Optimal Favoritism in Contests with Identity-Contingent Prizes*

Jingfeng Lu† Zhewei Wang‡ Lixue Zhou§

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Abstract

We investigate optimal favoritism using identity-contingent prizes in a two-player Tullock model. Besides the usual balance effect, prize allocation has an extra efficiency effect: One additional unit of prize tends to induce more effort, if it is used as the winning prize for the stronger player whose marginal cost is lower. We find that a total-effort-maximizing (contest) designer should offer a larger prize to the strong player if and only if the contest is sufficiently noisy. Our paper provides a more complete analysis on identity-contingent prizes, which completes the conventional insight on levelling battle field for effort maximization in contests with asymmetric players.

JEL Classification: C72; D72; D74.

Keywords: Balance effect; efficiency effect; favoritism; identity-contingent prizes.

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†Jingfeng Lu: Department of Economics, National University of Singapore, 10 Kent Ridge Crescent, SINGAPORE, 119260; Tel: (65)65166026, Fax: (65)67752646, Email: ecsljf@nus.edu.sg.

‡Zhewei Wang: School of Economics, Shandong University, 27 Shanda Nanlu, Jinan, CHINA, 250100; Tel: (86)53188369992, Fax: (86)53188571371, Email: zheweiwang@sdu.edu.cn.

§Lixue Zhou: School of Economics, Shandong University, 27 Shanda Nanlu, Jinan, CHINA, 250100; Tel: (86)53188369992, Fax: (86)53188571371, Email: sdzhoulixue@outlook.com.

1 Introduction

It is well known that the competitive intensity in a contest crucially depends on the competitive balance between contestants. For instance, in a two-player contest in which the underdog is much weaker than the favorite, one can expect a low level of competition in the sense that the contestants' total effort is low, because the underdog only invests little effort since his winning chance is quite low, and in equilibrium this also leads to a low level of effort from the favorite. Thus, the (contest) designer, who often seeks to maximize total effort, may prefer a biased contest, in which contestants receive unequal treatment and thus a more balanced competition is induced.

In the real world, many competitive balance policies, such as affirmative action policies, are implemented based on ethical concerns. For instance, in the US, applicants from certain underrepresented racial and ethnic groups are preferred in college admissions in many states; gender quotas in political organizations can be observed worldwide. In these situations, a set of contestants with a certain identity are favored in the winner-selection process. This literature on favoritism using biased winner-selection process has been well established, including the contributions of Konrad (2002); Che and Gale (2003); Fu (2006); Siegel (2009, 2014); Pastine and Pastine (2012); Li and Yu (2012); Kirkegaard (2012); Franke et al. (2014); Fu and Wu (2020), and Deng et al. (2021), among many others.[1] The conventional wisdom of leveling the playing field is discovered and repeatedly confirmed under different settings in this literature.[2] Intuitively, this is because when the underdog is incentivized and invests more effort, the favorite's best response is to expend more effort as well.

In this paper, we study optimal favoritism in contests by focusing on identity-contingent prizes rather than biased winner-selection process. As argued by Gürtler and Kräkel (2010), identity-contingent prizes are widely observed. For instance, Baker et al. (1994) and Dohmen et al. (2004) show that there is a considerable variation in pay on each hierarchy level in real organizations, which implies that candidates with different identities may face heterogenous prizes in promotion contests. In recruitment contests, a successful applicant can get different salary offer, which often is associated with his/her ability and background, etc. In some developed countries, foreign employees from developing countries often earn lower

[1]See Chowdhury et al. (2020) for a comprehensive survey of biased contests.

[2]Franke et al. (2013) demonstrate that the optimality of fully leveling the playing field holds in a two-player model, but cannot be extended to a model with more than two players. Drugov and Ryvkin (2021) show in a more general contest setting that the robustness of this finding depends on how players' heterogeneity is modelled. Brown and Chowdhury (2017) show that total effort and sabotage are greater in biased contests.

salaries even when they are better trained or more capable. In universities of Singapore, international graduate students receive lower financial support than their local counterparts though international students typically are more competent.[3] In many sports, the value of the prize may vary substantially across individuals. In a typical UFC (Ultimate Fighting Championship) fight, the winner's real prize consists of a common-value win bonus and an individual-specific pay raise—usually, the more popular/able fighter gets a higher pay raise after his/her victory.

Moreover, it is common to see that an innovative company creates internal competition among multiple research teams, to solve the same technological problem or develop an innovative product prototype.[4] In these situations, a company can set the value of the reward conditional on the team's identify—e.g., the reward for the stronger team with more resources can be smaller or larger than that for the weaker team with less resources. In Lichtenberg (1988), a design competition begins when a federal agency such as the Department of Defense (DoD) issues a formal request for proposals. Typically, a small number of firms exert effort and submit detailed proposals. The firm with the best proposal wins the competition—she gets not only the current (initial) contract but also a sequence of follow-up contracts for R&D, production, spare parts, maintenance and training, etc. over a number of years. These design competitions can be viewed as contests with identity-contingent prizes, since DoD can offer different (initial and follow-up) contracts to different winning firms.[5]

In this paper, we study a contest in which two asymmetric contestants compete for prizes which are identity-contingent. We assume that the designer's expected payoff equals her benefit, which increases with the total effort invested by contestants, less her expected expenditure of allocating prizes to contestants. First, we show that given any expected prize expenditure $\Gamma > 0$, there exists an optimal prize allocation that induces the highest level of total effort in equilibrium. Based on this result, the optimal prize allocation with no constraints on the expected prize expenditure can be obtained by identifying the optimal Γ.[6]

[3] Although they look like examples of affirmative actions, our analysis shows that this practice can be rationalized from a perspective of effort maximization when the winner selection process is accurate enough.

[4] For instance, Telstar Communications had two research teams competing for developing a middleware technology platform (Birkinshaw, 2001). IBM fosters competition between teams for would-be product ideas and problem-solving solutions (Peters and Waterman Jr., 2003). Similar internal competitions between research teams can be found in other tech giant companies, such as Apple and Tencent, when their original product prototypes (of iPhone and Wechat) were developed.

[5] In addition, Lichtenberg (1990) finds that the DoD subsidizes the winning firm's expenditures on private military R&D substantially (over 40% on average) in the long run. This further implies that the true value of the prize for a winning firm can be different, because the amount of the subsidies can vary significantly among firms.

[6] For a similar approach, see Drugov and Ryvkin (2020, Section 4.3), in which a Lazear-Rosen tournament

Surprisingly, by analyzing the optimal prize allocation for a given expected prize expenditure Γ, we find that leveling the playing field is only effective when the contest is sufficiently accurate—that is, when the contest is sufficiently noisy a larger prize should be offered to the strong player, and thus the competition becomes even less balanced in this sense. Our finding echoes that of Drugov and Ryvkin (2017) who show that biased contests (in which one player has an advantage in the winner-selection process) of symmetric players can be optimal under certain conditions, in the sense that making the competition more unbalanced can improve overall performance.[7]

By checking the expression of each player's equilibrium effort, we can see that as the conventional wisdom tells us, there is a tendency for the designer to increase the prize for the weak player, which incentivizes himself and further motivates the strong player. This is referred to as the "balance effect." In the meanwhile, there is a tendency for the designer to increase the prize for the strong player, because a one-unit increase of the prize for the strong player corresponds to a larger increase of his individual effort, since it is less costly for the strong player to exert the same amount of effort compared with his opponent. This is referred to as the "efficiency effect."

Our result suggests that there is an optimal effort-maximizing level of balance between the two effects (i.e., the balance and the efficiency effects). When the contest is sufficiently noisy, the contest itself "overlevels" the playing field in the sense that the weak player's equilibrium winning probability is sufficiently large.[8] Intuitively, the high level of noise (relatively) motivates the weak player better, which weakens the effectiveness of the balance effect when increasing the weak player's individual prize. In this case, the balance effect is dominated by the efficiency effect, and thus it is optimal for the designer to give a larger prize to the more able player to achieve the optimal balance.

In contrast, when the contest is sufficiently accurate, the contest "unlevels" the playing field in the sense that the strong player's equilibrium winning probability is sufficiently large. Intuitively, the low level of noise (relatively) motivates the strong player better, which strengthens the effectiveness of the balance effect when increasing the weak player's individual prize. In this case, the balance effect dominates the efficiency effect, and thus it is optimal for the designer to offer a larger prize to the less able player to achieve the optimal

with a flexible budget is studied.

[7]Fu et al. (2012) show that when an innovation contest involves substantial difficulty, it is optimal to preferentially subsidize the strong player. In a contest where one's output depends on ability and effort, Bastani et al. (2020) show that total effort can be larger when players' abilities get more heterogeneous.

[8]It can be shown that in the initial case with $V_1 = V_2$, the weak player's equilibrium winning probability increases when the contest gets noisier.

balance.[9]

This paper is closely related to Fu and Wu (2020), who provide a general analysis of biased winner selection in N-player contests. In their model, leveling the playing field is always preferable when $N = 2$, although this does not extend to cases with $N \geq 3$. Specifically, with $N = 2$, they find that the total-effort-maximizing contest perfectly levels the playing field in the sense that each player's winning probability is exactly one-half in equilibrium.[10] In contrast, we focus on offering unequal prizes to contestants, and keeping the non-discriminatory winner-selection process intact. Our analysis shows that the two instruments have significantly different implications on optimal favoritism.

Our paper is also closely related to Mealem and Nitzan (2014) who study contests with discriminatory taxation, in which a designer taxes (or subsidizes) players' individual-specific prizes, subject to a requirement of zero taxation on expectation. In their model, players are asymmetric in their pre-tax valuations of winning; while in our model, players are asymmetric in their cost functions. Their analysis in a Tullock contest setting shows that it is optimal to level the playing field without reversing the initial order of their relative strength for effort maximization.[11] In contrast, our analysis shows that it is optimal to offer a larger prize to the strong player (i.e., to unlevel the playing filed) if and only if the contest is sufficiently noisy. For any given eligible prizes satisfying the budget constraint in our setting, the equilibrium analysis can be equivalently obtained from the Mealem and Nitzan model for properly set taxations. However, in general these taxations do not satisfy the zero expected taxation constraint in their setting, which means that the feasible sets of prize structures differ across the two models.[12] Therefore, it is not surprising that optimal designs would diverge across the two settings.

Our paper is most closely related to Gürtler and Kräkel (2010), who also study contests with identity-contingent prizes.[13] They find that the weak player should be offered a larger

[9] See more details about the intuition behind the results after Figure 1.

[10] Epstein et al. (2011) are the first to compare (two-player) all-pay contests and Tullock contests under optimal discrimination in the winner-selection process. Technically, the Tullock contest model of Epstein et al. (2011) with $\gamma = 0$ in the designer's objective function (i.e., the designer only aims to maximize total effort and ignores contestants' payoffs) is equivalent to the two-player model of Fu and Wu (2020).

[11] Please refer to Proposition 2 of Mealem and Nitzan (2014) for details. Note that in their setting, player 1 is assumed to be the strong player in the sense that he has a higher initial valuation of winning the contest, equation (14) and Proposition 2 imply that the optimal taxation scheme requires to lower (resp. raise) the strong (resp. weak) player's prize value, but the strong player still has a higher post-tax value of winning.

[12] More details are available from the authors upon request.

[13] In both Gütler and Kräkel (2010) and this paper, a player gets zero when someone else wins the contest. In contrast, Linster (1993) considers a multi-player Tullock contest with $r = 1$, in which a player may get a non-zero prize when he is not the winner, and the value of the prize depends on the identity of the winning

prize under regularity conditions. This differs from our result, which says that this is only true when the competitive environment is sufficiently accurate. The differences in the two papers' results lie in the differences of the two contest models. Gürtler and Kräkel (2010) study a tournament model, in which a player wins if the players' *effort difference* plus a random noise is strictly greater than zero. We analyze a Tullock contest model, in which, by the interpretation from a noisy-performance-ranking perspective (Fu and Lu, 2012), a player wins if the *difference in logarithms* of players' impact functions plus a random noise is strictly greater than zero. Technically, the result of Gürtler and Kräkel (2010) means that in our model the sum of *logarithms* of players' impact functions is maximized when the weak player is favored. However, this does not mean the sum of equilibrium efforts is maximized at the same time. Let the impact function take a power form with $r \in (0, 1)$ being the power. Let $y_i = \ln(x_i^r)$, $i \in \{1, 2\}$, in which x_i denotes player i's equilibrium effort. Then the sum of efforts is $[\exp(y_1)]^{1/r} + [\exp(y_2)]^{1/r}$. Note that function $[\exp(y)]^{1/r}$ is convex, and the degree of its convexity increases dramatically as r converges to zero. Therefore, for a small r, to maximize the sum of equilibrium efforts subject to a constant value of $(y_1 + y_2)$, one should instead increase the difference between y_1 and y_2. This means that when r is small, to induce a higher level of total effort, it can be more effective to incentivize the stronger player to exert greater effort.

Pérez-Castrillo and Wettstein (2016) study a contest model with incomplete information about players' abilities—i.e., each player's ability is his private information, in which one's perceivable output equals the sum of his effort and ability. They show that with ex ante identical contestants, a discriminatory contest where the reward depends on the identity of the winner (i.e., contests with identity-contingent prizes) can dominate the non-discriminatory contest. In contrast, we study a Tullock contest model with heterogenous contestants under complete information—i.e., each player's ability is common knowledge among all contestants and the (contest) designer.

In market-based tournaments (Waldman, 2013), firms use tournament outcomes to learn about workers' abilities and offer different wages to them. Heterogeneity in workers' prior ability distributions would lead to identity-contingent prizes. Waldman (2013) assumes that firms are unable to commit to future compensation levels or rules concerning whom to promote. In contrast, we study classic tournaments/contests with identity-contingent prizes, in which the prize structure arises from commitment.

opponent.

2 Model

Two contestants, denoted by 1 and 2, compete in a Tullock contest by exerting irreversible effort. Given players' effort entries (x_1, x_2), player i, $\forall i \in \{1, 2\}$, wins the contest with probability

$$P_i(x_1, x_2) = \begin{cases} \dfrac{x_i^r}{x_1^r + x_2^r} & \text{if } x_1 + x_2 > 0, \\ \dfrac{1}{2} & \text{if } x_1 + x_2 = 0, \end{cases} \tag{1}$$

where the parameter $r \in (0, 1]$, which is often referred to as the discriminatory power, can also be interpreted as the accuracy level of the contest. We focus on $r \in (0, 1]$ for two reasons. First, as will be shown later, it is a sufficient condition to ensure the existence and uniqueness of a pure-strategy equilibrium; Second, it makes a direct comparison between this paper and Fu and Wu (2020), in which $r \in (0, 1]$.

Let player i's cost of exerting effort $x_i \geq 0$ be $c_i x_i^b$, where $c_i > 0$ and $b \geq 1$, $\forall i \in \{1, 2\}$. Clearly, this cost function is (weakly) convex, differentiable, and $c(0) = 0$. Without loss of generality, assume that $c_1 \leq c_2$, which implies that player 1 (resp. 2) is the more (resp. less) able player. Define the ability ratio of the two players as

$$c = \frac{c_1}{c_2},$$

and, clearly, $c \in (0, 1]$ since $c_1 \leq c_2$. The two players are symmetric when $c = 1$, and player 1 becomes relatively more able than player 2 when c gets smaller.

The contest designer benefits from greater effort induced from the players, and suffers from higher expected expenditure on prizes paid to the players. Let $\psi(TE)$ be the designer's benefit function, which increases in TE, where TE denotes the contestants' total effort induced in equilibrium. The designer's expected payoff, denoted by Π, equals her benefit $\psi(TE)$ less the expected cost of allocating prizes, denoted by Γ, so we write

$$\Pi = \psi(TE) - \Gamma,$$

where

$$\Gamma = P_1^e V_1 + P_2^e V_2,$$

in which P_i^e is the probability that player i wins and gets prize V_i in equilibrium, $\forall i \in \{1, 2\}$.[14]

[14]Later on, we show that once c_1, c_2, and r are given, for any prize allocation $\mathbf{V} = (V_1, V_2)$ with an expected prize expenditure Γ, there exists a unique pure-strategy equilibrium for any $r \in (0, 1]$, in which the two players' effort outlays x_1^e and x_2^e and the corresponding winning probabilities P_1^e and P_2^e can be derived.

The sequence of moves in this contest game is as follows. In stage 1, the designer chooses an identity-contingent prize allocation, denoted by $\mathbf{V} = (V_1, V_2)$, and announces it publicly. In stage 2, two players exert effort simultaneously, and the winner is rewarded as previously announced in stage 1.

We aim to identify the optimal prize allocation, denoted by $\mathbf{V}^{**} = (V_1^{**}, V_2^{**})$, such that it maximizes the designer's expected payoff Π. In the following analysis, we first show that for any given expected prize expenditure $\Gamma > 0$, there exists a unique optimal prize allocation $\mathbf{V}^*(\Gamma) = (V_1^*(\Gamma), V_2^*(\Gamma))$ that yields the highest level of total effort. That is to say, for any given $\Gamma > 0$, the designer's maximized payoff is given by

$$\Pi(\Gamma) = \psi\left(TE(\mathbf{V}^*(\Gamma))\right) - \Gamma.$$

Thus, $\Pi(\Gamma)$ can be seen as a function of a single variable Γ. We further show that there exists a unique $\Gamma = \Gamma^* > 0$ that maximizes the designer's payoff. Notice that once Γ^* is determined, the optimal identity-contingent prize allocation $\mathbf{V}^{**} = \mathbf{V}^*(\Gamma^*)$ can be immediately identified.

3 Analysis

We first identify the optimal identity-contingent prize allocation $\mathbf{V}^*(\Gamma)$ for an arbitrary expected prize expenditure $\Gamma > 0$.

We solve this two-stage game by backward induction. In stage 2, for a given prize allocation $\mathbf{V} = (V_1, V_2)$, player i's expected payoff is expressed as

$$\pi_i = P_i(x_1, x_2)V_i - c_i x_i^b,$$

where $P_i(x_1, x_2)$ is given by (1), $\forall i \in \{1, 2\}$. The first-order condition of player i's maximization problem implies that

$$rV_i x_i^r x_j^r = bc_i x_i^b (x_i^r + x_j^r)^2, \tag{2}$$

$\forall i, j \in \{1, 2\}$, and $i \neq j$. When a pure-strategy equilibrium exists, we must have $x_1 > 0$ and $x_2 > 0$.[15]

Using (2), we derive that when a pure-strategy equilibrium exists, the equilibrium effort

[15] There exists no equilibrium in which $\min\{x_1, x_2\} = 0$, because any player will have an incentive to secure the prize by exerting an arbitrarily small amount of effort when his opponent's effort is zero,

levels, x_1^e and x_2^e, are given by

$$x_1^e = \left[\frac{r\left(c\frac{V_2}{V_1}\right)^{\frac{r}{b}}}{b\left[1+\left(c\frac{V_2}{V_1}\right)^{\frac{r}{b}}\right]^2} \left(\frac{V_1}{c_1}\right) \right]^{\frac{1}{b}},$$
$$x_2^e = \left[\frac{r\left(c\frac{V_2}{V_1}\right)^{\frac{r}{b}}}{b\left[1+\left(c\frac{V_2}{V_1}\right)^{\frac{r}{b}}\right]^2} \left(\frac{V_2}{c_2}\right) \right]^{\frac{1}{b}}. \tag{3}$$

To facilitate future analysis, we define

$$t = \left(\frac{c_1 V_2}{c_2 V_1}\right)^{\frac{1}{b}}. \tag{4}$$

By (3), we derive that in equilibrium the two players' effort ratio equals t (as defined above):

$$\frac{x_2^e}{x_1^e} = t. \tag{5}$$

This is intuitive, because (5) can be written as $\frac{x_2^e}{x_1^e} = \left(\frac{c_1 V_2}{c_2 V_1}\right)^{\frac{1}{b}}$, which means that player i's equilibrium effort becomes relatively larger than his opponent's when his individual prize (V_i) increases or/and his cost parameter (c_i) decreases.

Next, we show that for any $r \in (0, 1]$, a pure-strategy equilibrium indeed exists.

Lemma 1 *For any prize allocation* $\mathbf{V} = (V_1, V_2)$*, there always exists a unique pure-strategy equilibrium for any* $r \in (0, 1]$*.*

Proof. See Appendix. ■

Using (4) and $c = c_1/c_2$, we have

$$\frac{V_1}{V_2} = \frac{c}{t^b}. \tag{6}$$

From (5) and (6), the following result can be obtained.

Lemma 2 *For a prize allocation* $\mathbf{V} = (V_1, V_2)$ *and* $r \in (0, 1]$*, the two players' effort ratio* x_2^e/x_1^e *always equals* t *in equilibrium, where* t *is given by (4). It can be derived that* $t^b > c$ *if* $V_1 < V_2$*,* $t^b = c$ *if* $V_1 = V_2$*, and* $t^b < c$ *if* $V_1 > V_2$*.*

Notice that in a standard Tullock contest with asymmetric players, it is often assumed that the two players with different abilities (i.e., marginal costs of exerting effort) compete for the same value of prize.[16] This is equivalent to a special case of the current model—i.e., in which $V_1 = V_2$ $(= \Gamma)$ and $b = 1$, and by the above lemma, we have $t = c$ in this case.

More importantly, the above lemma also shows that when prizes are allowed to be identity contingent—i.e., when the cases in which $V_1 \neq V_2$ are also considered, relatively speaking, each player has an incentive to increase (resp. decrease) his effort more than his opponent when his individual prize gets larger (resp. smaller). This result is intuitive, because compared with the benchmark case in which $V_1 = V_2$ and $t = c$, a relatively larger V_i means that player i is better incentivized, and thus x_i becomes relatively larger—e.g., $V_2 > V_1$ (resp. $V_1 > V_2$) means that player 2 (resp. 1) is better motivated than his opponent and thus the equilibrium effort ratio t (recall that $x_2^e/x_1^e = t$ in equilibrium) becomes larger (resp. smaller), which explains why $t^b > c$ (resp. $t^b < c$) in this case.

Next, we seek to identify the optimal prize allocation that maximizes the expected total effort among all prize allocations that have an expected prize expenditure $\Gamma > 0$—i.e., we focus on those prize allocations $\mathbf{V} = (V_1, V_2)$ such that in equilibrium, V_1 and V_2 satisfy

$$P_1^e V_1 + P_2^e V_2 = \Gamma, \tag{7}$$

where P_i^e is the probability that player i wins and gets prize V_i in equilibrium.

In stage 1, among all prize allocations satisfying (7), the designer needs to choose the one that maximizes the equilibrium total effort $TE = x_1^e + x_2^e$, where by (3),

$$TE = \left[\frac{r\left(c\frac{V_2}{V_1}\right)^{\frac{r}{b}}}{b\left[1+\left(c\frac{V_2}{V_1}\right)^{\frac{r}{b}}\right]^2} \right]^{\frac{1}{b}} \left[\left(\frac{V_1}{c_1}\right)^{\frac{1}{b}} + \left(\frac{V_2}{c_2}\right)^{\frac{1}{b}} \right]. \tag{8}$$

Using (4), (7) can be rewritten as

$$\frac{1}{1+t^r}V_1 + \frac{t^r}{1+t^r}V_2 = \Gamma. \tag{9}$$

Notice that once t is given, the corresponding prize allocation $\mathbf{V} = (V_1, V_2)$ satisfying both (4) and (9) is determined, where

$$\begin{aligned} V_1 &= \frac{c(1+t^r)}{c+t^{b+r}}\Gamma, \\ V_2 &= \frac{t^b(1+t^r)}{c+t^{b+r}}\Gamma. \end{aligned} \tag{10}$$

[16]With risk-neutral players, the model with the same ability (i.e., constant marginal cost) but heterogeneous valuations of the prize is equivalent to the model with heterogeneous abilities but the same valuation of the prize. See Wang (2010) and Fu and Wu (2020) for further discussions.

Using (4), (8), and (10), the expression of total effort can be rewritten as

$$TE = (1+t)\left[\frac{rt^r}{bc_2\left(t^r+1\right)\left(c+t^{b+r}\right)}\Gamma\right]^{\frac{1}{b}}, \tag{11}$$

which is a function of a single variable t, given the values of exogenous variables c_1, c_2, r, and Γ.

It should be emphasized that, for any $t \in (0, \infty)$, one can uniquely pin down V_1 and V_2, as in (10). Therefore, the designer's original maximization problem in stage 1—i.e., maximizing (8) subject to (7) by choosing a prize allocation $\mathbf{V} = (V_1, V_2)$ optimally—is equivalent to maximizing (11) by choosing a value of t optimally.

Proposition 1 *For any $\Gamma > 0$, there exists a unique value of t^* such that the expected total effort (11) is maximized at $t = t^*$, where t^* is uniquely determined by*

$$c = \frac{\left(t^*\right)^{b+r}\left(b\left(\left(t^*\right)^r+1\right)+r\left(t^*\right)^r\left(t^*+1\right)\right)}{bt^*\left(\left(t^*\right)^r+1\right)+r\left(t^*+1\right)}. \tag{12}$$

It can be shown that
(i) the value of t^*, which only depends on c and r, has no relation with Γ;
(ii) t^* increases with c, $t^* \in (0,1]$ when $c \in (0,1]$, and $t^* = 1$ if and only if $c = 1$.

Proof. See Appendix. ■

By (10) and (12), we can see that for any given Γ, there is a one-to-one relationship between the optimal equilibrium effort ratio t^* and the optimal prize allocation $\mathbf{V}^* = (V_1^*, V_2^*)$, in which t^* is independent of Γ, and both V_1^* and V_2^* are linearly related to Γ.[17]

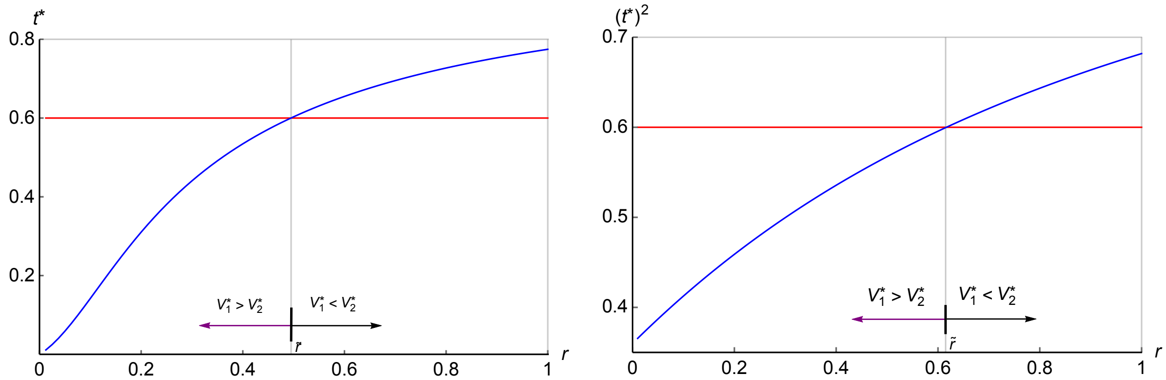
The following is the main result of this paper.

Proposition 2 *When the two players are asymmetric (i.e., when $c < 1$), there exists a unique threshold $\tilde{r}$, where $\tilde{r} \in (0,1]$, such that $V_1^* > V_2^*$ when $r \in (0, \tilde{r})$, $V_1^* = V_2^*$ when $r = \tilde{r}$, and $V_1^* < V_2^*$ when $r \in (\tilde{r}, 1]$; when the two players are symmetric (i.e., when $c = 1$), we always have $V_1^* = V_2^*$ for any $r \in (0,1]$.*

Proof. See Appendix. ■

Figure 1 exemplifies the result of the above proposition through a contest with $c = 0.6$. In this case, it can be derived that $\tilde{r} \approx 0.495$ when $b = 1$, $\tilde{r} \approx 0.615$ when $b = 2$, and we

[17] In Drugov and Ryvkin (2020) also study a tournament model with a flexible budget (Section 4.3), in which they also show that the structure of optimal prizes does not depend on the size of the budget.



(a) $b = 1$ (b) $b = 2$

Figure 1: The relationship between $(t^*)^b$ and r (when $c = 0.6$)

can see that for $r \in (0, 1]$, $V_1^* > V_2^*$ (resp. $V_1^* < V_2^*$) when $r < \tilde{r}$ (resp. $r > \tilde{r}$)—i.e., it is optimal to offer a larger prize to the strong (resp. weak) player if the contest is sufficiently noisy (resp. accurate).

Next, we try to offer some intuition behind the above results. We focus our attention on the case in which $V_1 = V_2$: We seek to see that starting from this case, whether or not the designer has an incentive to favor the weak (resp. strong) player by increasing V_2 and decreasing V_1 (resp. increasing V_1 and decreasing V_2) subject to an expected prize expenditure Γ.

Using (3), player i's equilibrium effort, $\forall i \in \{1, 2\}$, can be rewritten as

$$(x_i^e)^b = \frac{r}{b}\left[p_1(1-p_1)\right]\left(\frac{V_i}{c_i}\right). \tag{13}$$

In the following discussions, $p_1(1-p_1)$ is referred to as the middle term and V_i/c_i as the last term of (13), respectively.

From (13), one can see that a larger V_2 (resp. V_1) will increase (resp. decrease) the two players' effort levels by increasing the middle term of (13), $p_1(1-p_1)$, because it can be shown that for prize allocations $\mathbf{V}$ in which $V_1 \geq cV_2$, $p_1(1-p_1)$ is increasing in V_2 and decreasing in V_1.[18] From this perspective, there is a tendency for a total-effort-maximizing designer to increase V_2 and decrease V_1, which, as noted before, is called the "balance effect." This is because, intuitively, a larger V_2 implies that the weak player (player 2) can be better incentivized, which can further motivate the strong player (player 1)—i.e., a larger V_2 (or a smaller V_1) may increase both players' effort, as the battlefield is more balanced.[19] The

[18] When $V_1 = V_2$, clearly, we have $V_1 \geq cV_2$, as $c \in (0, 1]$.

[19] The balance effect is also evident from the fact that $\partial x_1^e/\partial V_1 > 0$, $\partial x_2^e/\partial V_1 < 0$, $\partial x_1^e/\partial V_2 > 0$, $\partial x_2^e/\partial V_2 >$

middle term is also referred to as the balance-effect term.

Next, we focus on the change in the last term V_i/c_i rather than that in the middle term. From the initial case in which $V_1 = V_2$, a one-unit increase in V_1 will increase player 1's effort by $1/c_1$, while a one-unit increase in V_2 can increase player 2's effort by $1/c_2$ and, clearly, $1/c_2 < 1/c_1$. From this perspective, there is a tendency for the designer to increase V_1 (rather than V_2), which, as noted before, is called the "efficiency effect." The last term is also referred to as the efficiency-effect term.

Notice that with a given Γ, an increase/decrease in V_1 must correspond to a decrease/increase in V_2, in order to make (7) hold. Therefore, starting from the initial case with $V_1 = V_2$, for a given Γ, the balance effect favors increasing V_2/V_1, while the efficiency effect favors decreasing V_2/V_1. In the proof of Proposition 2, we show that V_2^*/V_1^* is increasing in r, where (recall that) V_1^* and V_2^* denote the optimal prize allocation for a given Γ. This result indicates that to maximize total effort, the designer will offer the weak (resp. strong) player a relatively bigger (resp. smaller) prize when the contest becomes more accurate.

Next, we try to further elaborate on the intuition of the optimal prize allocation for different valuations of r. Despite the fact that both the balance and efficiency effects get weaker when r gets smaller—the first term of (13) is exactly r, the optimal prize allocation depends on which effect is relatively larger when r varies. In the following analysis, we focus on the middle term $p_1(1 - p_1)$, which varies with r, rather than the last term V_i/c_i, which remains unchanged when r changes.

In the initial case in which $V_1 = V_2$, it can be shown that in equilibrium $p_1 = 1/(1 + c^{\frac{r}{b}})$, which increases with r. Thus, a smaller r corresponds to a smaller p_1, which leads to a larger $p_1(1 - p_1)$, as $p_1 \geq 1/2$. In other words, the middle term—i.e., the balance effect term—is sufficient large (resp. small) when r is sufficient small (resp. large).

When r is sufficiently low, the noisy contest itself overlevels the playing field in the sense that the strong (resp. weak) player's winning probability p_1 (resp. p_2) is sufficiently small (resp. large), due to the fact that p_1 increases in r (as shown above). Thus, other things equal, the middle term $p_1(1 - p_1)$ is sufficiently large, which further implies that there isn't much room for this balance-effect term to grow. Intuitively, the high level of noise motivates the weak player better (relatively), which weakens the effectiveness of the balance effect when increasing the weak player's individual prize. The above explains why the balance effect is dominated when r is low.

When r is sufficiently high, the accurate contest itself unlevels the playing field in the sense

0, which says that x_1^e increases but x_2^e decreases in V_1; in contrast, both x_1^e and x_2^e increase in V_2.

that the strong (resp. weak) player's winning probability p_1 (resp. p_2) is sufficiently large (resp. small). Thus, other things equal, the middle term $p_1(1-p_1)$ is sufficiently small, which further implies that there is a lot of room for this balance-effect term to grow. Intuitively, the low level of noise motivates the strong player better (relatively), which strengthens the effectiveness of the balance effect when increasing the weak player's individual prize. The above explains why the balance effect dominates when r is high.

In summary, our result suggests that there is an optimal effort-maximizing level of balance between the two (balance and efficiency) effects in a contest with identity-contingent prizes. So if a noisy contest itself "overbalances" the playing field, the designer has the luxury of giving a larger prize to the more able player to achieve the optimal balance.[20]

By (11), we can see that for a given $\Gamma > 0$, the highest total effort is induced if and only if $t = t^*$, where t^* is determined by (12), and thus has no relation with Γ. Then, using (11), for any given Γ, the designer's maximized payoff Π can be written as

$$\Pi(\Gamma) = \psi\left((1+t^*)\left[\frac{r\,(t^*)^r}{bc_2\left((t^*)^r+1\right)\left(c+(t^*)^{b+r}\right)}\Gamma\right]^{\frac{1}{b}}\right) - \Gamma,$$

which shows that for given values of c_1, c_2, and r, Π is a function of a single variable Γ.

The following assumption is a sufficient (but not necessary) condition for the existence and uniqueness of Γ^* that maximizes the designer's expected payoff.

Assumption 1 *The designer's benefit function $\psi(\cdot)$ is twice differentiable, with $\psi(0) = 0$, $\psi'(x) > 0$, $\psi''(x) < 0$ for any $x > 0$, $\lim_{x\to 0^+} \psi'(x) = \infty$, and $\lim_{x\to\infty} \psi'(x) < 1$.*

Under this assumption, it can be shown that Γ^* is uniquely determined by $\Pi'(\Gamma^*) = 0$. The following proposition offers the optimal prize allocation that maximizes the designer's expected payoff, with no constraint on the expected prize expenditure Γ.

Proposition 3 *Under Assumption 1, there exists a unique $\Gamma^* > 0$ such that $\mathbf{V}^{**} = \mathbf{V}^*(\Gamma^*) = (V_1^*(\Gamma^*), V_2^*(\Gamma^*))$ is the optimal prize allocation that maximizes the designer's expected payoff.*

4 Concluding Remarks

Contests with identity-contingent prizes are commonplace in the real world—e.g., pay variation within hierarchy levels in organizations, and different winning prizes for individual

[20]We thank an anonymous reviewer who pointed this out in a clear and organized fashion.

athletes in sports.[21] We study the optimal design of identity-contingent prizes in Tullock contests, and find that the strong/weak player should be offered a larger prize if the contest is sufficiently noisy/accurate, which implies that the conventional wisdom of leveling the playing field does not hold in general. Our results are in contrast to the findings obtained from contest models with biases in the winner-selection process, in which leveling the playing field is always preferable in a two-player setting (Fu and Wu, 2020). Our findings can be intuitively explained by a balance effect and an efficiency effect when setting identity-contingent prizes.

In this paper, we have been focusing on an environment with two contestants, since we mainly aim to reveal whether the weaker player should always be favored in such a setting. A more general analysis with multiple players is technically challenging, but it could be rewarding in terms of revealing other interesting findings. We leave this to future work.

5 Appendix

5.1 Proof of Lemma 1

In equilibrium, given $x_1 = x_1^e$ and $x_2 = x_2^e$, the two players' expected payoffs are given by

$$\begin{aligned} \pi_1^* &= \left[\frac{b\,(t^r+1) - rt^r}{b\,(t^r+1)\,(c+t^{b+r})}\right] c\Gamma, \\ \pi_2^* &= \left[\frac{b\,(t^r+1) - r}{b\,(t^r+1)\,(c+t^{b+r})}\right] t^{b+r}\Gamma. \end{aligned}$$

$\pi_1^* \geq 0$ and $\pi_2^* \geq 0$ are equivalent to that $b\,(t^r+1) \geq rt^r$ and $b\,(t^r+1) \geq r$—i.e.,

$$\begin{aligned} b\,(t^r+1) &\geq rt^r, \\ b\,(t^r+1) &\geq r. \end{aligned}$$

Because $r \leq 1$ and $b \geq 1$, we always have $\pi_1^* \geq 0$ and $\pi_2^* \geq 0$.

The first and second derivatives are expressed as follows:

[21] More examples can be found inside organizations. For instance, a company may run an R&D contest to promote innovations, in which multiple innovator are involved—e.g., an inside innovator and an outside innovator (Neyer et al., 2009), and the prize value can be different, depending on the winner's identity.

$$\frac{\partial \pi_1}{\partial x_1} = \frac{rV_1 x_1^r x_2^r - bc_1 x_1^b \left(x_1^r + x_2^r\right)^2}{x_1 \left(x_1^r + x_2^r\right)^2}$$

$$= \frac{x_1^{r-1}}{\left(x_1^r + x_2^r\right)^2} \left[rV_1 x_2^r - \underbrace{bc_1 x_1^{b-r} \left(x_1^r + x_2^r\right)^2}_{\text{increasing in } x_1} \right], \tag{14}$$

$$\frac{\partial \pi_2}{\partial x_2} = \frac{rV_2 x_1^r x_2^r - bc_2 x_2^b \left(x_1^r + x_2^r\right)^2}{x_2 \left(x_1^r + x_2^r\right)^2}$$

$$= \frac{x_2^{r-1}}{\left(x_1^r + x_2^r\right)^2} \left[rV_2 x_1^r - \underbrace{bc_2 x_2^{b-r} \left(x_1^r + x_2^r\right)^2}_{\text{increasing in } x_2} \right], \tag{15}$$

$$\frac{\partial^2 \pi_1}{\partial x_1^2} = \frac{rV_1 x_1^{r-2} x_2^r}{\left(x_1^r + x_2^r\right)^3} \underbrace{\left[(r-1)\, x_2^r - (1+r)\, x_1^r\right]}_{<0} - \underbrace{b\,(b-1)\, c_1 x_1^{b-2}}_{\geq 0}, \tag{16}$$

$$\frac{\partial^2 \pi_2}{\partial x_2^2} = \frac{rV_2 x_2^{r-2} x_1^r}{\left(x_1^r + x_2^r\right)^3} \underbrace{\left[(r-1)\, x_1^r - (1+r)\, x_2^r\right]}_{<0} - \underbrace{b\,(b-1)\, c_2 x_2^{b-2}}_{\geq 0}. \tag{17}$$

Given $x_2 = x_2^e$, using the above expressions of the first and second derivatives and the expressions of equilibrium effort, it can be shown that $\frac{\partial^2 \pi_1}{\partial x_1^2} < 0$ for any $x_1 \in [0, \infty)$, given any $r \in (0, 1]$, $b \in [1, +\infty)$. To see this, using (16) with $x_2 = x_2^e > 0$, we obtain that $\frac{\partial^2 \pi_1}{\partial x_1^2} < 0$ for any $x_1 \in [0, \infty)$, since $\left[(r-1)\, x_2^r - (1+r)\, x_1^r\right] < 0$ and $b\,(b-1)\, c_1 x_1^{b-2} \geq 0$. By equation (2), $\frac{\partial \pi_1}{\partial x_1} = 0$ if $x_1 = x_1^e$. Using (14), we derive that $\frac{\partial \pi_1}{\partial x_1} > 0$ as $x_1 \to 0^+$, because, for any $r \in (0, 1]$, we have

$$\lim_{x_1 \to 0^+} \frac{\partial \pi_1}{\partial x_1} = \lim_{x_1 \to 0^+} \frac{1}{x_1^{1-r}} \left(\frac{1}{x_2^r}\right) rV_1 > 0;$$

and $\frac{\partial \pi_1}{\partial x_1} < 0$ when x_1 is sufficiently large, because $rV_1 x_2^r - bc_1 x_1^{b-r} \left(x_1^r + x_2^r\right)^2 < 0$ for sufficiently large x_1. The above results imply that $\frac{\partial \pi_1}{\partial x_1} > 0$ if $x_1 < x_1^e$, $\frac{\partial \pi_1}{\partial x_1} = 0$ if $x_1 = x_1^e$, and $\frac{\partial \pi_1}{\partial x_1} < 0$ if $x_1 > x_1^e$. Also, we can see that given $x_2 = x_2^e$, $\pi_1(x_1 = 0) = 0$ and $\pi_1(x_1 = x_1^e) > 0$, it is safe to conclude that given $x_2 = x_2^e$, for $x_1 \in [0, \infty)$, $\pi_1(x_1)$ reaches its global maximum at $x_1 = x_1^e$.

Following a similar procedure, we can also derive that given $x_1 = x_1^e$, $\frac{\partial \pi_2}{\partial x_2} > 0$ if $x_2 < x_2^e$, $\frac{\partial \pi_2}{\partial x_2} = 0$ if $x_2 = x_2^e$, and $\frac{\partial \pi_2}{\partial x_2} < 0$ if $x_2 > x_2^e$. Also notice that given $x_1 = x_1^e$, $\pi_2(x_2 = 0) = 0$ and $\pi_2(x_2 = x_2^e) > 0$. The above results imply that for $x_2 \in [0, \infty)$, $\pi_2(x_2)$ reaches its global maximum at $x_2 = x_2^e$. Therefore, (x_1^e, x_2^e) constitutes a pure strategy Nash equilibrium.

5.2 Proof of Proposition 1

We can derive that

$$\frac{dTE}{dt} = \frac{c_2 t^{-r-1}}{r\Gamma}\left[\frac{rt^r\Gamma}{bc_2\left(t^r+1\right)\left(t^{b+r}+c\right)}\right]^{1+\frac{1}{b}} \times \left[bt\left(t^r+1\right)+r(t+1)\right]\left(c-f(t,r)\right),$$

where

$$f(t,r)=\frac{t^{b+r}\left(b\left(t^r+1\right)+rt^r(t+1)\right)}{bt\left(t^r+1\right)+r(t+1)}, \tag{18}$$

which is a function of t and r. Note that $0<c\leq 1$ and $f(t,r)>0$. We derive that

$$\frac{\partial f}{\partial t} = \frac{(b+r)t^{b+r-1}}{\left(bt\left(t^r+1\right)+r(t+1)\right)^2} \times \left[\begin{array}{c} b\left(b-1+r\right)\left(t^{2r+1}+2t^{r+1}+t\right) \\ +\left(br+2r^2\right)\left(t^{r+2}+t^r\right) \\ +\left(4r^2+brt^{r+1}\right)t^{r+1}+br \end{array}\right]>0.$$

That is to say, $f(t,r)$ is increasing in t. Both $f(t=0)=0$ and $f(t=1)=1$ imply that $c-f(t=0)>0$ and $c-f(t=1)<0$. Thus, there exists a unique $t=t^*$, where $t^*\in(0,1]$, such that

$$\frac{dTE}{dt} \gtreqless 0 \text{ if } t \lesseqgtr t^*.$$

Clearly, TE reaches its maximum at $t=t^*$.

5.3 Proof of Proposition 2

Given c and r, we have a unique t^*. Next, we analyze the relationship between t^* and r. From the proof of Proposition 1, t^* is determined by $c=f(t^*,r)$, where

$$f(t^*,r)=\frac{(t^*)^{b+r}\left[b(t^*)^r+b+r(t^*)^{r+1}+r(t^*)^r\right]}{b\left((t^*)^{r+1}+t^*\right)+r\left(t^*+1\right)}. \tag{19}$$

We further derive that

$$\frac{\partial f(t^*,r)}{\partial r}=\frac{t^{b+r}[g_1((t^*)^{r+1}-1)+g_2\log(t^*)]}{\left[bt\left(t^r+1\right)+r(t+1)\right]^2},$$

where

$$\begin{aligned} g_1 &= b(t^*+1)((t^*)^r+1), \\ g_2 &= 2r^2(t^*+1)^2(t^*)^r + b^2 t^*((t^*)^r+1)^2 \\ &\quad +br(t^*+1)(2(t^*)^{r+1}+(t^*)^{2r+1}+2(t^*)^r+1). \end{aligned}$$

It can be verified that when $c = 1$, we always have $t^* = 1$. Next, we turn to the more interesting case in which $c < 1$. For any given $c < 1$, we have $t^* < 1$. We can further derive that $\frac{\partial f(t^*,r)}{\partial r} < 0$ for any $c < 1$, because $g_1 > 0$, $(t^*)^{r+1} - 1 < 0$, $g_2 > 0$ and $\log(t^*) < 0$. Based on the fact that $c = f(t^*, r)$, $\frac{\partial f(t^*,r)}{\partial t^*} > 0$, and $\frac{\partial f(t^*,r)}{\partial r} < 0$, total differentiation of $f(t^*, r)$ implies that $\frac{dt^*}{dr} > 0$ for a given $c < 1$.

When $r \to 0^+$, (19) becomes

$$c = \lim_{r\to 0^+} f(t^*, r) = \lim_{r\to 0^+} (t^*)^{b+r-1} = \lim_{r\to 0^+} (t^*)^{b-1}.$$

Thus, we further derive that

$$\lim_{r\to 0^+} (t^*)^b = c^{\frac{b}{b-1}} < c. \tag{20}$$

When $r = 1$, (12) becomes $c = (t^*)^{b+1}(b + t^*) / (bt^* + 1)$, which implies that

$$(t^*)^b|_{r=1} = \frac{bt^*+1}{bt^*+(t^*)^2} c > c. \tag{21}$$

Given any $c < 1$, we have shown $\frac{dt^*}{dr} > 0$, (20) and (21) imply that there exists a unique value of $r = \tilde{r}$, where $\tilde{r} \in (0, 1]$, such that $(t^*)^b = c$ when $r = \tilde{r}$, and $\tilde{r} = 1$ if and only if $c = 1$; $t^* < c$ when $r < \tilde{r}$; and $t^* > c$ when $r > \tilde{r}$.

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