Auction design with shortlisting when value discovery is covert

Murali Agastya∗ Xin Feng†
University of Sydney Nanjing University
Jingfeng Lu‡
National University of Singapore

May 2021

Abstract

This paper studies optimal auction design when buyers’ value discovery investment is covert but essential for mutually beneficial trade between seller and buyers. Since selling mechanisms contingent on value discovery (e.g. ex ante fees charged upon information acquisition) are not feasible, a framework of second price auctions with reserves contingent on number of actual bidders ex post is adopted. Under a regularity condition of monotone hazard rate, we find that the optimal reserve depends on the number of shortlisted bidders, but not on the number of actual bidders. Depending on the value discovery cost, the seller shortlists either the socially efficient number of buyers or one more bidder. The comparison between the two options of the seller is completely resolved. The optimal reserve depends discontinuously and non-monotonically on the value discovery cost. In the former case, equilibrium information acquisition is efficient but ex post allocation is inefficient, while in the latter case it is the opposite.

Keywords: Exclusive bidding; Covert information acquisition; Endogenous market size; Optimal auctions; Revenue maximization.

JEL Classifications: D44, D45, D82.

∗Murali Agastya, Economics Discipline, H04 Merewether Building, University of Sydney, Sydney NSW 2006, AUSTRALIA. E-mail: m.agastya@econ.usyd.edu.au.
†Xin Feng, School of Economics, Nanjing University, 22 Hankou Road, Nanjing, Jiangsu 210093, CHINA. E-mail: a0078175@u.nus.edu.
‡Jingfeng Lu, Department of Economics, National University of Singapore, SINGAPORE 117570. E-mail: ecsljf@nus.edu.sg.
1 Introduction

There are many situations where gains from trade between seller and buyers have to be created by information acquisition investment when the traders are not perfectly informed about their values. Without incurring the investment to discover his true value, each buyer’s expected value can be lower than the seller’s value, meaning there are no ex-ante gains from trade. However, a buyer can make a fixed investment and discover his true value, which follows a distribution and thus can be higher than the seller’s value with a positive probability. Whenever the buyer’s expected payoff from his investment on information acquisition is higher than his investment cost, a trading opportunity can be created by the value discovery of buyers. Dasgupta (1990), Tan (1992), Fullerton and McAfee (1999), Che and Gale (2003), Jehiel and Lamy (2015), Sogo, Bernhardt and Liu (2016), Li (2019), Gershkov, Moldovanu, Strack and Zhang (2020) among others explore auctions with value discovery or endogenous values.

On one hand, investment on information acquisition is the source for mutually beneficial gains from trade. In this sense, sufficient incentive should be provided to encourage buyers on discovering their values. On the other hand, since value discovery is costly and the marginal gain generated is generally decreasing, there is a finite level of efficient information acquisition that maximizes the total gains from trade. Both these two considerations suggest the importance of seller’s control over buyers’ information acquisition on improving the total gains from trade as well as the seller’s revenue.

Besides selecting the auction format, one natural way for the seller to control the information acquisition decisions of buyers is by shortlisting eligible buyers, which has been illustrated by Levin and Smith (1994), Taylor (1995), Ye (2007), Li and Zheng (2009), Moreno and Wooders (2010), Lu and Ye (2013), Bhattacharya, Roberts and Sweeting (2014), Sweeting and Bhattacharya (2015), Quint and Hendricks (2018), and Lu, Ye and Feng (2021) in different environments among others. Intuitively, less eligible buyers would provide each of them more incentive to discover his values, while a lower number of eligible buyers also tends to restrict total investment on value discovery. Given the number of shortlisted, buyers’ investment decisions also depend on the auction format, which the seller uses to extract surplus from buyers and generate revenue. The seller’s optimal strategy to maximize revenue involves a shortlist of eligible buyers as well as the subsequent auction format.

Levin and Smith (1994) were the first to offer general (and elegant) results for

---

In a closely related literature, the seller also needs to determine the optimal number of bidders, since the seller needs to incur costs to search for bidders. See Crémer, Spiegel and Zheng (2009), Szech (2011), Doval (2018), and Lee and Li (2020).
auctions with entry when investment cost is uniform across buyers. They showed that for any given number of buyers, as long as a second price auction with a reserve equal to seller’s value induces a symmetric mixed-strategy information acquisition equilibrium, the said auction would be revenue-maximizing among all auctions that induce symmetric information acquisition equilibrium. Moreover, they find that the optimal revenue would decrease with the number of buyers for such cases. As such, their results can then be reinterpreted to offer a solution the optimal shortlist but with two important caveats. First, auctions and entry costs are restricted so as to ensure that the participation choice of the shortlisted buyers is necessarily a mixed strategy equilibrium. Second, the seller can observe the value discovery costs.

This paper effectively removes both the aforementioned caveats. Value discovery of buyers is covert and non-verifiable. This feature of our setup is in the spirit of Persico (2000), Bergemann and Välimäki (2002), Shi (2012), Li (2019), Gershkov, Moldovanu, Strack and Zhang (2020). The assumption captures many situations where it is infeasible to prevent buyers from covertly pursuing an investigation of the qualities of the object on sale. An immediate implication of covert and non-verifiable value discovery is that in this case the eligible auction mechanisms cannot be contingent on the value discovery decisions of buyers. A further complication of covert value discovery lies in that no one can be blocked from participating in the auction even a bidder who does not discover his true value. It then remains an open question what the seller’s optimal selling procedure would be in this case. How many buyers should the seller shortlist at the optimum? How should the seller design the auction to optimally balance between providing the buyers incentive on value discovery and extracting information rent from them?

In our model, auction that follows a short-list can be any anonymous auctions that allocates the object to the actual bidder with the highest value subject, above a threshold. We focus on symmetric information acquisition equilibria in our analysis. Nevertheless, we allow heterogeneous minimum winning values that are contingent on the number of actual bidders. For tractability of analysis, a framework of second-price auction with reserves contingent on the number of actual bidders is adopted.

We find that the optimal auction design crucially depends on the number of buyers shortlisted. When the number of shortlisted is strictly greater than the efficient level,
the insight and analysis of Levin and Smith (1994) still hold and a uniform reserve equal to seller’s value is revenue maximizing. In this case, the auction induces random information acquisition and achieves ex post efficient allocation of the object. However, when the number of shortlisted is less than or equal to the efficient level, it turned out to be quite challenging to pin down the optimal reserves. A uniform reserve set at the seller’s value would induce every shortlisted bidder to make the investment, but generally it does not maximize the revenue. A higher reserve might still induce the same information acquisition but increase the revenue. As the reserve increases, it will eventually get into a range that would induce random information acquisition. One might ask whether a reserve in this range can be optimal. Moreover, the same random information acquisition can be induced by heterogeneous reserves contingent on the number of actual bidders, which further increases the difficulties for searching for the optimal reserves.

As one of our major results, we find that the well-adopted monotone hazard rate condition of the value distribution plays a significant role in tackling this challenging problem. With this condition, we find that for any information acquisition equilibrium, the optimality of a uniform reserve is guaranteed. One can thus focus on uniform reserves, which greatly simplifies the analysis of our auction design problem. We further find that a high reserve that induces random information acquisition can never be optimal. The intuition is that a high reserve would definitely hurt the total surplus, which must coincide with the seller’s revenue when information acquisition is random. Note that the virtual value function is regular when the monotone hazard rate condition holds. It is then clear that the optimal reserve should the Myerson reserve or the highest level that induces every bidder to acquire information, whichever is lower.

Given the above characterizations of the optimal auctions for all possible number of shortlisted buyers, one can compare them and pin down the optimal shortlisting. Clearly, the result of Levin and Smith (1994) means that the seller should shortlist at most one more buyer than the efficient level. However, how many should exactly be shortlisted? To answer this question, we first establish that the revenue from the optimal auctions strictly increases with the number of shortlisted until it reaches exactly the efficient level. Thus, at the optimum, the seller should shortlist either the efficient number of bidders or one more bidder. But which level is optimal? Again, the monotone hazard rate condition plays an important role in answering this question. We find that this condition guarantees that the revenue from the optimal auction with the efficient number of bidder is a concave function of the investment costs. On the other hand, the optimal revenue with one more bidder is a convex function of investment costs. Moreover, we find that while both revenue functions are decreasing
with investment costs, the auction with efficient number of buyers dominates when investment is in the high range, and the auction with one more bidder dominated when the investment cost is in a low range. The shapes of the two revenue functions guarantee that the two ranges are separated by a unique threshold of investment costs.

One can therefore find tipping points so that when the cost goes over one of these, it becomes optimal for the seller to exclude an extra buyer. At any such point, the optimal reserve jumps above the seller’s valuation and then decreases continuously to the seller’s value and remains there until the next tipping point. Furthermore, in between these tipping points, participation in the revenue maximizing auction by the eligible buyers changes from pure strategy to mixed strategy. Our finding thus echoes that of Shi (2012) who shows that optimal reserve should lie between the ex ante mean valuation of bidders and the Myerson reserve.⁴

This paper can be considered as a complementary paper of Levin and Smith (1994). While assuming the uninformed bidders can be blocked from bidding, they established the optimal auction when the number of buyers is strictly higher than the efficient level and showed for these cases that the seller’s revenue would drop with the number of bidders. While adopting roughly the same setting, we further study the revenue-maximizing auctions for the cases where the number of buyers is no greater than the efficient level. These additional analyses allow us to fully pin down the optimal shortlisting policy and optimal reservation prices in standard auctions.

The recent paper Chen and Kominers (2021) contains results that resemble some of ours where an additional (equilibrium) bidder fetches more revenue than an admission price. They consider a model in which the entry cost is private information. Here, the size of the cost is known, and it is a deliberate, and equally importantly a covert act on part of a potential bidder whether to incur that cost and enter the auction. In particular, an ex ante fee charged upon information acquisition of buyers is infeasible in our analysis. Nevertheless, in our setting, we provide complete comparative statics on the relative of an additional bidder i.e., the optimal shortlist as the entry cost varies over an arbitrary range. This setup captures many situations where it is infeasible to prevent the buyers from covertly pursuing an investigation of the qualities of the object. Consequently, a selling procedure cannot be contingent on the information acquisition decisions of the buyers. In particular, an ex ante fee charged upon information acquisition of buyers is infeasible in our analysis. While allowing heterogeneous minimum winning values that are contingent on the number of actual

⁴Our Assumption 1 means that the ex ante expected valuation of bidders is lower than the seller’s value. In Shi (2012), while bidders’ investment on information acquisition is continuous, no shortlisting of bidders is allowed.
bidders, the revenue-maximizing auction is established under a regularity condition of
monotone hazard rate property of value distribution. We find that the seller’s rev-
enue would increase until the number of buyers reaches the efficient level. We further
completely pin down the optimal shortlisting policy by comparing across the efficient
number of bidders and the case with one more buyer, which accomplishes the analysis
of Levin and Smith (1994) on shortlisting policy of seller.

Another key element in our analysis is that we drop the “serious bidder assumption”
that is adopted by Levin and Smith (1994) and Bulow and Klemperer (1996). Instead,
we assume that the ex ante expected value of a bidder is lower than the seller’s value,
and therefore there is no gains from trade without information acquisition of buyers.
We would like to stress that the dominance of shortlisting the efficient number of buyer
over a smaller number is not directly implied by the arguments of Bulow and Klemperer
(1996), which based on the observation that an optimal auction with any number of
informed buyers must be dominated by a second price auction without reserve but
with one more buyer. As has been illustrated by themselves by an example in their
paper, their arguments crucially reply on the “serious bidder assumption”. In Section
3.3.3, we will further elaborate on this issue.

The rest of the paper is organized as follows. Section 2 sets up the seller’s problem.
Section 3 first presents the revenue-maximizing auctions for each number of buyers
shortlisted. The optimal shortlisting policy is then completely characterized based on
the revenue-maximizing auctions derived. This section also discusses the issues on
ex ante subsidies/fees, the implications of dropping the “serious bidder assumption”,
and the functions of monotone hazard rate property. The conclusion is presented in
Section 4.

2 The seller’s problem

A seller owns a single object worth \( v_0 \) to her. There are a sufficiently large number
of potential buyers, each of whom initially knows only that his value for the object
is a random variable \( V_i \) that is continuously distributed according to a probability
distribution function \( F(\cdot) \) and a positive density \( f(\cdot) \) on an interval \([u, \bar{v}]\) where \( u \geq 0 \).
By incurring a cost \( c (> 0) \), any of these agents is able to discover his true value of
the object. We refer to \( c \) as the value discovery cost or information acquisition cost
following the literature. The seller and bidders are risk neutral. Bidder \( i \) thus values
the object at \( EV_i \) if he does not invest to identify his true value.

Let \( \eta = EV_i, \forall i \). Throughout the paper, we assume that there are no (expected)
gains from trade if no buyer incurs the cost to discover his value, i.e.,

**Assumption 1** $0 < \eta \leq v_0$.

Assumption 1 means that we drop the “serious bidder assumption” of Bulow and Klemperer (1996). Coey, Larsen and Sweeney (2018) also drop the “serious bidder assumption” in their empirical analysis of bidder exclusion effect. The importance of this restriction is that it forces a buyer to engage in value discovery for any trade to be mutually beneficial. A further discussion on the role of Assumption 1 is deferred to Section 3.3.3. To make mutually beneficial trade indeed feasible, we assume that it is socially desirable for at least one buyer to discover his value, i.e.

**Assumption 2** $E[\max\{V_i - v_0, 0\}] \geq c$.

The timing of the game is as following.

A selling method of the seller begins in **stage 0** with her shortlisting an integer $N$ buyers ($i = 1, \ldots, N$) who are eligible to bid in an auction for the object. We will refer to them as *eligible bidders*.

In **stage 1**, the auction format, which is allowed to be contingent on the number of actual bidders, is publicly announced.

In **stage 2**, with a public knowledge of both $N$ and the auction format, each of the eligible bidders make a _covert_ decision on whether to, perhaps probabilistically, incur the fixed cost $c > 0$ to discover their values for the object.

In **stage 3**, all eligible bidders simultaneously decide whether to become an actual bidder no matter they have incurred the discovery cost and discovered their values. The number of actual bidders is then publicly revealed.

In **stage 4**, the auction is held according to the announced format among all actual bidders.

The timing of the game is consistent with Assumption 4 in Levin and Smith (1994) in the sense that the number of eligible bidders $N$, the selling mechanism are revealed before the value discovery and participation stage, and the number of actual bidders is publicly revealed prior to the bidding stage.

Our setting however diverges from the Levin and Smith (1994) setup in the following two key aspects. First, in their setting the number of eligible bidders of stage 0 has to be sufficiently large, which must entail random value discovery at optimum.\(^5\)

\(^5\)Larsen (2020) presents related empirical evidence supporting Assumption 1.\(^6\)

\(^6\)This essentially requires that the number of shortlisted must be higher than the efficient number of buyers who carry out their value discovery.
Here, we would allow any number of bidders being shortlisted. Levin and Smith (1994) have shown an upper bound for the number of eligible buyers that the seller would like shortlisted. In this paper, we are to fully pin down the optimal number of being shortlisted. Allowing any number of shortlisted and characterizing the corresponding optimal selling mechanism become essential for this task. Second, Levin and Smith (1994) assume that an eligible buyer must incur the fixed cost and discover his value in order to participate. In our setting, any eligible buyer can participate even without discovering his value if he finds this is in his interest. This reflects the fact that the seller cannot discriminate eligible bidders based on their information acquisition decisions since these are covert. We would like to stress that in our setting an auction is implemented at stage 4 after the eligible bidders’ value discovery decisions, which also reflects the fact that covert value discovery makes it infeasible to have a value-discovery contingent auction mechanism.

Following Levin and Smith (1994), eligible mechanisms that are available to the seller to use in stage four require that a bidder wins and pays for the item only if his bid is the highest and moreover actual buyers’ bids are increasing in their values regardless of their information acquisition decisions. Furthermore, as in Levin and Smith (1994), we focus on symmetric information acquisition decisions across all eligible buyers induced by anonymous selling mechanisms. Here, by “anonymous”, we mean that the auction adopted at stage 4 is not contingent on the identities of the actual bidders. In view of these considerations and the complication of the bidding behavior of the bidders who do not incur the discovery cost, in this paper, the eligible selling mechanisms with $N$ eligible bidders being shortlisted are described by a vector $r_N = (r_1, \ldots, r_N)$ where $r_k \in [0, \bar{v}]$ is the common-knowledge reserve price at which a second price auction at stage four is run when there are $k$ actual bidders show up. Let $SPA_k(r)$ denote a second price auction with a reserve $r$ and $k$ actual bidders.

**Definition 1 (Selling Procedure)** A strategy for the seller (a selling procedure) is a tuple $\mu_N := (N, r_N)$ where $N$ is the number of agents given exclusive bidding rights and $r_N = (r_1, \ldots, r_k, \ldots, r_N)$ with $r_k \in [0, \bar{v}]$ is a vector of reserve prices such that $SPA_k(r_k)$ is held by the seller in the event there are $k$ actual bidders.

The selling procedure of Definition 1 covers the well-known revenue-maximizing auction derived by Levin and Smith (1994) as special cases, which takes a form of second price auction with a uniform zero reservation price, which equals the seller’s value in their setup. Our selling procedure further allows heterogeneous reserve prices contingent on the number of actual buyers. This flexibility grants the seller the options of using different combinations of reserves to induce the same information acquisition...
decisions of any $N$ eligible buyers. An interesting question immediately raises: Can auction design be improved by allowing contingent reserves? Moreover, there is a clear trade-off between rent extraction at stage 4 and trade-enhancing value discovery at stage 2, which the optimal reserves have to address. In our selling procedure, the reserves are not restricted to be higher than the seller’s value $v_0$. The seller has the options of setting lower reserves to provide the eligible buyers more incentive for their value discovery. One may ask whether setting some reserves lower than $v_0$ or even lower than $\eta$ can ever be in the benefit of the seller.

Note that the selling procedure of Definition 1 is contingent only on the number of actual bidders but not the value discovery decisions of eligible bidders. Recall that our focus is on situations where information acquisition is covert (as in Bergemann and Välimäki (2002)). If investment in value discovery is publicly verifiable, it becomes possible for the seller to directly influence the cost $c$ through the use of either a subsidy $s \geq 0$ or a fee $f \geq 0$ which transforms the cost to $c - s$ or $c + f$ respectively. Note that $s$ and $f$ are transfers that accrue to a buyer regardless of actual value but contingent on her making the information acquisition investment. (In the literature on auctions with information acquisition, these transfers would be referred to as ex-ante entry fee/subsidy.) In Section 3.3.1, we explain that allowing for subsidies has no impact on the results while the presence of fees may indeed change the outcome. In the remainder of the paper, due to the covert information acquisition decisions of bidders, the assumption is that value discovery decisions are unobservable and it is infeasible to charge ex-ante entry fees contingent on the bidders’ information acquisition decisions.

The seller is to derive the optimal selling mechanism with each $N$ shortlisted for the purpose of revenue maximization and then chooses the optimal $N^*$. 

Buyers’ decisions on being an actual bidder

The participation decision at Stage 3 is endogenous. Before we study the optimal selling mechanism for each $N$, we first pin down the eligible bidders’ decisions on being an actual bidder.

First, it is a weakly dominant strategy for each actual buyer to bid his true value in the stage 4 second price auction. The highest bidder wins if his bid is higher than the reserve and his payment is lower than his value. No payments are made by other actual buyers. This makes it a weakly dominant strategy for an informed buyer to participate. We assume that the informed bidders are endowed with hard evidence about their information acquisition (e.g. the details about the auctioned project) to
distinguish themselves from uninformed bidders. The seller can make the informed
bidders strictly prefer to become an actual bidder regardless of their discovered values
by providing an epsilon show-up reward, which does not change the stage 2 information
acquisition decisions and bidding behavior when the epsilon goes to zero in the limit.\footnote{Nevertheless, entry fees cannot be charged upon the stage 2 information acquisition decisions based on the informed bidders’ hard evidence on their value discovery, since it is not incentive compatible for these informed bidders to reveal these evidences.}

Second, an uninformed buyer who participates in the auction will bid as if his
valuation for the object is $\eta$. Given Assumption 1 for such a buyer to have any
chance of winning the object, the seller must be setting some reserves $r_k$ below $\eta$.
Therefore, when all reserves are above $\eta$, none of the uninformed buyers would have
incentive to become an actual bidder. When some reserves are below $\eta$, it is a weakly
dominant strategy for an uninformed buyer to become an actual bidder regardless of
the other uninformed bidders’ decisions on participation. The reason is as follows.
Suppose there exists $k_0$ such that $1 \leq k_0 \leq N$ and $r_{k_0} \in [0, \eta)$. In the event that $k_0$
actual bidders (including the concerned uninformed buyer) show up for an auction, the
uninformed bidder has a chance to gain a positive payoff, and in no event he would
suffer a negative payoff. In this paper, we focus on symmetric entry equilibrium in
stages 2 and 3. As a result, when some reserves are below $\eta$, it is the unique equilibrium
in stage 3 for all uninformed buyers to participate with probability 1.

The above participation behavior of informed and uninformed buyers is retained
throughout the paper. Therefore, the participation decision at Stage 3 would convey
information about a buyer’s value discovery status if and only if the minimum reserve
$\min\{r_k\}$ is above $\eta$.

**Notations**

We collect here some further frequently used notations. Recall $SPA_k(r)$ denotes a
second price auction with $k$ actual buyers and a reserve $r$. As shown in the subsection
above, when $r > \eta$, uninformed buyers would not participate and the $k$ actual buyers
must be informed. Assume a typical IPV setting where the values of $k$ buyers are
drawn according to $F(\cdot)$. We use $u_k(r)$ to denote the corresponding ex-ante payoff of
a typical bidder in auction $SPA_k(r)$ and $R_k(r)$ the seller’s expected revenue. Also let
$G_k(\cdot)$ denote the probability distribution of the random variable $Y_k = \max\{V_1, \ldots, V_k\}$,
i.e. the highest value among $k$ informed bidders. The total welfare under $SPA_k(r)$ is

$$W_k(r) = E[\max\{Y_k, r\}] - (r - v_0)G_k(r).$$
Recall that $S\mathcal{P}\mathcal{A}_k(v_0)$ is the VCG mechanism for the allocation of the object among $k$ informed buyers. Therefore, the payoff of a typical buyer in $S\mathcal{P}\mathcal{A}_k(v_0)$ is the change in welfare that is obtained by his addition. Therefore, the payoff of a typical buyer in $S\mathcal{P}\mathcal{A}_k(v_0)$ is given by

$$u_k(v_0) = W_k(v_0) - W_{k-1}(v_0).$$

(1)

More generally, the payoff of a typical buyer in $S\mathcal{P}\mathcal{A}_k(r)$ is given by

$$u_k(r) = E[\max\{Y_k, r\}] - E[\max\{Y_{k-1}, r\}].$$

Since $u_{k+1}(v_0) < u_k(v_0)$, $\lim_{k \to \infty} u_k(v_0) = 0$ and $c \leq u_1(v_0)$ (Assumption 2), there exists a unique integer $N_c \geq 1$ such that

$$u_{N_c+1}(v_0) < c \leq u_{N_c}(v_0).$$

(2)

In other words, $N_c$ is a decreasing step function of $c$ as $N_c = N$ if and only if $c \in (u_{N+1}(v_0), u_N(v_0)]$. As noted elsewhere in the literature $N_c$ represents the efficient amount of value discovery while allowing each potential bidder to make the discovery with any probability. See for example Lu (2008).

3 Revenue-maximizing selling procedure

Sub-optimality of reserves lower than $\eta$

**Proposition 1** $\forall N \geq 1$, for any selling procedure $(N, r_N)$ where $\min\{r_k\} < \eta$, there must exist a selling procedure $(N, \tilde{r}_N)$ where $\min\{\tilde{r}_k\} \geq \eta$, which dominates $(N, r_N)$ in terms of seller’s revenue.

**Proof.** For selling procedure $(N, r_N)$ where $\min\{r_k\} < \eta$, all $N$ eligible bidders participate in the auction no matter they are informed or not. Therefore, an auction $S\mathcal{P}\mathcal{A}_N(r_N)$ prevails. It is a dominant strategy for each of the uninformed buyers to bid $\eta$.

We first consider the case of $N \leq N_c$, i.e., $c \leq u_N(v_0) \leq u_N(\eta)$ using [2] and Assumption [1].

If $r_N \geq \eta$, then selling procedure $(N, r_N)$ is equivalent to selling procedure $(N, \tilde{r}_N)$ where $\tilde{r}_k = r_N, \forall k$: an uninformed buyer never wins, each informed bidder’s winning chance and payments are exactly the same across the two procedures no matter how many uninformed bidders submit bids. Therefore, the two selling procedures would
induce the same value discovery decisions of bidders. The seller’s revenue is also the same no matter how many uninformed bidders submit bids. As a result, the two procedures must be revenue equivalent.

Suppose \( r_N < \eta \). Let \( Y_{k-1} \) denote the second highest order statistic from the \( k \) random variables \( V_1, \ldots, V_k \). In the event that there are \( k \leq N - 2 \) informed buyers and at least two uninformed buyers, the seller’s payoff in this auction is

\[
G_k(\eta)\eta + (1 - G_k(\eta))E[\max\{\eta, Y_{k-1}\} \mid Y_k \geq \eta] = R_k(\eta) + (\eta - v_0)G_k(\eta).
\]

In the event that there are \( k = N - 1 \) informed buyers and one uninformed buyer, the seller’s payoff in this auction is

\[
G_k(\eta)r + (1 - G_k(\eta))E[\max\{\eta, Y_{k-1}\} \mid Y_k \geq \eta] = R_k(\eta) + (r_N - v_0)G_k(\eta).
\]

From Assumption 1 and the above results, it follows that whenever there is at least one uninformed buyer, the seller’s revenue is no more than \( R_k(\eta) \), which is smaller than \( R_N(\eta) \) using \( r_N < \eta \leq v_0 \).

When all \( N \) buyers are informed, the seller’s revenue is exactly \( R_N(r_N) \). Next, we show that \( R_N(r_N) \leq R_N(\eta) \) when \( r_N < \eta \leq v_0 \). In the event that all the \( N \) buyers are informed, when reserve price is \( r \), the expected payment of a bidder with value \( V \) is given by

\[
m(V) = rG_{N-1}(r) + \int_r^V yg_{N-1}(y)dy.
\]

The seller’s expected payoff equals

\[
R_N(r) = N \int_r^V m(V)f(V)dV + F^N(r)v_0
= NrG_{N-1}(r)(1 - F(r)) + N \int_r^V yg_{N-1}(y)(1 - F(y))dy + F^N(r)v_0.
\]

Differentiating this with respect to \( r \), we have

\[
\frac{d}{dr} R_N(r) = NG_{N-1}(r)(1 - F(r))[1 - (r - v_0) \frac{f(r)}{1 - F(r)}].
\]

Note that \( \frac{d}{dr} R_N(r) > 0 \), when \( r < v_0 \). This implies that an increase in \( r \) always leads to a higher revenue whenever \( r < v_0 \). As a result, \( R_N(r_N) \leq R_N(\eta) \), when \( r_N < \eta \leq v_0 \).

We are now ready to establish that the revenue from \((N, r_N)\) is lower than that from
solving procedure \((N, \tilde{r}_N)\) where \(\tilde{r}_k = \eta, \forall k\). Since \(c \leq u_N(\eta), c \leq u_k(\eta), \forall k \leq N\). In other words, with \((N, \tilde{r}_N)\), an informed bidder always makes a payoff which is higher than his value discovery cost no matter how many informed bidders are participating. It is thus a dominant strategy for each bidder to make the value discovery. It entails that a second price auction with \(N\) informed buyers and a reserve of \(\eta\) would prevail. The seller’s revenue is thus \(R_N(\eta)\).

We now turn to the case of \(N \geq N_c + 1\). Following same procedure with Levin and Smith (1994), one can show that a selling procedure \((N, \tilde{r}_N)\) with \(\tilde{r}_k = v_0 (> \eta), \forall k\) is efficient and revenue-maximizing among all mechanisms that implement symmetric value discovery across eligible bidders. The details will be provided when we formally present the revenue-maximizing selling procedure for \(N \geq N_c + 1\) in the proof of Proposition 2.

3.1 Revenue-maximizing selling procedure for given \(N\)

Based on Proposition 1 the seller needs only to consider reserves \(r_k \in [\eta, v]\). In this case, an eligible bidder becomes an actual bidder if and only if he incurs the cost and discover his value.

Given such a strategy \(\mu_N = (N, r_N)\) of the seller, the information acquisition decision of eligible bidder \(i = 1, \ldots, N\) can be described by a number \(p_i \in [0, 1]\), namely the probability of investing in value discovery. As our interest is in (equilibrium) situations where this decision is symmetric across the agents, in our paper \(p_i \equiv p\) for some \(p \in [0, 1]\). Given that each of his rivals invests in value discovery with probability \(p\), if any bidder incurs the cost and discover his value, then his payoff is

\[
U_N(p, r_N, c) = \sum_{k=0}^{N-1} \beta(k; p, N-1)u_{k+1}(r_{k+1}) - c, \tag{3}
\]

where \(\beta(k; p, N) = C_N^k p^k (1 - p)^N - k\) is the probability that there are \(k\) other bidders who discover their values and participate in the auction. His payoff is zero if he does not invest in value discovery. The symmetric information acquisition equilibrium is defined as follows:

**Definition 2 (Information Acquisition Equilibrium)** \(p_e(\mu_N; c) \in [0, 1]\) is said to be a symmetric information acquisition equilibrium of a selling procedure \(\mu_N = (N, r_N)\) if at \(p = p_e(\mu_N; c)\) one of the following conditions hold: a) \(U_N(p, r_N, c) = 0\); b) \(U_N(p, r_N, c) > 0\) and \(p = 1\); or c) \(U_N(p, r_N, c) < 0\) and \(p = 0\). The equilibrium is
said to be a full value-discovery equilibrium if \( p_e(\mu_N; c) = 1 \) and a partial or random value-discovery equilibrium if \( p_e(\mu_N; c) \in (0, 1) \).

Note that \( U_N(p, r_N, c) \) is continuous in \( p \). The existence of symmetric information acquisition equilibrium for any selling procedure \( \mu_N = (N, r_N) \) can thus be easily established.

Given \( \mu_N = (N, r_N) \) where \( r_k \geq \eta, \forall k \), if every eligible bidder discovers his value with probability \( p \), then the seller’s revenue is

\[
R_N(p, r_N) = \sum_{k=0}^{N} \beta(k; p, N)R_k(r_k),
\]

and the total expected surplus of seller and bidders is

\[
S_N(p, r_N, c) = \sum_{k=0}^{N} \beta(k; p, N)(W_k(r_k) - kc).
\]

**Lemma 1** The following holds for all \( N, p, c \):

\[
S_N(p, r_N, c) = R_N(p, r_N) + NpU_N(p, r_N, c), \quad \forall r_N,
\]

\[
\frac{\partial S_N(p, v_0, c)}{\partial p} = NU_N(p, v_0, c),
\]

where \( v_0 = (v_0, \ldots, v_0) \).

**Proof.** We note that

\[
\sum_{k=0}^{N} \beta(k; p, N)k[u_k(r_k) - c] = \sum_{k=1}^{N} \frac{N!}{k!(N-k)!} p^k (1-p)^{N-k}[u_k(r_k) - c] = Np \sum_{k=1}^{N} \frac{(N-1)!}{(k-1)!(N-k)!} p^{k-1} (1-p)^{N-k}[u_k(r_k) - c]
\]

Since \( W_k(r_k) - kc = R_k(r_k) + k[u_k(r_k) - c] \), it readily follows that \( S_N(p, r_N, c) = R_N(p, r_N) + NpU_N(p, r, c) \), namely (6), which is first shown by Moreno and Wooders (2010). (7) is shown in Levin and Smith (1994) for the case of \( v_0 = 0 \). (7) with a general \( v_0 \) can be verified directly using (1). The details are available from the authors. \( \square \)
Suppose $\mu_N$ induces symmetric value-discovery equilibrium $p_e$, the seller’s revenue is $R_N(p_e, r_N)$. When $p_e \in (0, 1)$, we have that the eligible bidders’ payoffs are zero. Therefore seller’s revenue $R_N(p_e, r_N)$ must equal the total surplus $S_N(p_e, r_N, c)$.

**Definition 3 (Standard Selling Procedure)** A selling procedure $\mu_N := (N, r_N)$ is said to be standard if $r_N = (r_k)$ is such that $r_k \equiv r$, $\forall k$ and will be denoted by $(N, r)$.

Recall that most generally, a seller’s strategy allows her the possibility of setting a reserve that depends on the number of buyers that actually participate in auction. In an anonymous selling procedure, the seller forgoes this possibility. As we will soon establish, a monotone hazard rate condition of the value distribution guarantees that the seller needs only consider a standard selling procedure for revenue maximization.

### 3.1.1 When $N \geq N_c + 1$

**Proposition 2** (Levin and Smith, 1994) For $N \geq N_c + 1$, under Assumptions [1] and [2], standard selling procedure $(N, v_0)$ is revenue-maximizing.

**Proof.** The proof is available for the case of $v_0 = 0$ while assuming the object can never be to allocated to a bidder who does not discover his value. Their proof applies to our setting with $v_0 \geq \eta$. Nevertheless, the following proof adapted to our setting is included so that the readers do not have to resort to Levin and Smith (1994). Note that in our setting, it is rather incentive compatible for a bidder not to bid in $(N, v_0)$ if he does not discover his value.

Given any exogenous symmetric information acquisition $p \in [0, 1]$, the maximum total surplus achievable is $S_N(p, v_0, c)$ where $v_0 = (v_0, ..., v_0)$. This happens when the allocation is always ex post efficient.

We first identify the optimal $p^*$ that maximizes $S_N(p, v_0, c)$. We have

$$\partial S_N(p, v_0, c)/\partial p = N(U_N(p, v_0, c)),$$

by Lemma [1]. Since $U_N(1, v_0, c) = u_N(v_0) - c < 0$ and $U_N(0, v_0) = u_1(v_0) - c > 0$, it follows that the $p^* = \arg \max_p S_N(p, v_0, c)$ must be an interior point and that $U_N(p^*, v_0, c) = 0$. Thus for $\mu_N = (N, v_0)$, we have the seller’s revenue is $R_N(p^*, v_0) = S_N(p^*, v_0, c)$ using [6]. At any equilibrium $p_e$ of $\mu_N = (N, r_N)$, we have

$$R_N(p_e, r_N) \leq S_N(p_e, v_0, c) \leq S_N(p^*, v_0, c) = R_N(p^*, v_0).$$
The first inequality is due to the fact that at equilibrium \( p_e \), bidders’ expected payoffs might be non-negative.

3.1.2 When \( N \leq N_c \)

Optimality of standard selling procedure

**Assumption 3** \( H(\cdot) = \frac{1-F(\cdot)}{f(\cdot)} \) is decreasing.

**Proposition 3** (Optimality of A Uniform Reserve) Under Assumption 3, for any \( N \leq N_c \), the equilibrium revenue in an arbitrary selling procedure \( (N, r_N) \) where \( r_k \geq \eta \) is bounded above by the equilibrium revenue of a standard selling procedure \( (N, r) \) for some \( r \geq \eta \), which induces the same information acquisition equilibrium.

**Proof.** Let \( \mu_N = (N, r_N) \) be given where \( r_k \in [\eta, \overline{r}], \forall k \). Suppose that the information acquisition equilibrium is \( p_e(\mu_N; c) \in [0, 1] \). If \( p_e(\mu_N; c) = 0 \), it continues to be an equilibrium of standard procedure \( (N, \overline{r}) \) and the revenue is the same. If \( p_e(\mu_N; c) = 1 \), it continues to be an equilibrium of standard selling procedure \( (N, r_N) \) and the revenue is the same.

We now consider the case \( p_e(\mu_N; c) \in (0, 1) \). Consider the program \( \mathcal{P} \) where,

\[
\mathcal{P} := \max_{r_N \in [\eta, \overline{r}]} S_N(p_e(\mu_N; c), r_N, c) \quad \text{s.t.} \quad U_N(p_e(\mu_N; c), r_N, c) = 0.
\]

This program solves the revenue-maximizing reserves which implements equilibrium \( p_e(\mu_N; c) \in (0, 1) \). Let us then analyze \( \mathcal{P} \). Clearly it must have a solution \( r^*_N \), since all the involved functions are continuous and the domain is nonempty and compact.

Since \( r^*_N \) induces partial value-discovery equilibrium and \( N \leq N_c \), it must not be the case that \( r^*_k \leq v_0, \forall k \). Otherwise, we must have \( p_e(\mu_N; c) = 1 \) as \( u_k(r^*_k) \geq u_k(v_0) \geq c, \forall k \leq N \leq N_c \).

For problem \( \mathcal{P} \), set up the Lagrangian:

\[
L(r, \lambda) = S_N(p_e(\mu_N; c), r_N, c) + \lambda( -U_N(p_e(\mu_N; c), r_N, c) )
\]

One can verify that \( u'_k(r) = -[1 - F(r)]G_{k-1}(r) \) and

\[
W'_k(r) = (v_0 - r)G'_k(r) = k(v_0 - r)G_{k-1}(r)f(r).
\]
With (3) and (5), we then have

\[
\varphi_k(r_k) := \frac{\partial L((r_k, r_{-k}), \lambda)}{\partial r_k} = \beta(k - 1; p_c(\mu_N; c), N - 1) G_{k-1}(r_k) \\
\times \left( Np_c(\mu_N; c)(v_0 - r_k)f(r_k) + \lambda(1 - F(r_k)) \right),
\]

using \( \beta(k - 1; p, N) = \frac{Np_c(\mu_N; c)(v_0 - r_k)f(r_k) + \lambda(1 - F(r_k))}{\partial L((r_k, r_{-k}), \lambda)} \).

Note that \( \varphi_k(\overline{v}) < 0 \), it cannot be the case that \( r^*_k = \overline{v} \) for any \( k \). Bases on the above results, there exists a \( k_0 \leq N \) such that \( r^*_{k_0} \in (v_0, \overline{v}) \). For this \( r^*_{k_0} \), the first order condition for an interior optimum requires that \( \varphi_{k_0}(r^*_{k_0}) = 0 \), i.e.

\[
Np_c(\mu_N; c)(v_0 - r^*_{k_0})f(r^*_{k_0}) + \lambda[1 - F(r^*_{k_0})] = 0.
\]

Therefore \( \lambda = \frac{Np_c(\mu_N; c)(v_0 - r^*_{k_0})f(r^*_{k_0})}{1 - F(r^*_{k_0})} > 0 \).

It remains to prove that \( r^*_k = r^*_{k_0}, \forall k \). Using the identified value for \( \lambda \), for any \( r^*_k \in [\eta, \overline{v}], k \neq k_0 \), we may rewrite \( \varphi_k(r^*_k) \) as

\[
\varphi_k(r_k) = p_c(\mu_N; c)N \beta(k - 1; p_c(\mu_N; c), N - 1) G_{k-1}(r_k) \left[1 - F(r_k)\right] \\
\times \left( \frac{f(r^*_k)}{1 - F(r^*_k)}(r^*_k - v_0) - \frac{f(r_k)}{1 - F(r_k)}(r_k - v_0) \right).
\]

The term in the large brackets is positive for \( r_k < r^*_{k_0} \) and negative for \( r_k > r^*_{k_0} \) due to the monotone hazard rate, which means that at optimum \( r^*_k = r^*_{k_0}, \forall k \). \( \square \)

**Optimal reserve** Given Proposition 3, the seller only needs to consider uniform reserve price, which does not depend on the number of actual bidders for revenue maximization. With a uniform reserve, the impact of auction mechanism on total surplus can be clearly analyzed based the following key Lemma 2. This would greatly facilitate the search for the optimal reserve in terms of revenue based the connection between revenue and total surplus.

**Lemma 2** \( u_k(r) < W_k(r) - W_{k-1}(r), \forall r > v_0, \forall k \).

**Proof.** Recall that \( u_k(r) = E[\max\{Y_k, r\}] - E[\max\{Y_{k-1}, r\}] \) and \( W_k(r) = E[\max\{Y_k, r\}] - (r - v_0)G_k(r) \). By direct calculation, \( W_k(r) - W_{k-1}(r) = E[\max\{Y_k, r\}] - E[\max\{Y_{k-1}, r\}] + (r - v_0)(G_{k-1}(r) - G_k(r)) > u_k(r) \) as \( r > v_0 \) and \( G_{k-1}(r) > G_k(r) \). \( \square \)

17
Definition 4 Let \( r^c_N \) to be solution of \( u_N(r^c_N) = c \); Let \( r^m \) to be the solution of \( J(r^m) = r^m - \frac{1 - F(r^m)}{f(r^m)} = v_0 \), i.e., \( r^m \) is the Myerson (1981) optimal reserve in a standard IPV setting with any number of buyers.

Proposition 4 Under Assumption 3, for any \( N \leq N_c \), the optimal uniform reserve \( r^*_N \) is \( \min\{r^c_N, r^m\} \), which is higher than \( v_0 \). This selling procedure induces a full value discovery equilibrium.

Proof. Under Assumption 3 virtual value function \( J(\cdot) \) is increasing. Reserve \( r^m \) is thus the Myerson optimal reserve in a standard IPV setting with \( N \) buyers.

If \( u_N(r^m) \geq c \), then at \( r^m \), every bidder acquires information, and the maximum revenue is achieved by the optimal Myerson mechanism.

If \( u_N(r^m) < c \), then \( r^c_N < r^m \). It follows that \( J(\cdot) < 0 \) for \( v \leq r^c_N \). Therefore, any reserve \( r \in [\eta, r^c_N) \) is dominated by \( r^c_N \). A reserve \( r > r^c_N \) must induce partial value discovery equilibrium \( p_e < 1 \) as \( u_N(r) < c \). Note that at equilibrium \( p_e \) and reserve \( r \), the revenue is bounded by the total surplus \( \sum_{k=0}^{N} \beta_k(p_e, N)[W_k(r) - kc] \), which is smaller than \( \sum_{k=0}^{N} \beta_k(p_e, N)[W_k(r^c_N) - kc] \) as \( W_k(\cdot) \) decreases to the right of \( v_0 \). Note that \( \forall k \leq N \), we have

\[
[W_k(r^c_N) - kc] - [W_{k-1}(r^c_N) - (k-1)c] = [W_k(r^c_N) - W_{k-1}(r^c_N) - u_k(r^c_N)] + [u_k(r^c_N) - c] > 0,
\]

according to Lemma 2 and \( u_k(r^c_N) > u_N(r^c_N) = c \). Therefore,

\[
\sum_{k=0}^{N} \beta_k(p_e, N)[W_k(r^c_N) - kc] < W_N(r^c_N) - N c = W_N(r^c_N) - N u_N(r^c_N),
\]

which is the seller’s expected revenue when reserve is uniformly \( r^c_N \). □

3.2 Optimal shortlisting

Proposition 5 (i) (Levin and Smith, 1994) Optimal revenue \( R_N(p^e_N, v_0) \) strictly decreases with \( N \) when \( N \geq N_c + 1 \) if \( N_c \geq 1 \); (ii) Optimal revenue \( R_N(r^*_N) \) increases with \( N \) when \( N \leq N_c \), if \( N_c \geq 1 \).

Proof. Part (i) is established by Propositions 1 and 9 in Levin and Smith (1994).
To show part (ii), it is sufficient to show that $R_{N-1}(r_{N-1}^*) < R_N(r_N^*), \forall N \leq N_c$. Note that $r_{N-1}^* \geq r_N^*$ as $r_{N-1}^* > r_N^*$. Recall that $r_N^* = \min\{r_N^c, r_m^c\}$ by Proposition 4.

If $r_N^* = r_m^c$, then $r_{N-1}^* = r_m^c$. In this case, it must be true that $R_{N-1}(r_{N-1}^*) < R_N(r_N^*)$.

If $r_N^* = r_N^c < r_m^c$, then $r_{N-1}^* > r_N^c$ and thus $u_N(r_{N-1}^*) < u_N(r_N^c) = c$. Note that

$$R_{N-1}(r_{N-1}^*) \leq \sum_{k=0}^{N-1} \beta_k(p_e, N-1)[W_k(r_{N-1}^*) - kc]$$

$$\leq \sum_{k=0}^{N-1} \beta_k(p_e, N-1)[W_k(r_N^c) - kc].$$

The first inequality holds since the revenue is bounded by the total surplus, and the second inequality holds since $r_{N-1}^* > r_N^c > v_0$.

Analogous to the proof of Proposition 4 $\forall k \leq N$, we have

$$[W_k(r_N^c) - kc] - [W_{k-1}(r_N^c) - (k-1)c]$$

$$= [W_k(r_N^c) - W_{k-1}(r_N^c) - u_k(r_N^c)] + [u_k(r_N^c) - c] > 0,$$

according to Lemma 2 and $u_k(r_N^c) > u_N(r_N^c) = c$. It implies that

$$R_{N-1}(r_{N-1}^*) \leq \sum_{k=0}^{N-1} \beta_k(p_e, N-1)[W_k(r_N^c) - kc]$$

$$< W_{N-1}(r_N^c) - (N-1)c$$

$$< W_N(r_N^c) - Nc$$

$$= W_N(r_N^c) - N u_N(r_N^c)$$

$$= R_N(r_N^c)$$

$$= R_N(r_N^*).$$

The last equality holds since $r_N^* = r_N^c$ and the second last equality holds since all buyers participate with probability 1 and $u_N(r_N^c) = 0$.

**Value of a bidder**

It is useful to offer a more intuitive interpretation of Proposition 5 in terms of the “value of an additional bidder”. For any market size $N > N_c + 1$, both at $N$ and $N - 1$ the Information Acquisition Equilibrium is necessarily random for the revenue-
maximizing selling mechanism. Therefore, the “market tightness principle” of Levin and Smith (1994) is directly applicable and the seller gains from dropping a bidder.

One implication of Lemma 2 is that \( u_k(r) \geq c \Rightarrow W_k(r) - W_{k-1}(r) \geq c \) whenever \( r \geq v_0 \). In other words, when an additional bidder is added to \( SPA_{k-1}(r) \), the change in that bidder’s utility is \( u_k(r) \). The above implication says that if this is enough to cover the cost of value discovery for that marginal agent, the total social welfare must increase from \( W_{k-1}(r) - (k - 1)c \) to \( W_k(r) - kc \). Intuitively, one would expect at least part of this rise to be transferable to the seller. From this perspective, seller’s revenue should increase with \( N \) when \( N \leq N_c \), i.e., when \( u_N(r_N^*) \geq c \).

When \( N \leq N_c \), there is a \( r \geq v_0 \) such that \( u_N(r) = c \). If \( r > r^m \), then \( u_N(r^m) > c \). In this case, clearly dropping a bidder cannot increase the seller’s revenue. Assume \( r \leq r^m \). Note that the seller is able to get all the buyers participate with probability one by setting \( r_N = (r, \ldots, r) \). By doing this, she also drives the entire rent of a buyer to zero and gets the entire social surplus \( W_N(r) - Nc \) as her revenue. By dropping a bidder, if the seller induces an equilibrium in which all \( N - 1 \) buyers participate with probability one, the highest possible payoff of the seller is the resulted total expected social surplus \( W_{N-1}(r') - (N - 1)c \), where \( u_{N-1}(r') \geq c \) and \( r' \in (r, r^m) \). Therefore, the change in revenue is higher than

\[
[W_N(r) - Nc] - [W_n(r') - (N - 1)c] = W_N(r) - W_{N-1}(r') - c = W_N(r) - W_{N-1}(r') - u_N(r) > W_{N-1}(r) - W_{N-1}(r'), \quad \text{(using Lemma 2)}
\]

which is positive as \( v_0 < r < r' \). To complete the “additional competition is valuable” principle of Bulow and Klemperer (1996), however, one needs to also establish that lower revenue is achieved in an Information Acquisition Equilibrium where entry is in mixed strategies. For this, Proposition 3 is needed to conclude that a constant reserve would be sufficient. After this, an argument in terms of efficiency as in the proof of Proposition 5 applies.

The above arguments reveal that maximal revenue for a given \( N \) is first increasing in the market size \( N \) until \( N_c \) and decreasing to the right of \( N_c + 1 \). Therefore, the optimal number of bidders must be \( N_c \) or \( N_c + 1 \).

Note that Assumption 2 means that we have \( c \leq u_1(v_0) \). Since \( u_K(v_0) \) monotonically drops to zero when \( K \) approaches to infinity. There exists a unique \( K \geq 1 \) such

\[ A \text{ reserve } r' \text{ lower than } r \text{ cannot lead to higher revenue with one less bidder.} \]
that \( c \in (u_{K+1}(v_0), u_K(v_0)) \). Note that we have \( K = N_c \) by definition of \( N_c \). Proposition 5 thus means that \( \forall K \geq 2 \) and \( c \in (u_{K+1}(v_0), u_K(v_0)) \), we have that optimal number of bidders must be \( K \) or \( K + 1 \). We next fully pin down the optimum.

**Proposition 6** Under Assumption 3, \( \forall K \geq 2 \) and \( c \in (u_{K+1}(v_0), u_K(v_0)) \), there exists a unique \( \hat{c}_K \in (u_{K+1}(v_0), u_K(v_0)) \) such that the optimal selling procedure is \((K, r^*_K)\) if \( c \in (\hat{c}_K, u_K(v_0)) \); and the optimal selling procedure is \((K + 1, v_0)\) if \( c \in (u_{K+1}(v_0), \hat{c}_K) \). The two procedures generate the same revenue when \( c = \hat{c}_K \). Thus, if \( c \in (u_{K+1}(v_0), \hat{c}_K) \), the seller shortlists \( K + 1 \) buyers, and if \( c \in (\hat{c}_K, u_K(v_0)) \), the seller shortlists \( K \) buyers.

**Proof.** The proof consists of three steps. It uses Lemma 1.

The revenue under procedure \( \mu_1 = (K + 1, v_0) \) is \( S(c) = S_{K+1}(p_e(\mu_1; c), v_0, c) \), where equilibrium value discovery \( p_e(\mu_1; c) \) is interior of \([0, 1]\). The first step in the remainder of the proof consists of showing that \( S(\cdot) \) is a decreasing convex function. Next, let \( R(c) \) denote the seller’s revenue under \( \mu_2 = (K, r^*_K) \). Since \((1 - F(v))/f(v)\) is assumed to be decreasing, under \( \mu_2 \), we have \( r^*_K = \min\{r^*_K, r_m^*\} \) where \( r_m^* \) is the unconstrained Myerson reserve defined by Definition 4. Therefore, \( R(c) = R_K(r^*_m) \) for all \( c \leq u_K(r^*_m) \).

When \( c \in [u_K(r^*_m), u_K(v_0)] \), \( r^*_K = r^*_K \) and therefore \( R(c) = W_K(r^*_K) - Kc \). The second step in the proof involves showing that \( R(\cdot) \) is also decreasing in this region, just as \( S(\cdot) \), but that it is concave. The final step involves showing that \( R(\cdot) \) intersects \( S(\cdot) \) from below at some \( \hat{c}_K \in (u_{K+1}(v_0), u_K(v_0)) \) and moreover such a \( \hat{c}_K \) must be unique.

**Step 1.** Note that the expectation of the number of participants is

\[
\sum_{k=0}^{K+1} \beta(k, p_e(\mu_1; c), K + 1)k = (K + 1)p_e(\mu_1; c).
\]

Recall that \( p_e(\mu_1; c) \) maximizes \( S_{K+1}(\cdot, v_0, c) \), the envelope theorem, the derivative of \( S(\cdot) \) is

\[
\frac{dS(c)}{dc} = \frac{\partial S_{K+1}(p_e(\mu_1; c), v_0, c)}{\partial c} = -(K + 1)p_e(\mu_1; c) < 0.
\]

Therefore, \( S(\cdot) \) is decreasing in \( c \). To see that it is convex, we need to verify that \( \frac{d^2 S(c)}{dc^2} < 0 \). By Milgrom-Shannon Theorem, it is equivalent to show that \( S_{K+1}(p_e(c), c) := S_{K+1}(p_e(\mu_1; c), v_0, c) \) obeys single crossing condition. In particular, it suffices to show that \( \frac{\partial^2}{\partial p_e(c)} S_{K+1}(p_e(c), c) \geq 0 \), which holds as \( \frac{\partial^2}{\partial p_e(c)} S_{K+1}(p_e(c), c) = \frac{\partial}{\partial p_e} \left( (K + 1)p_e(c) \right) = K + 1 \geq 0 \). Therefore, \( \frac{d^2 S(c)}{dc^2} < 0 \) follows.

**Step 2.** Without loss of generality, we assume \( u_{K+1}(v_0) \leq u_K(r^*_m) \). On interval \([u_{K+1}(v_0), u_K(r^*_m)]\), \( R(c) = R_K(r^*_m) \) is constant.
On interval \([u_K(r^m), u_K(v_0)]\), \(R(c) = W_K(r^r_K) - Kc\), which we need to show is decreasing and concave. Taking \(u_K(r^r_K) = c\) to be an identity and noting that \(u'_K(r) = -(1 - F(r))G_{k-1}(r)\) give

\[
\frac{dr^r_K}{dc} = \frac{1}{-(1 - F(r_K))(G_{K-1}(r_K))} < 0.
\]

Recall \(W'_K(r) = (v_0 - r)G'_K(r) = k(v_0 - r)G_{k-1}(r)f(r)\) (equ. (8)), we have

\[
\frac{dR(c)}{dc} = K\frac{(r^r_K - v_0)f(r^r_K)}{1 - F(r^r_K)} - K \\
= K\frac{f(r^r_K)}{1 - F(r^r_K)}[J(r^r_K) - v_0].
\]

\(J(r^r_K) < J(r^m) = v_0\) since \(r^r_K < r^m\) and hence \(\frac{dR(c)}{dc} < 0\). Moreover, since \(r^r_{N_c} > v_0\) and \(f(\cdot)/(1 - F(\cdot))\) is increasing, the fact that \(r^r_K\) is decreasing implies that the first term in (9) is decreasing in \(c\), i.e., \(\frac{dR(c)}{dc} < 0\). Hence, \(R(\cdot)\) is concave.

**Step 3.** We will first argue that \(R(u_{K+1}(v_0)) < S(u_{K+1}(v_0))\) and \(R(u_K(v_0)) > S(u_K(v_0))\). Recall that \(R(c) = R_K(r^m)\) for all \(c \leq u_K(r^m)\), and \(R(c) = W_K(r_c) - Kc\) for all \(c \in [u_K(r^m), u_K(v_0)]\).

By (2), when \(c = u_{K+1}(v_0)\), \(N_c = K + 1\). In other words, \(K+1\) bidders discover their values and participate in the auction, and the resulting total surplus is \(S(u_{K+1}(v_0)) = W_{K+1}(v_0) - (K + 1)c\), which is the highest total welfare for \(K + 1\) bidders.

Using (1) and \(c = u_{K+1}(v_0)\), we have

\[
S(u_{K+1}(v_0)) = W_{K+1}(v_0) - (K + 1)c \\
= E[\max\{Y_{K+1}, v_0\}] - (K + 1)c \\
= E[\max\{Y_K, v_0\}] - (K + 1)c + E[\max\{Y_{K+1}, v_0\}] - E[\max\{Y_K, v_0\}] \\
= E[\max\{Y_K, v_0\}] - Kc - c + u_{K+1}(v_0) \\
= W_K(v_0) - Kc \\
> W_K(r_c) - Kc \\
= R(u_K(v_0)).
\]

By Step 2, \(R(c)\) is a decreasing function. Therefore, \(R(u_{K+1}(v_0)) < R(u_K(v_0)) < S(u_{K+1}(v_0))\).

Likewise, by (2), when \(c = u_K(v_0)\), we have \(N_c = K\). In other words, there are
$K$ bidders who discover their values and participate in the auction, and the resulting expected revenue is $R(u_K(v_0)) = W_K(v_0) - Kc$, which is the highest social welfare for $K$ bidders.

Analogously, using (1) and $c = u_K(v_0)$, we have

$$S_{K+1}(u_K(v_0)) = W_{K+1}(v_0) - (K + 1)c = E[max\{Y_{K+1}, v_0\}] - (K + 1)c = E[max\{Y_K, v_0\}] - Kc + E[max\{Y_{K+1}, v_0\}] - E[max\{Y_K, v_0\}] - c = E[max\{Y_K, v_0\}] - Kc + u_{K+1}(v_0) - u_K(v_0) < W_K(v_0) - Kc.$$  

Therefore, $R(u_K(v_0)) = W_K(v_0) - Kc$ is greater than the social welfare $S(u_K(v_0)) = S_{K+1}(u_K(v_0))$. From the above reasoning, the function $d(c) = R(c) - S(c)$ is such that $d(u_{K+1}(v_0)) < 0$ and $d(u_K(v_0)) > 0$. Since it is continuous, there is $\hat{c}_K \in (u_{K+1}(v_0), u_K(v_0))$ such that $d(\hat{c}_K) = 0$. From Step 1 and Step 2, $d(\cdot)$ is concave and hence such a $\hat{c}_K$ must be unique. □

In Proposition 6, we deal with the case when there are at least three buyers in the pool. In Proposition 7, we solve for the optimal selling procedure when there are exactly two potential buyers. The result in Proposition 7 is analogous to that in Proposition 6. However, unlike Proposition 6, when $c$ equals cutoff $u_1(v_0)$, the two procedures $(1, r^*_1)$ and $(2, v_0)$ generate the same revenue.

**Proposition 7** Under Assumption 3, $\forall c \in (u_2(v_0), u_1(v_0)]$, there exists a unique $\hat{c}_1 \in (u_2(v_0), u_1(v_0))$ such that the optimal selling procedure is $(1, r^*_1)$ if $c \in (\hat{c}_1, u_1(v_0))$; and the optimal selling procedure is $(2, v_0)$ if $c \in (u_2(v_0), \hat{c}_1)$. The two procedures generate the same revenue when $c = \hat{c}_1$ or $u_1(v_0)$. Thus, if $c \in (u_2(v_0), \hat{c}_1)$, the seller shortlists 2 buyers, and if $c \in (\hat{c}_1, u_1(v_0))$, the seller shortlists 1 buyer.

**Proof.** The proof resembles that of Proposition 6. The only difference is that when $c = u_1(v_0)$, the two procedures $(1, r^*_1)$ and $(2, v_0)$ generate the same revenue of $v_0$. It thus suffices to further show that $\frac{dR_1(r^*_1)}{dc}|_{c=u_1(v_0)} < \frac{dR_2(p^*_2, v_0)}{dc}|_{c=u_1(v_0)} \leq 0$. We next establish this property.

When $c \in (\hat{c}_1, u_1(v_0))$ is in a small neighborhood of $u_1(v_0)$, we have

$$R_1(r^*_1) = v_0F(r^*_1) + r^*_1(1 - F(r^*_1)).$$
where \( f^\delta_{r_1^*} (v - r_1^*) f(v) dv = c \). Note \( r_1^* = v_0 \) when \( c = u_1(v_0) \).

We thus have \( \frac{dr_1^*}{dc} |_{c = u_1(v_0)} = -\frac{1}{1 - F(v_0)} \), and \( \frac{dR_1(r_1^*)}{dc} |_{c = u_1(v_0)} = -1. \)

When \( c \in (\hat{c}_1, u_1(v_0)) \) is in a small neighborhood of \( u_1(v_0) \), we have

\[
R_2(p_c^2, v_0) = (p_c^2)^2 R_2(v_0) + 2p_c^2 (1 - p_c^2) R_1(v_0) + (1 - p_c^2)^2 R_0(v_0),
\]

where \( p_c^2 \) is the entry equilibrium and \( R_k(v_0) \) stands for the expected seller revenue in a standard second price auction with \( k \) bidders and a reservation price \( v_0 \). Note \( R_1(v_0) = R_0(v_0) = v_0 \), and \( p_c^2 = 0 \) when \( \hat{c}_1 \).

The entry equilibrium is given by \( p^2_t u_2(v_0) + (1 - p^2_t) u_1(v_0) = c \). We thus have

\[
\frac{d\hat{r}_1^*}{dc} |_{c = u_1(v_0)} = -\frac{1}{u_1(v_0) - u_2(v_0)} \text{ and } \frac{dR_2(p_c^2, v_0)}{dc} |_{c = u_1(v_0)} = \frac{2[R_1(v_0) - R_0(v_0)]}{u_1(v_0) - u_2(v_0)} = 0.
\]

We thus have \( \frac{dR_1(r_1^*)}{dc} |_{c = u_1(v_0)} < \frac{dR_2(p_c^2, v_0)}{dc} |_{c = u_1(v_0)} \leq 0 \), which further means \( R_1(r_1^*) > R_2(p_c^2, v_0) \) when \( c \in (\hat{c}_1, u_1(v_0)) \). \(\square\)

By Propositions 6 and 7 if \( c \in (u_{K+1}(v_0), \hat{c}_K) \), the seller shortlists \( K + 1 \) buyers and runs a second price auction that is ex-post efficient (i.e., the reserve is set at \( v_0 \)). Each of these \( K + 1 \) buyers invests in value discovery with equilibrium value discovery probability \( p_c \). If \( c \in (\hat{c}_K, u_K(v_0)) \), the seller shortlists \( K \) buyers and runs a second price auction that is ex-post inefficient (i.e., the reserve is higher than \( v_0 \)). Each of the \( K \) buyers invests in value discovery with equilibrium value discovery probability 1.

Recall when \( c \in (u_{K+1}(v_0), u_K(v_0)) \), we have \( N_c = K \). Proposition 6 thus yields two important insights. First, the proposition highlights the close connection between the socially efficient value discovery and the extent of value discovery that occurs in the revenue maximizing selling procedure. The proposition also shows that when \( c \in (\hat{c}_K, u_K(v_0)) \), i.e., the costs are relatively high, the extent of value discovery that occurs under revenue maximization is also the socially optimal investment in value discovery. On the other hand, if the cost is less significant, value discovery is random. However the object is assigned in a socially optimal manner among the informed buyers. We collect these observations as the following corollary

**Corollary 1 (Optimal shortlisting & efficient value discovery)** When the cost \( c \) is such that:

1. When \( c \in (\hat{c}_K, u_K(v_0)) \), the equilibrium value discovery is socially efficient. However, the consequent allocation of the object among the informed buyers is ex-post inefficient.
2. When \( c \in (u_{K+1}(v_0), \hat{c}_K) \), the equilibrium value discovery is socially inefficient.
However, the consequent allocation of the object among the informed buyers is necessarily ex-post efficient.

Second, note that Proposition 6 shows that, whenever \( c \in (u_{K+1}(v_0), u_K(v_0)) \), there is a \( \hat{c}_K \in (u_{K+1}(v_0), u_K(v_0)) \) such that the optimal reserve is \( v_0 \) if \( c \leq \hat{c}_K \) and the optimal reserve \( r_K^* \geq v_0 \) otherwise. From the definition \( r_K^* = \min\{r_K^c, r_m^c\} \), it is clear that \( r_K^* \) is (weakly) decreasing in \( c \) on \( c \in (\hat{c}_K, u_K(v_0)) \) with \( r_K^* = v_0 \) when \( c = u_K(v_0) \). Therefore we have the following observation.

**Corollary 2 (Comparative statics of the optimal reserve)** As \( c \) varies in each interval \((u_{K+1}(v_0), u_K(v_0))\), \( K \geq 1 \), the optimal reserve price is first the constant \( v_0 \) to the right of \( u_{K+1}(v_0) \), then jumps up (discontinuously) and then decreases continuously to \( v_0 \).

Corollary 2 is a characteristic of the seller’s ability to directly control the number of buyers. Corollary 2 can therefore form a basis for an empirical inquiry on exclusive bidding.

The following example illustrates our main results.

**Example 1** Suppose that \( v_0 = 0.5 \) and a buyer’s value follows uniform distribution over \([0, 1]\), i.e., \( F(x) = x \) on \([0, 1]\).

Using (1), direct calculation gives \( u_K(v_0) = \frac{K}{K+1} + \frac{1}{K+1}r^{K+1} - \frac{K-1}{K} - \frac{1}{K+1}r^K \). From Propositions 6 and 7 for \( c \in (u_{K+1}(v_0), u_K(v_0)) \), the optimal selling procedure is either \((K+1, v_0)\) or \((K, r_K^*)\). As \( c \) varies, we plot the revenues that result from the selling mechanisms \((K+1, v_0)\) and \((K, r_K^*)\) respectively in Figure 1, for \( K = 1, 2, 3, 4, 5 \).

Figure 1 further confirms our theoretical results. For \( K \geq 2 \), Proposition 6 indicates that the two revenue curves, which are induced by the two aforementioned mechanisms, cross only once. To the left of the crossing point, i.e. when the entry cost is in the lower range of the concerned interval, shortlisting \( K + 1 \) bidders is optimal; otherwise, shortlisting \( K \) bidders is optimal. For \( K = 1 \), Proposition 7 indicated that the two revenue curves cross twice at \( u_1(v_0) = 0.0125 \) (the right end of the interval) and \( \hat{c}_1 \approx 0.0875 \). Similarly, if and only if the entry cost is in the lower range of the concerned interval, shortlisting \( K + 1 \) bidders is optimal.

The maximum of the two revenues thus gives the optimal revenue as a function of entry cost. Not surprisingly, it is decreasing and continuous.

In Figure 2, we further plot the optimal reservation price as a function of the value discovery cost \( c \in (u_{K+1}(v_0), u_K(v_0)) \), \( K = 1, 2, 3, 4, 5 \). The flat solid line describes
the reservation price $v_0$ when $K + 1$ is the optimal number shortlisted; The piecewise decreasing broken line describes the optimal reservation price when $K$ is the optimal number shortlisted. We can see that each segment of the broken lie starts from a lower level compared to its left neighbour and decreases continuously to $v_0$ as entry cost $c$ increases.

Figure 1: Revenue Curves

Figure 2: Optimal Reservation Price
3.3 Discussion

3.3.1 On value discovery subsidies

We now return to the issues raised in Section 2 (see the paragraphs following Definition 1) on how the results change if value discovery investment is publicly verifiable.

One might now allow the seller to provide a subsidy to offset the value discovery costs that is paid contingent on incurring them. That is, along with committing to the vector of reserves \( r \), the seller could also announce a subsidy \( s \geq 0 \) to be paid to a potential buyer if she makes the investment. From a buyer’s point of view, this has the same impact as reducing the value discovery cost from \( c \) to \( c - s \). More formally, let a selling procedure now be the tuple \( \mu = (N, r, s) \) where \( s \geq 0 \). In the selling procedure \( \mu = (N, r, s) \), if the buyers discover their values with a probability \( p \), the payoff of the seller is \( R_N(p, r) - Nps \) while a buyer will engage in value discovery only if \( U_N(p, r, c) \geq -s \). With a consequent relaxing of the incentives to engage in value discovery, it may be speculated that the seller can gain as she can then set a higher reserve. We have the following result.

**Proposition 8** In any revenue maximizing selling procedure \( \hat{\mu} = \hat{N}, \hat{r}_N, \hat{s} \), the seller sets a zero subsidy on value discovery, i.e., \( \hat{s} = 0 \). The choice of \( \hat{N} \) and \( \hat{r}_N \) is thus exactly as in Proposition 6.

**Proof.** Now suppose \( p_e \) is in fact the equilibrium value discovery for \( \hat{\mu} = (\hat{N}, \hat{r}_N, \hat{s}) \). If \( \hat{N} \geq N_e + 1 \), then by the proof of Proposition 2 the seller’s revenue is dominated by the total surplus under standard procedure \((N, v_0)\), which is the highest possible revenue achieved by \((N, v_0)\). Reserves \( \hat{r}_N(\neq v_0) \) or subsidy \( \hat{s}(>0) \) will lead to a suboptimal value discovery equilibrium or ex post inefficient allocations. In both cases, the highest revenue will not be achieved.

Now assume \( \hat{N} \leq N_e \). We claim that \( U_N(p_e, \hat{r}_N, c) + \hat{s} = 0 \) must hold if \( \hat{s} > 0 \). This clearly holds if \( p_e \in (0, 1) \). If \( p_e = 1 \), \( \hat{s} \) can be reduced and we still have \( U_N(1, \hat{r}_N, c) + \hat{s} = 0 \) holds such that \( p_e = 1 \) is still induced. But the revenue will be higher. This contradicts to the assumption that \( \hat{\mu} \) is revenue-maximizing.

When \( \hat{N} \leq N_e \), we know that \( r^c_{\hat{N}} \geq v_0 \) and also that \( u_k(r^c_{\hat{N}}) > c \) for all \( k < \hat{N} \). Should it be the case that \( \hat{r}_k \leq r^c_{\hat{N}} \) for all \( k \leq \hat{N} \), it would then mean \( U_N(p_e, \hat{r}_N, c) > 0 \) and the only way for \( U_N(p_e, \hat{r}_N, c) + \hat{s} = 0 \) to hold would be to have \( \hat{s} < 0 \). Since \( \hat{s} > 0 \), we must have \( \hat{r}_k > r^c_{\hat{N}} \) for some \( k \).

We can now prove that \( \hat{s} = 0 \). Otherwise, pick one \( \hat{r}_k > r^c_{\hat{N}} \) and reduce it slightly, which turn increases \( U_N(p_e, \hat{r}_N, c) \), but since \( \hat{s} > 0 \), it can also be reduced.
to offset this increase and keep the information acquisition equilibrium intact. Since 
\( S_N(p, \cdot, \hat{r}_N, s), c \) is decreasing to the right of \( v_0 \), the above change immediately con-
tradicts that \( (\hat{r}_N, \hat{s}) \) is revenue-maximizing. Note the expected payoffs of bidders are 
zero, thus revenue and total surplus coincide.

The reason why such subsidies cannot help is fairly intuitive. Given \( N \), both 
subsides and reserves may be used to influence participation. Setting a non-trivial 
reserve is distortive, it results in an ex-post inefficient allocation. To the extent that 
participation can be guaranteed with a lower reserve and a correspondingly lower 
subsidy, the seller gains from the increased efficiency.

3.3.2 On ex-ante fees

The situation with entry fees as opposed to subsidies is different. We know from 
Proposition 2 (or Proposition 6 in Levin and Smith (1994) that if the seller initially 
selects a market size \( N \geq N_c + 1 \), entry fees contingent on value discovery do not have 
any role. However, the seller can do better by selecting the market size \( N_c \) and proper 
entry fees. More formally, let a selling procedure now be the tuple \( \mu = (N, r_N, f) \) 
where \( f \geq 0 \). All other details are just as in the case of subsidies.

Proposition 9 If the seller can charge entry fees contingent on value discovery, the 
revenue maximizing strategy for the seller is to shortlist \( N_c \) buyers, runs an ex-post 
efficient auction and charges an entry fee of \( f = u_{N_c}(v_0) - c \). Equilibrium value 
discovery is socially efficient and the seller gets the entire social surplus 
\( W_{N_c}(v_0) - N_c c \).

The proof is clear – given the seller’s strategy, it is a pure strategy equilibrium for 
each of the \( N_c \) buyers to invest in value discovery. For buyers, following the suggested 
action yields them a payoff of \( u_{N_c}(v_0) - c - f = 0 \) which makes them indifferent to 
taking any other action. The seller obviously cannot do any better since 
\( W_{N_c}(v_0) - N_c c \) is the maximum total social surplus that can be generated while allowing all possible 
value discovery behavior of a large pool of potential buyers.

In summary, when ex ante fees can be charged contingent on value discovery, then 
the seller would shortlist efficient number of buyers and charge appropriate fees to 
extract all the surplus. This result is independent of the value distribution as long as 
Assumption 1 holds.
3.3.3 On the role of Assumption 1

Assumption 1 is motivated by the fact that in many situations, a buyer needs to discover whether there are gains from trade. Such investment in value discovery should of course be induced at the right amount if it is inefficient to trade without such investment. Though one can have ex-post inefficiency even if Assumption 1 does not hold, but when such investment is not observable, this assumption make it incentive compatible that buyers who do not discover their values do not participate in the optimal auctions at stage four.

One can drop Assumption 1 and instead assume (as in Levin and Smith (1994)) that value discovery investment is publicly observable together with an exogenous restriction that an uninformed buyer is not allowed to participate. In this case, the analysis will be exactly the same as in our paper. However, if we drop the assumption that uninformed bidders can be blocked from bidding, then the analysis can be quite different from both Levin and Smith (1994) and our results. We will explore this direction in our future research.

Contrary to Assumption 1, Bulow and Klemperer (1996) adopt a "serious bidder" assumption, which says that the bidders’ minimum value is higher than the seller’s value. With this assumption and no value-discovery issue involved, they showed that a simple second price auction with no reserve but \( N + 1 \) bidders always dominates the Myerson optimal auction with \( N \) bidders. When the "serious bidder" assumption is dropped, i.e., when Assumption 1 kicks in, they showed explicitly by providing one example that their claim no longer holds.

With Assumption 1 and costly value discovery, Levin and Smith (1994)’s result means that the number of the shortlisted bidders should not go beyond one more bidder than the efficient level. The remaining question is what exactly is the optimal number of bidders the seller should invite? Our result further answers this question by showing that under monotone hazard rate condition, no less than the efficient level should be shortlisted. On one hand, our finding generalizes the insight of Bulow and Klemperer (1996) in the sense that more bidders are better as long as there is room for total social welfare to be improved. On the other hand, our result differs from Bulow and Klemperer (1996) significantly. Unlike in Bulow and Klemperer (1996), the dominance of one more bidder in our setting cannot be shown by looking at the comparison of the optimal auction with \( N \) bidders and a simple second price auction with \( N + 1 \) bidders because of the adoption of Assumption 1. Much more involved arguments must be employed instead, which established that the optimal auction with \( N + 1 \) bidders must dominate the optimal auction with \( N \) bidders. This is one of the
major contributions of this paper.

3.3.4 On the monotone hazard rate condition

The monotone hazard rate condition adopted in Assumption 3 is the key condition that drives our result on the dominance of one more bidder when \( N \) is lower than the efficient level \( N_c \). This condition guarantees the optimality of a uniform reserve when ex ante fees contingent on value discovery cannot be charged. This result is a key finding that further makes the comparison across different \( N \) feasible when \( N \leq N_c \).

The monotone hazard rate condition also plays a key role in our comparison between \( N_c \) and \( N_c + 1 \). This condition guarantees the concavity of optimal revenue \( R_{N_c}(r_{N_c}^*) \) as a function of cost \( c \), which further entails a clear cut comparison between shortlisting \( N_c \) and \( N_c + 1 \) buyers in Proposition 6.

4 Concluding remarks

In an environment where creation of any gains from trade is only possible if buyers engage in costly information acquisition and these investments on information discovery are covert, we have studied revenue maximizing auctions within a framework of second price auctions with reserves contingent on number of actual bidders, and the role of granting exclusive bidding rights by seller. We obtained several interesting insights concerning these issues: a) The close connection is discovered between the social desirability of investment in value discovery of an additional bidder and the value of a bidder to the seller. The seller would optimally shortlist the efficient number of buyers or one more buyer. Moreover, the comparison between the two options are completely sorted out. To our best knowledge, this is the first time in the literature, the optimal shortlisted is exactly pinned down when ex ante fees upon information acquisition are infeasible. b) The seller may actually prefer to induce random value discovery at the optimum. c) Under a regularity condition of monotone hazard rate, there is no loss of generality to consider uniform reserve for optimal auction design, no matter how many buyers are shortlisted. d) The optimal reserve as a function of the value discovery cost is non-monotonic and discontinuous.

There are two important assumptions in our setup that underline our main results. i.e. trade is inefficient without value discovery, and the monotone hazard rate condition of value distributions. When either of them is dropped, our analysis will no longer completely hold. If trade can be efficient even without value discovery, a reserve equal to the seller’s value would induce the uninformed bidder to make a bid, which is not
incentive compatible in our setting. Dropping the monotone hazard rate condition would get involved the issue of the optimality of contingent reserves, which would introduce additional technicalities in the analysis. While it is interesting to investigate how our main result would change if the two assumptions are not in place, the analysis is beyond the scope of this paper. We will further study these issues in future research.

References


