2$. We next show that given $A(1)$ bids v, player $B(1)$'s best response is 0. Standard argument reveals that $B(1)$'s best response must be 0 or preemptive bid $v + \frac{\varphi v}{k}$. Bidding zero leads to zero payoff. Preemptive bidding leads to $v - (v + \frac{\varphi v}{k})k < 0$.

Scenario III: Team A places its stronger player in the second period, and team B places its strong player in the first period.

According to Lemma A0, there are altogether three possibilities: Both two players $A(1)$ and $B(1)$ bid zero; $A(1)$ bids zero and $B(1)$ bids φv; and $A(1)$ bids $\frac{v}{k}$ and $B(1)$ bids 0. Note that the proof of Theorem 1 has shown that when one player bids zero at period 1, the other must bid zero or preempt.

We first show that it cannot be an equilibrium that $A(1)$ bids zero and $B(1)$ bids φv. This is clearly, otherwise, $B(1)$ gets negative payoff since $\varphi > 1$.

We next show that it cannot be an equilibrium that $A(1)$ bids $\frac{v}{k}$ and $B(1)$ bids 0. If $B(1)$ bids zero, then $A(1)$'s best response must be zero rather than $\frac{v}{k}$. Preemption gives him a payoff $\varphi v - \frac{v}{k} \cdot k = (\varphi - 1)v$, strictly lower than the payoff $(1 - \frac{1}{2\varphi k})\varphi v = \varphi v - \frac{v}{2k}$ from bidding zero because $k > 2$.

Finally, we show that it is an equilibrium that both $A(1)$ and $B(1)$ bid zero. Recall if $B(1)$ bids zero, then $A(1)$'s best response must be zero rather than $\frac{v}{k}$. If $A(1)$ bids zero, then $B(1)$'s best response must be zero rather than φv. Preemption gives him a negative payoff.

Scenario IV: Both two teams place their weak players in the first period.

According to Lemma A0, there are altogether three possibilities: Both two players $A(1)$

and $B(1)$ bid zero; $A(1)$ bids zero and $B(1)$ bids φv; and $A(1)$ bids v and $B(1)$ bids 0. Note that the proof of Theorem 1 has shown that when one player bids zero at period 1, the other must bid zero or preempt.

We first show that it cannot be an equilibrium that $A(1)$ bids zero and $B(1)$ bids φv. This is clearly, otherwise, $B(1)$ gets negative payoff.

We next show that it cannot be an equilibrium that $A(1)$ bids v and $B(1)$ bids 0. Given $B(1)$ bids 0, player $A(1)$ chooses between bidding 0 and v. Bidding v leads to a payoff of $\varphi v - kv$. Bidding zero leads to $(1-\frac{1}{2\varphi})\varphi v = \varphi v - \frac{1}{2}v > \varphi v - kv$.

Finally, we show that it is an equilibrium that both $A(1)$ and $B(1)$ bid zero. Recall if $B(1)$ bids zero, then $A(1)$'s best response must be zero rather than v. If $A(1)$ bids zero, then $B(1)$'s best response must be zero rather than φv. Recall preemption gives him a negative payoff.

This completes the proof. ■

Proof of Theorem 7

Proof. As stated in the text, there are four possible scenarios for early player placement. In this proof, we only verify the equilibrium plays in the four subgames. The rest is well stated in the text.

Scenario 1: Both teams place their weaker players in the early period.

In this case, neither player places positive bid in the first period, as in the baseline model.

Scenario 2: Team A (the stronger team) places its weaker player in the early period, while Team B (the weaker team) assigns its stronger player to this slot.

In this case, player $A(1)$'s payoff function is

$$\pi_{A(1)}(x_{A(1)}, x_{B(1)}) = \begin{cases} \left[1 - \frac{1}{2\varphi v^2}\left(v^2 - \Delta^2\right)\right]\varphi v - kx_{A(1)} & \text{if } x_{A(1)} \in (x_{B(1)}, x_{B(1)} + v); \\ \left(1 - \frac{1}{2\varphi}\right)\varphi v - kx_{A(1)} & \text{if } x_{A(1)} \in [x_{B(1)} - \varphi v + v, x_{B(1)}]; \\ \left[\frac{1}{2\varphi v^2}\left(\varphi^2 v^2 - \Delta^2\right)\right]\varphi v - kx_{A(1)} & \text{if } x_{A(1)} \in (x_{B(1)} - \varphi v, x_{B(1)} - \varphi v + v). \end{cases} \tag{7}$$

It can be verified that the marginal payoff is negative. Thus, player $A(1)$ must bid zero. We then consider player $B(1)$'s effort choice in the early period. A bid b would give him a lead of b. He can maximally bid v, so he cannot preempt the competition. Suppose $b \geq \varphi v - v$. His net payoff in the second period would be $b + v - \varphi v$, and his net payoff in the two-period contest is $v - \varphi v < 0$. Suppose $b < \varphi v - v$. His net payoff in the second period would be zero, and his net payoff in the two-period contest is $-b$. Hence, player $B(1)$ has no incentive to place positive bid in the first period.

Scenario 3: Team A (the stronger team) places its stronger player in the early period, while Team B (the weaker team) assigns its weaker player to this slot.

By Lemmata 1-3, we have the following three subcases for player $B(1)$'s payoff function, which correspond to the three ranges of Δ.[12]

$$
\begin{aligned}
&\pi_{B(1)}(x_{B(1)}, x_{A(1)}) \\
= &\begin{cases}
\left[1 - \frac{1}{2\varphi v^2}\left(\varphi^2 v^2 - \Delta^2\right)\right] v - k x_{B(1)} & \text{if } x_{B(1)} \in (x_{A(1)}, x_{A(1)} + \varphi v); \\
\left(1 - \frac{\varphi}{2}\right) v - k x_{B(1)} & \text{if } x_{B(1)} \in [x_{A(1)} - v + \varphi v, x_{A(1)}]; \\
[\frac{1}{2\varphi v^2}\left(v^2 - \Delta^2\right)] v - k x_{B(1)} & \text{if } x_{B(1)} \in (x_{A(1)} - v, x_{A(1)} - v + \varphi v).
\end{cases}
\end{aligned}
\tag{8}
$$

The marginal payoff would be

$$
\begin{aligned}
&\frac{\partial \pi_{B(1)}(x_{B(1)}, x_{A(1)})}{\partial x_{B(1)}} \\
= &\begin{cases}
-\frac{1}{\varphi v}\Delta - k & \text{if } x_{B(1)} \in (x_{A(1)}, x_{A(1)} + \varphi v); \\
-k & \text{if } x_{B(1)} \in [x_{A(1)} - v + \varphi v, x_{A(1)}]; \\
\frac{1}{\varphi v}\Delta - k & \text{if } x_{B(1)} \in (x_{A(1)} - v, x_{A(1)} - v + \varphi v).
\end{cases}
\end{aligned}
\tag{9}
$$

Bidding zero would be a dominant strategy for the player. The marginal payoff can be positive only if $\frac{1}{\varphi v}\Delta - k > 0$, which is impossible because this requires $\Delta > k\varphi v$; player $A(1)$ has no incentive to bid more than φv.

Given that player $B(1)$ bids zero, we can explore player $A(1)$'s effort choice. A bid b leads to a lead b. If $b > v$, he will win the contest for sure, and his net payoff in the second period would be φv, and his net payoff in the two-period contest is $\varphi v - b$. If $b \leq 0$, his net payoff in the second period would be $b + \varphi v - v$, and his net payoff in the two-period contest is $\varphi v - v$. That is, he is indifferent across any bid $b \in [0, v]$. We then end up with a continuum of equilibria for this subgame: In the early period, player $B(1)$ bids zero with probability 1, while player $A(1)$ places a constant bid $b \in [0, v]$.

Scenario 4: Both teams place their strong players in early spots.

The subgame becomes a two-player sequential contest à la Hirata (2014). There exists a continuum of equilibria: In the early period, player $B(1)$ bids zero with probability 1, while player $A(1)$ places a constant bid $b \in [0, v]$. ■

5.3 Proof of Theorem 8

It should be noted that the second-stage bidding is unaffected by the nuance, and Lemmata 1-3 remain valid. However, now Δ is defined as $\Delta = d(x_{j(1)} - x_{i(1)})$. The payoff function is

[12] Note that the following ranges for $x_{i(1)}$ and $x_{j(1)}$ might cover negative effort, which is not eligible. For notational conveniency, we ignore this issue for the time being. This issue will be well taken care of, however, when we derive the equilibrium.

then given by

$$
\begin{aligned}
&\pi_{i(1)}(x_{i(1)},x_{j(1)}) = p_i\varphi_i v - k_{i(1)}x_{i(1)} \\
= &\begin{cases}
\left[1-\frac{1}{2v_{i(2)}v_{j(2)}}\left(v_{j(2)}^2-\Delta^2\right)\right]\varphi_i v - k_{i(1)}x_{i(1)} & \text{if } x_{i(1)}\in(x_{j(1)},x_{j(1)}+\frac{v_{j(2)}}{h}); \\
\left(1-\frac{v_{j(2)}}{2v_{i(2)}}\right)\varphi_i v - k_{i(1)}x_{i(1)} & \text{if } x_{i(1)}\in[x_{j(1)}-\frac{v_{i(2)}}{h}+\frac{v_{j(2)}}{h},x_{j(1)}]; \\
[\frac{1}{2v_{i(2)}v_{j(2)}}\left(v_{i(2)}^2-\Delta^2\right)]\varphi_i v - k_{i(1)}x_{i(1)} & \text{if } x_{i(1)}\in(x_{j(1)}-\frac{v_{i(2)}}{h},x_{j(1)}-\frac{v_{i(2)}}{h}+\frac{v_{j(2)}}{h}).
\end{cases}
\end{aligned}
\tag{10}
$$

We then obtain early players' marginal payoff functions for $\Delta\in(-\frac{v_{j(2)}}{d},\frac{v_{i(2)}}{d})$:

$$
\begin{aligned}
\frac{d\pi_{i(1)}}{dx_{i(1)}} &= \frac{dp_i}{dx_{i(1)}}\varphi_i v - k_{i(1)} \\
&= \begin{cases}
-\frac{hk_{i(2)}}{v_{j(2)}}\Delta - k_{i(1)} & \text{if } x_{i(1)}\in(x_{j(1)},x_{j(1)}+\frac{v_{j(2)}}{h}); \\
-k_{i(1)} & \text{if } x_{i(1)}\in[x_{j(1)}-\frac{v_{i(2)}}{h}+\frac{v_{j(2)}}{h},x_{j(1)}]; \\
\frac{hk_{i(2)}}{v_{j(2)}}\Delta - k_{i(1)} & \text{if } x_{i(1)}\in(x_{j(1)}-\frac{v_{i(2)}}{h},x_{j(1)}-\frac{v_{i(2)}}{h}+\frac{v_{j(2)}}{h});
\end{cases}
\end{aligned}
\tag{11}
$$

and

$$
\frac{d\pi_{j(1)}}{dx_{j(1)}} = \begin{cases}
-\frac{k_{j(2)}}{v_{i(2)}}\Delta - k_{j(1)} & \text{if } x_{j(1)}\in(x_{i(1)}-\frac{v_{j(2)}}{h},x_{i(1)}); \\
-k_{j(1)} & \text{if } x_{j(1)}\in[x_{i(1)},x_{i(1)}+\frac{v_{i(2)}}{h}-\frac{v_{j(2)}}{h}]; \\
\frac{k_{j(2)}}{v_{i(2)}}\Delta - k_{j(1)} & \text{if } x_{j(1)}\in(x_{i(1)}+\frac{v_{i(2)}}{h}-\frac{v_{j(2)}}{h},x_{i(1)}+\frac{v_{i(2)}}{h}).
\end{cases}
\tag{12}
$$

The fundamental trade-off remains in place. Lemma A0 also continues to hold. So the analysis does not differ from that of Theorem 1. We only need to check when team A's stronger player is placed in the early slot, whether he has an incentive to preempt. There are two relevant cases.

Case 1 Both teams place their stronger players in early slots To preempt, player $A(1)$ must bid $\frac{v}{hk}$. Then his payoff is $\varphi v-\frac{v}{hk}$. First, assume $\varphi\geq k$. Then if he bids zero, his teammates can win with a probability $1-\frac{1}{2\varphi}$. Then the payoff is $\varphi v-\frac{v}{2}$, which is strictly more than $\varphi v-\frac{v}{hk}$ if $hk<2$.

Case 2 Team A places the stronger player in early slot, while team B does the opposite. To preempt, player $A(1)$ must bid $\frac{v}{h}$. His payoff is $\varphi v-\frac{v}{h}$. First, assume $\varphi\geq k$. Then in the second stage, his teammate wins with a probability $1-\frac{k}{2\varphi}$ if he bids zero. He has a payoff $\varphi v-\frac{kv}{2}$. $\varphi v-\frac{v}{h}<\varphi v-\frac{kv}{2}$ must hold if and only if $hk<2$. Second, assume $\varphi<k$. Then in the second stage, if he bids zero, his teammate wins with a probability $\frac{\varphi}{2k}$. His payoff is $\frac{\varphi^2}{2k}v$. We now claim $\varphi v-\frac{v}{h}<\frac{\varphi^2}{2k}v$, which is equivalent to $2\varphi kh-2k<\varphi^2h\Leftrightarrow h(2\varphi k-\varphi^2)<2k\Leftrightarrow h(2\varphi k-\varphi^2-k^2)<2k-k^2h\Leftrightarrow k(kh-2)<h(k-\varphi)^2$, which is obvious because $kh-2<0$.

As a result, under the condition $kh<2$, the stronger player on team A would not place a large bid to preempt. A similar analysis to that of theorem 1 would imply that he would bid zero when he is placed in the early slot. As a result, in the equilibrium, both teams would place their stronger players in the late slots.

References

[1] Alcalde, J., and Dahm, D., 2010. Rent Seeking and Rent Dissipation: A Neutrality Result. *Journal of Public Economics*, 94, 1-7.

[2] Aoyagi, M., 2010. Information Feedback in a Dynamic Tournament. *Games and Economic Behavior,* 70, 242-260.

[3] Baik, K.H., 1993. Effort levels in contests: The public-good prize case. *Economics Letters*, 41(4), 363-367.

[4] Baik, K.H., 2008. Contests with group-specific public-good prizes. *Social Choice and Welfare*, 30(1), 103–117.

[5] Barbieri, S., Malueg, D.A., and Topolyan, I. (2014), The Best-Shot All-Pay (Group) Auction with Complete Information. *Economic Theory*, 57, 603-640.

[6] Barbieri, S., Konrad, D.A., and Malueg, D.A. (2018), Preemption Contests between Groups. Working paper.

[7] Chowdhury, S.M., Lee, D. & Topolyan, I. (2017). The Max-Min Group Contest: Weakest-link (Group) All-pay Auction. *Southern Economic Journal.* 84 (2), 474-483.

[8] Chowdhury, S.M., and Topolyan, I. (2016). The Attack-and-Defense Group Contests: Best-shot versus Weakest-link. *Economic Inquiry*, 54 (1), 548-557.

[9] Denter, P., and Sisak, D. (2015). Do Polls Create Momentum in Political Competition? *Journal of Public Economics*, 130, 1-14.

[10] Ederer, F., 2010. Feedback and Motivation in Dynamic Tournaments. *Journal of Economics and Management Strategy.* 19, 733-769.

[11] Eliaz, K., and Wu, Q, 2018. A Simple Model of Competition Between Teams. *Journal of Economic Theory.* 176, 372-392.

[12] Esteban, J., and Ray, D., 2001. Collective Action and the Group Paradox. *American Political Science Review.* 95, 663-672.

[13] Esteban, J., and Ray, D., 2008. On the Salience of Ethnic Conflict. *American Economic Review* 98. 2185–2202.

[14] Fu, Q., Lu, J., and Pan, Y., 2015. Team Contests with Multiple Pairwise Battles, *American Economic Review* 105(7), 2120-2140.

[15] Fudenberg, D., and Tirole, J., 1991. *Game Theory*. Cambridge, Massachusetts: the MIT Press.

[16] Gelder, A., 2014. From Custer to Thermopylae: Last Stand Behavior in Multi-Stage Contests. *Games and Economic Behavior*, 87, 442–466.

[17] Gelder, A., Kovenock, D., and Roberson, B., 2015. All-Pay Auctions with Ties, working paper.

[18] Gelder, A., Kovenock, D., and Sheremeta, R., 2015. Behavior in All-Pay Auctions with Ties, working paper.

[19] Gershkov, A. and Perry, M., 2009. Tournaments with Midterm Reviews. *Games and Economic Behavior,* 66, 162-190.

[20] Glicksberg, I. L. 1952. A further generalization of the Kakutani fixed point theorem with application to Nash equilibrium points. *Proceedings of the National Academy of Sciences* 38: 170-174.

[21] Goltsman, M. and Mukherjee, A., 2011. Interim Performance Feedback in Multistage Tournaments: the Optimality of Partial Disclosure. *Journal of Labor Economics,* 29, 229-265.

[22] Gürtler, O. and Harbring, C., 2010. Feedback in Tournaments under Commitment Problems: Experimental Evidence. *Journal of Economics and Management Strategy,* 19, 771-810.

[23] Häfner, S., 2017. A Tug of War Team Contest. Games and Economic Behavior. 104, 372-391. working paper.

[24] Hamilton, J., and Romano, R., 1998. "Equilibrium Assignments in Pairwise Team Contests: How to Form Political Slates and Tennis Teams. *European Journal of Political Economy* 14, 101-114.

[25] Harris, C., and Vickers, J., 1987. Racing with Uncertainty. *Review of Economic Studies,* 54, 1-21.

[26] Hirata, D., 2014. A Model of a Two-Stage All-Pay Auction. *Mathematical Social Sciences*, 68, 5–13.

[27] Joffrion, J., and Parreiras, J.O., 2014. A Tortoise And A Hare Race. working paper.

[28] Kawamura, K., and Moreno de Barreda, I., 2014. Biasing Selection Contests with ex ante Identical Agents. *Economics Letters*, 123(2), 240-243.

[29] Konrad, K.A., 2002. Investment in the Absence of Property Rights; The Role of Incumbency Advantages. *European Economic Review*, 46(8), 1521-1537.

[30] Konrad, K., 2009. *Strategy and Dynamics in Contests*. Oxford, UK: Oxford University Press.

[31] Konrad, K., and Kovenock, D., 2006a. Multi-Battle Contests. *Games and Economic Behavior* 66, 256-274.

[32] Konrad, K., and Kovenock, D., 2009. The alliance formation puzzle and capacity constraints, *Economics Letters*, 103(2), 84-86.

[33] Konrad, K., and Kovenock, D., 2010. Multi-Stage Contests with Stochastic Ability. *Economic Inquiry,* 48, 89-103.

[34] Konrad, K., and Leininger, W., 2007. The Generalized Stackelberg Equilibrium of The All-Pay Auction with Complete Information, *Review of Economic Design*, 11, 165-174.

[35] Klumpp, T., and Polborn, M.K., 2006. Primaries and the New Hamshire Effect. *Journal of Public Economics,* 90, 1073-1114.

[36] Lazear, E.P. and Rosen, S., 1981. Rank-Order Tournaments as Optimum Labor Contracts. *Journal of Political Economy*, 1981, 89, 841-864.

[37] Li, S., and Yu, J., 2012. Contests with Endogenous Discrimination. *Economics Letters*, 117(3), 834-836.

[38] Münster, J., 2007. Simultaneous Inter- and Intra-Group Conflicts. *Economic Theory,* 32, 333-352.

[39] Münster, J., 2009. Group Contest Success Functions. *Economic Theory* 41, 345-357.

[40] Nitzan, S., 1991. Collective Rent Dissipation. *Economic Journal* 101, 1522-1534.

[41] Nitzan, S., and Ueda, K., 2009. Collective Contests for Commons and Clubs. *Journal of Public Economics,* 93, 48-55.

[42] Nitzan, S., and Ueda, K., 2011. Prize Sharing in Collective Contests. *European Economic Review,* 55(5), 678-687.

[43] Pastine, I. and Pastine, T., 2012. Incumbency Advantage and Political Campaign Spending Limits. *Journal of Public Economics*, 96(1-2), pp.20-32.

[44] Siegel, R., 2014. Asymmetric Contests with Head Starts and Nonmonotonic Costs. *American Economic Journal: Microeconomics*, 6(3), 59-105.

[45] Skaperdas, S., 1998. On the Formation of Alliances in Conflicts and Contests. *Public Choice* 96, 25-42.

[46] Varian, H., 1994. Sequential contributions to public goods, *Journal of Public Economics*, 53(2), 165-186.

[47] Yildirim, H., 2005. Contests with Multiple Rounds, *Games and Economic Behavior* 51 (1), 213-227.