Information sharing decisions in all-pay auctions with correlated types*

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In many real-life competitions, contestants may not be aware of their own type (e.g., value or ability) prior to the contest. Furthermore, contestants' types, which are observed privately after entering the competition, are frequently correlated with one another. We examine a two-stage competition that involves two players with correlated (binary) types. In the first stage, players decide simultaneously or sequentially on the probabilities they use to disclose or conceal their type, which will become their private information later on. In the second stage, each player privately observes their own type and commits to disclosing or concealing it, after which they compete in an all-pay auction. We discover that information sharing does not occur when players' types are negatively correlated. However, when players' types are positively correlated, information is partially shared in all equilibria examined in this study. In an asymmetric pure strategy equilibrium, one player shares his information with probability one and the other player with probability zero. In a symmetric mixed strategy equilibrium, each player shares his information with the same positive probability.

Keywords: All-pay auctions; Contests; Correlated types; Information disclosure, Pure/Mixed strategy equilibrium.

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1 Introduction

In many real-life competitions, a contestant may only become aware of their type (which could be their valuation of winning or their ability to exert effort) after they have entered the competition. Furthermore, contestants' types can be positively or negatively correlated in various competitive environments. In these situations, it is typically assumed that all contestants, including the contest designer, share a prior belief about the contestants' types before the competition.

For instance, during the early stages of a pandemic outbreak, two or more pharmaceutical companies may compete to develop an effective vaccine. Since the cause of the pandemic is new, they may not know their capabilities prior to the competition. If both companies use traditional (vector) vaccine technology, the effectiveness of their vaccines may be positively correlated. However, if one company uses traditional technology while the other pursues a new (mRNA) technology, the effectiveness of their vaccines may be negatively correlated.[1] Moreover, consider a typical internal innovation competition between two (or multiple) research teams within a company.[2] If both teams adopt similar scientific approaches after entering the competition, when one team observes that he is a high-performing (high-type) team, it is natural for him to anticipate that the other team is also likely to be high-performing. However, if the two teams adopt contrasting scientific approaches, one team's high performance implies that the other team, which adopts a different approach, is likely to have low performance.

Establishing an information disclosure policy prior to a competition can be observed in various real-world events. For example, in a typical innovation competition involving several private firms, a firm can commit to disclose information about its future research capability by utilizing public funds. In such cases, the firm is subject to specific considerations regarding disclosure and transparency, while other firms do not face the same level of public scrutiny. In the next-generation electric-vehicle competition, Tesla's patent open-source strategy can be viewed as a commitment to revealing its re-

[1] We thank an anonymous referee for providing this example.

[2] Such competitions are commonplace in high-tech companies nowadays. For example, in the early 2000s, Microsoft had several research teams working on developing a new user interface for their operating system, Windows. The winning team's interface became the basis for the new Windows Vista operating system. Similarly, when Apple and Tencent developed the original prototypes of the iPhone and WeChat, respectively, they were engaged in internal innovation competitions.

search ability. By making all of its current and future patents open and available for any firm, Tesla can signal its ability at an early stage of the competition, in which other significant electric-vehicle manufacturers, like BYD and Volkswagen, are also involved.

In the current competition within the field of generative artificial intelligence (AI) among major technology companies, including OpenAI, Google, Meta, IBM, and others, two primary scientific approaches can be observed: the open-source approach and the closed-source approach. For example, OpenAI (with Chat GPT) and Google (with Bard) adopt the closed-source approach, whereas Meta (with LLaMA) embraces the open-source approach.[3] In a typical AI race focused on developing a successful next-generation model, companies adopting the open-source approach need to reveal their research capabilities at an early stage of the competition due to the transparency of their open-source nature. As a result, selecting the open-source approach can be regarded as a commitment to a disclosure policy during the ex ante stage of the competition. On the other hand, companies choosing the closed-source approach can keep their research abilities private throughout the competition. [4]

In this paper, we study a two-player contest model, in which every player chooses an information disclosure policy between "disclosure" (denoted by D) and "concealment" (denoted by C) before they observe their own type. We assume that if a player chooses D, the information about his type, which will become available to himself at a later stage, can be verified by a third party (e.g., his opponent or the designer) with zero cost. Thus, when a player chooses D, he has no incentive to misreport his type. Also, when a player chooses C, his type will remain being his private information throughout the contest. Another key assumption we make in this study is that contestants' types are correlated. The correlation between players' types is measured by a single parameter ρ, where $\rho \in (-1, 1)$.[5]

The timing of the game is as follows: In stage 1, two players choose their own disclosure policies between D and C, sequentially or simultaneously. When contestants

[3]According to an internal document widely circulated within Google and other companies in Silicon Valley (Love et al., 2023), open-source AI models currently exhibit comparable performance and, in certain aspects, even outperform closed-source AI models developed by Google and OpenAI.

[4]The key difference between open-source AI models and closed-source AI models lies in that open-source models, such as Meta's LLaMA, offer public access to the model weights (i.e., the parameters obtained during the training process) whereas closed-source models do not provide such access.

[5]$\rho = 1$ (resp. $\rho = -1$) corresponds to the case with a perfectly positive (resp. negative) correlation.

chose their policies sequentially, without loss of generality, let player 1 be the leader who moves first and player 2 the follower who moves after observing player 1's disclosure policy. In stage 2, each player first observes his own type privately, and then discloses or conceals it as committed before; after that, two players compete in an all-pay auction under an information structure determined in stage 1. Notice that for different disclosure policies chosen in stage 1, the two players' equilibrium bidding strategies in stage 2 can be quite different, since they compete under different information structures.

To analyze players' policy choices in stage 1, it is essential to first conduct an equilibrium analysis of an all-pay auction held in stage 2, which offers a player different expected payoffs under different policies determined in stage 1. Our equilibrium analysis of stage 2 shows that under a full disclosure policy (D, D), there is a unique mixed strategy equilibrium of Hillman and Riley (1989). Under a partial disclosure policy (D, C), there exists a unique mixed strategy equilibrium following Konrad (2009). Under no disclosure policy (C, C), if players' types are mildly correlated, a unique monotone equilibrium in mixed strategies can be constructed following the approach of Siegel (2014). However, if players' types are sufficiently positively or negatively correlated, there are no papers in the literature which we can directly rely on to characterize players' bidding behavior.[6] We thus construct a unique bidding equilibrium in mixed strategies for cases where players' types are sufficiently (positively or negatively) correlated. This equilibrium is nonmonotonic in the sense that bid intervals for the two types overlap. In this sense, the paper makes a technical contribution to the literature by presenting a novel approach for identifying a nonmonotonic equilibrium when players' types are sufficiently correlated such that the monotonic equilibria of Siegel (2014) do not exist.[7]

After conducting an analysis of stage 2 under all possible information structures, we move backward to stage 1 to analyze players' disclosure policies in equilibrium. Our analysis shows that when players move sequentially in stage 1: If players' types are

[6]Siegel's (2014) procedure is not applicable since the conditions required are not satisfied.

[7]Specifically, we use Lemma 1 from Siegel (2014) to show that in any possible equilibrium, the bidding strategies of both types are continuous over their individual intervals. Then, we determine the equilibrium (expected) payoffs for each type—we show that when the players' types are sufficiently positively correlated, both types' equilibrium payoffs are zero. When the players' types are sufficiently negatively correlated, the l type's equilibrium payoff is zero, and the h type's equilibrium payoff is equal to the difference between their prize valuations. Finally, we derive the equilibrium cumulative distribution function (CDF) for each type using the payoff equations that must hold in equilibrium.

positively correlated, there is a unique subgame perfect Nash equilibrium (henceforth: SPNE) denoted by (C, D), in which the leader chooses C and the follower chooses D in stage 1; in stage 2, both players compete in an all-pay auction under one-sided asymmetric information—in the sense that one player's type is commonly known and the other's remains his private information. If the players' types are negatively correlated, there is a unique SPNE denoted by (C, C), in which both players choose concealment in stage 1; in stage 2, they compete in an all-pay auction under two-sided asymmetric information in the sense that each player's type is his private information. When players move simultaneously in stage 1, similar results are obtained.

The intuition behind the above results is as follows. If players' types are positively correlated, with no information disclosure (when (C, C) is chosen), the battlefield is balanced in the sense that each player faces an opponent who is likely to be the same type. Thus, the competition in the all-pay auction is fierce. This implies that the expected total effort level is high, and each player's expected payoff is low. In contrast, with partial information disclosure (when (C, D) is chosen), the battlefield is less balanced and competition in the all-pay auction is less fierce, because there is a considerable chance that the player choosing C will find that his opponent choosing D is a different type. Intuitively, the above explains why (C, D) is chosen in equilibrium, in which the leader, who has a first-mover advantage, gets a larger expected payoff than the follower.[8] Note that (D, D) and (C, D) correspond to all-pay auctions with complete information and one-sided asymmetric information, respectively. With positively correlated types, the competition in the all-pay auction under (D, D) is more fierce than that under (C, D), since on average the battlefield under (D, D)—in which every player is likely to see an opponent who is the same type—is more balanced than that under (C, D). This explains why (D, D) is always dominated by (C, D) from either player's perspective, and thus cannot be chosen in equilibrium.

If the players' types are negatively correlated, with no disclosure policy (when (C, C) is chosen), the battlefield is already imbalanced in the sense that each player

[8]In the SPNE, player 1 (the leader) anticipates that if he chooses C, it is optimal for player 2 (the follower) to choose D after observing his disclosure policy—because from player 2's perspective, choosing D leads to a less balanced playing field and thus a lower level of competition intensity, which leads to a higher payoff than that from choosing C.

faces an opponent who is likely to be a different type. Thus, the competition in the all-pay auction in stage 2 is relatively less fierce, which implies that the expected total effort level is relatively low and each player's expected payoff is relatively high. Intuitively, this explains why the two players "coordinate" on (C, C) in equilibrium in stage 1.

Besides the above pure strategy equilibria (i.e., both the partial-disclosure and no-disclosure equilibria), we also analyze a symmetric mixed strategy equilibrium (SMSE) in which each player commits to disclose his type with a fixed positive probability in stage 1.[9] We further show that such an SMSE exists if players' types are positively correlated, and it does not exist if the types are independently distributed or negatively correlated. When players' types are positively correlated, each player's disclosing probability (i.e., the probability of sharing his private information with his opponent) first increases and then decreases when players' types become more positively correlated in SMSE. We also show that at the threshold level of correlation ($\rho = \bar{\rho}$), each player's disclosing probability reaches its global maximum 1/2, which implies that the probability that information sharing occurs (i.e., the probability of at least one player sharing information) in the contest game is 3/4.[10] Thus, our insights obtained from the previous analysis of pure strategy equilibria are robust to this generalization about mixed strategy equilibria.

Relation to the literature. This paper belongs to the literature on the comparison of information disclosure policies in auctions and contests, which has been relatively extensively studied. Milgrom and Weber (1982) establish the linkage principle, which states that an auctioneer ex ante should commit to fully disclosing all available information to all bidders. Baye et al. (1993, 1996) present a characterization of the equilibrium set for all-pay auctions with complete information. Hurley and Shogren (1998) compare prize allocation efficiency across different information structures in Tullock contests. Ganuza (2004) focuses on an auctioneer's incentive to release information about the object's characteristics to bidders, and finds that more information should be revealed when the competition gets fiercer. Morath and Münster (2008) compare full disclosure

[9] We show that there does not exist an asymmetric mixed strategy equilibrium, in which the probability of choosing a policy (C or D) differs across the two agents. This is also the reason we focus on symmetric mixed strategy equilibria rather than asymmetric ones.

[10] In this case, the probability of exactly one player sharing information is 1/2, and the probability of both players sharing information is 1/4.

and concealment policies for standard auctions,[11] and find that full concealment policy induces greater total effort in all-pay auctions than other standard auctions.[12] Chen (2019) compares full disclosure and concealment policies in all-pay auctions with affiliated types, and finds that revenue ranking between the two policies depends on the affiliation of bidders' types and the number of bidders.[13] In Chen (2019), the decision to disclose or conceal all players' types is determined by the designer. However, in our study, we grant each player the autonomy to make independent decisions regarding their information disclosure policies.

Zhang and Zhou (2016) study disclosure policies in Tullock contests from a Bayesian persuasion perspective. In a two-player model with one-sided asymmetric information, they show that there is no loss of generality to restrict to full disclosure and concealment policies for total effort maximization. Moreover, Kuang, Zhao and Zheng (2019) study information disclosure policies in all-pay auctions from a Bayesian persuasion perspective, in which contestants' ex ante symmetric type distributions are assumed to be correlated.[14] Chen and Chen (2022) study two-player all-pay auctions with one-sided private information and interdependent valuations, in which the information disclosure policy can be designed through Bayesian persuasion with respect to the player with private information. They show that the optimal information disclosure policy depends on players' relative strengths and the degree of valuation dependence. A notable distinction between Chen and Chen (2022) and this paper is that the designer in their paper opts to disclose an optimal level of information (none, partial, or full) regarding the private information of the player to his opponent. In contrast, our study examines scenarios in which each player independently makes decisions regarding information disclosure.

Our paper is closely related to that of Kovenock, Morath and Münster (2015) in the sense that both studies analyze a model in which contestants make decisions on disclosure policies prior to competing in all-pay auctions.[15] This paper differs from theirs

[11] Full concealment policy is referred to as "no disclosure policy" in this paper.

[12] Fu et al. (2014) generalize Morath and Münster's analysis by considering a multi-prize model.

[13] Chen and Serena (2022) study disclosure policies for players' types in a two-player all-pay auction with a uniform exogenously given bid cap.

[14] Chen, Kuang and Zheng (2022) also study a two-player all-pay auction model with one-sided asymmetric information, in which they completely characterize the designer's optimal Bayesian persuasion signal for different orders of (sequential and simultaneous) moves.

[15] Wu and Zheng (2017) study a similar question in a setting of lottery contests with independent types, and find that information sharing arises (resp. is strictly dominated) in equilibrium if types are

mainly because they assume that contestants have independent types, while we consider correlated types. They show that with independent types, when players make decisions independently, D is always dominated by C and (C, C) is a unique equilibrium.[16] In contrast, in our setting with correlated types, (C, C) is a unique equilibrium if players' types are negatively correlated, and (C, D) is a unique equilibrium if players' types are positively correlated. Our results imply that information sharing is indeed possible if players' types are positively correlated, which contrasts with Kovenock, Morath and Münster (2015), in which information sharing never occurs in equilibrium.[17]

Lu, Ma and Wang (2018) study an all-pay contest in which each player's valuation follows a binary distribution independently. The contest designer, who observes players' valuations ex post, commits to an information policy among the following four policies ex ante—full concealment, full disclosure, full disclosure if both types are high, and full disclosure if both types are low.[18]

Antsygina (2022) analyzes a situation in which the designer commits to disclosing (privately or publicly) or concealing players' values first, and then two contestants decide whether to share their information if it arrives privately. To ease the analysis, Antsygina (2022) imposes regularity conditions on the value distribution, and focuses on symmetric information sharing equilibrium in the subgame where the designer discloses players' values privately to each of them. In our analysis, no restrictions are imposed on value distribution, we allow for asymmetric information sharing and characterize all possible equilibria. Nevertheless, the findings of Antsygina (2022) on equilibrium information sharing are consistent with ours.

concentrated enough (resp. sufficiently dispersed).

[16]Besides the case in which players decide independently whether to disclose their private information, they also study information exchange in industry-wide agreements, where firms can enter binding agreements and either all firms share information or no firm does, and show that an industry-wide agreement to share information may arise as an equilibrium.

[17]Kovenock, Morath and Münster (2008) show that for any player, C is a dominant strategy when players make decisions independently in the first stage. This result contrasts ours which are obtained with zero correlation: A player is indifferent between D and C given his opponent's choice being C. Technically, this difference is caused by the two (binary and continuous) distributions adopted in the two papers. We study a binary-distribution case in which players' types follow a binary distribution; they consider a continuous-distribution case in which one's type follows a continuous function defined on a closed interval. See Footnote 18 of Kovenock, Morath and Münster (2008) for details on how their results (Proposition 2) would change in the case of a binary distribution.

[18]Note that the last two information policies are type-contingent. To our best knowledge, Serena (2022) is the first to study type-contingent information disclosure in contest literature.

2 Model setup

We consider a single prize all-pay auction with two players, in which each player can be one of two types. The type space for each player i is $\Theta_i = \{h, l\}$, $i \in \{1, 2\}$, in which h denotes the high type and l denotes the low type. A high type has a high winning value or a low bidding cost, which are strategically equivalent. In this paper, without loss of generality, we assume that if $\theta_i = h$, the value of the prize for player i is v_h; if $\theta_i = l$, the value of the prize for player i is v_l, where $v_h > v_l$; the marginal bidding cost of each player is normalized to 1.

A symmetric probability function $f : \Theta_1 \times \Theta_2 \longrightarrow \mathbb{R}$ depicts the joint distribution of the two players' types. This prior type distribution is common knowledge. For brevity, we let $f(\theta_i) = \sum_{-i\in\Theta_{-i}} f(\theta_1, \theta_2)$, and denote by $f(\theta_i|\theta_{-i}) = f(\theta_i, \theta_{-i})/f(\theta_{-i})$ the conditional probability of player i's type given player $(-i)$'s type θ_{-i}, where player $(-i)$ refers to player $3 - i$, who is the opponent of player i.[19] The correlation coefficient ρ is defined as

$$\begin{aligned}\rho &= f(\theta_i = h|\theta_{-i} = h) - f(\theta_i = h|\theta_{-i} = l)\\ &= f(\theta_i = l|\theta_{-i} = l) - f(\theta_i = l|\theta_{-i} = h),\end{aligned}$$

which denotes the correlation level of the two players' types.[20] The second equality of the above equation holds due to the fact that $f(h|h) + f(l|h) = f(l|l) + f(h|l) = 1$.

Denote the effort (or bid) of player i by x_i. In an all-pay auction, the winning probability of player i is specified as follows:

$$P_i(x_i, x_{-i}) = \begin{cases} 1 & \text{if } x_i > x_{-i}, \\ 0 & \text{if } x_i < x_{-i}, \end{cases}$$

and if $x_i = x_{-i}$, the tie-breaking rule is typically endogenously determined as a part of the equilibrium with $P_1(x_1, x_2) + P_2(x_1, x_2) = 1$.

Assume that both players do not know their own types before the all-pay auction

[19] It is clear that if $i = 1$, player i's opponent is player 2, since $3 - i = 2$; if $i = 2$, player i's opponent is player 1, since $3 - i = 1$ in this case.

[20] Note that ρ can be expressed as $\rho = f(h|h) - f(h|l) = f(l|l) - f(l|h)$ for brevity.

takes place—i.e., ex ante each player only knows the prior type distribution given above. Specifically, we study the following two-stage game. In stage 1, two players choose their own disclosure policies between concealment (C) and disclosure (D), sequentially or simultaneously. In particular, when players move sequentially, without loss of generality, let player 1 be the leader who moves first and player 2 the follower who moves after observing player 1's disclosure policy. In stage 2, an arbitrary player i first observes his own type θ_i privately, $i \in \{1, 2\}$, and then discloses or conceals his type as committed. After that, two players compete in an all-pay auction, in which they simultaneously choose their nonnegative effort outlays (x_1, x_2) based on their updated beliefs about their opponents' types. Notice that for different disclosure policies chosen in stage 1, contestants' bidding behaviors (e.g., their expected effort levels) in stage 2 can be quite different, since they compete under different information structures.

3 Bidding strategies in all-pay auctions (stage 2)

Given the disclosure policies chosen in stage 1, in stage 2 the two players first disclose/conceal their types as committed, then compete in an all-pay auction. We solve the two-stage game by backward induction. In this section, we analyze different bidding equilibria in an all-pay auction of stage 2 under all possible disclosure policies, which are determined in stage 1, respectively.

3.1 Full disclosure policy of (D, D)

If both players choose to disclose their type, either player knows exactly his opponent's type, and thus the two players bid in an all-pay auction with complete information. For each type profile, a unique mixed strategy equilibrium can be derived by Hillman and Riley (1989). The following proposition summarizes these results. The proofs are omitted to save space.

Proposition 1. *With full disclosure policy (D, D), the (unique) equilibrium mixed strategies are as follows:*

(i) For type profile $\{h, h\}$, both players bid uniformly on $[0, v_h]$ with a zero equilibrium

expected payoff.

(ii) For type profile $\{l, l\}$, both players bid uniformly on $[0, v_l]$ with a zero equilibrium expected payoff.

(iii) For type profile $\{h, l\}$, the h type bids uniformly on $[0, v_l]$, and the l type bids uniformly on $(0, v_l]$ with a mass bid at zero with probability $\frac{v_h - v_l}{v_h}$. The equilibrium expected payoff is $v_h - v_l$ for the h type and zero for the l type.

3.2 Partial disclosure policy of (D, C)

Consider the case in which one player chooses D and the other player chooses C. Without loss of generality, let player 1 be the player choosing to disclose his type, and player 2 be the one choosing to conceal his type.[21] Konrad (2009) characterizes a mixed strategy equilibrium in an all-pay auction under one-sided asymmetric information with discrete types. By Konrad (2009), the equilibrium is presented in the following proposition. The proofs are also omitted.

Proposition 2. *Under policy (D, C), the (unique) equilibrium mixed strategies of two players are as follows.*

(i) If player 1 is the l type, he bids on $[0, v_l]$ with a mass point at 0 with probability $\frac{f(h|l)(v_h - v_l)}{v_h}$, and the equilibrium mixed strategy for player 1 is

$$F_l(x) = \begin{cases} \frac{x}{v_l} + \frac{f(h|l)(v_h - v_l)}{v_h}, & \forall x \in [0, f(l|l)v_l), \\ \frac{x}{v_h} + \frac{v_h - v_l}{v_h}, & \forall x \in [f(l|l)v_l, v_l]. \end{cases} \tag{1}$$

The equilibrium expected payoff of player 1 (of l type) is zero. [22]

For player 2, who is the l type, upon observing that player 1 is the l type, bids uniformly on $[0, f(l|l)v_l]$ and the equilibrium mixed strategy for player 2 (of l type) is

$$F_{ll}(x) = \frac{x}{f(l|l)v_l}, \quad \forall x \in [0, f(l|l)v_l]. \tag{2}$$

[21] In the case of (D, C) in which player 1 chooses D and player 2 chooses C, the probability distribution in Proposition 2 is $f(\theta_2|\theta_1)$, and recall that $f(\theta_2|\theta_1) > 0$ by assumption. In the case of (C, D) in which player 1 chooses C and player 2 chooses D, the equilibrium can also be characterized in a similar way as in Proposition 2, where the probability distribution is $f(\theta_1|\theta_2)$ accordingly.

[22] It can be verified that there is no mass point at $x = f(l|l)v_l$.

The equilibrium expected payoff of the player 2 (of l type) is $\frac{f(h|l)(v_h - v_l)v_l}{v_h}$.

For player 2, who is the h type, upon observing that player 1 is the l type, bids uniformly on $[f(l|l)v_l, v_l]$ and the equilibrium mixed strategy for player 2 (of h type) is

$$F_{lh}(x) = \frac{x - f(l|l)v_l}{f(h|l)v_l}, \quad \forall x \in [f(l|l)v_l, v_l]. \tag{3}$$

The equilibrium expected payoff of player 2 (of h type) is $v_h - v_l$.

(ii) If player 1 is the h type, he bids on $[0, f(l|h)v_l + f(h|h)v_h]$ and the equilibrium mixed strategy for player 1 is

$$F_h(x) = \begin{cases} \frac{x}{v_l}, & \forall x \in [0, f(l|h)v_l), \\ \frac{x}{v_h} + \frac{f(l|h)(v_h - v_l)}{v_h}, & \forall x \in [f(l|h)v_l, f(l|h)v_l + f(h|h)v_h]. \end{cases} \tag{4}$$

The equilibrium expected payoff of player 1 (of h type) is $f(l|h)(v_h - v_l)$. [23]

For player 2, who is the l type, upon observing that player 1 is the h type, bids uniformly on $[0, f(l|h)v_l]$ with a mass point at 0 with probability $\frac{v_h - v_l}{v_h}$, and the equilibrium mixed strategy for player 2 (of l type) is

$$F_{hl}(x) = \frac{x}{f(l|h)v_h} + \frac{v_h - v_l}{v_h}, \quad \forall x \in [0, f(l|h)v_l]. \tag{5}$$

The equilibrium expected payoff of player 2 (of l type) is zero.

If player 2 is the h type, upon observing that player 1 is the h type, bids uniformly on $[f(l|h)v_l, f(l|h)v_l + f(h|h)v_h]$ and the equilibrium mixed strategy for player 2 (of h type) is

$$F_{hh}(x) = \frac{x - f(l|h)v_l}{f(h|h)v_h}, \quad \forall x \in [f(l|h)v_l, f(l|h)v_l + f(h|h)v_h]. \tag{6}$$

The equilibrium expected payoff of player 2 (of h type) is $f(l|h)(v_h - v_l)$.

There are two things which may deserve our more attention. First, in an equilibrium of Proposition 2, given player 1 (who discloses his type) being the h type, his expected payoff (ex ante) is $f(l|h)(v_h - v_l)$; in the meanwhile, if player 2 (who conceals his type) is also the h type, his expected payoff is also $f(l|h)(v_h - v_l)$. Second, in an equilibrium

[23] It can be verified that there is no mass point at $x = f(l|h)v_l$.

of Proposition 2, when both players are the h type, the one who conceals his type (i.e., player 2) has an informational advantage in the sense that his expected payoff is higher. Intuitively, when both players are the h type, the one who discloses his type (D) bids lower on average than the one who conceals his type (C), because the one choosing D does not know which type he faces but the one choosing C knows that he faces an h type for certain. The above results do not contradict each other, because in the former case, player 1, who is the h type and chooses D, expects player 2's type by the prior distribution; in the latter case, player 2, who is the h type and chooses C, knows that player 1 is the h type with probability one.

3.3 No disclosure policy of (C, C)

If both players choose to conceal their types,[24] each player's belief about the opponent's type is the same as the common prior distribution—i.e., they compete under two-sided asymmetric information. In this section, we assume that the probability distribution has full support, which means that $f(\theta_i|\theta_{-i}) > 0$ for each player $i = 1, 2$.

Next, according to the magnitude of the correlation coefficient, we characterize the equilibria for the following three cases.

Case (a)**.** The two players' types are mildly (positively and negatively) correlated in the sense that $\underline{\rho} < \rho < \bar{\rho}$.

Case (b)**.** The two players' types are sufficiently positively correlated in the sense that $\bar{\rho} \leq \rho < 1$.

Case (c)**.** The two players' types are sufficiently negatively correlated in the sense that $-1 < \rho \leq \underline{\rho}$.

For this standard asymmetric-information all-pay auction with discrete signals, since the full support assumption is satisfied, we only need to verify the monotonicity conditions required by Siegel (2014). When the monotonic conditions are satisfied (i.e., in case (a)), we derive a unique monotonic equilibrium following the constructive approach of Siegel (2014).

[24]In this paper, (C, C) is referred to as the "no disclosure policy," which is also called the "full concealment policy" in many other studies.

Following Siegel (2014), the monotonic conditions are

$$\begin{cases} f(h|h)v_h > f(h|l)v_l, \\ f(l|h)v_h > f(l|l)v_l, \end{cases} \tag{7}$$

which can be reduced to $-\frac{f(h|l)(v_h - v_l)}{v_h} < \rho < \frac{f(l|l)(v_h - v_l)}{v_h}$. To facilitate our analysis, denote $\bar{\rho} = \frac{f(l|l)(v_h - v_l)}{v_h}$ and $\underline{\rho} = -\frac{f(h|l)(v_h - v_l)}{v_h}$, and the above inequality becomes $\underline{\rho} < \rho < \bar{\rho}$.

When the monotonic conditions are not satisfied (i.e., in both cases (b) and (c)), the procedure described by Siegel (2014) is no longer applicable. To characterize a nonmonotonic equilibrium in cases (b) and (c), we still use Lemma 1 of Siegel (2014), which offers a key result regarding the continuity of bidding strategies in any equilibrium, including nonmonotonic ones in cases (b) and (c). Formally, we present this result in the following lemma.

Lemma 1. *In any possible equilibrium, the bidding strategies of both types are continuous on their individual supports.*

Next, we uniquely determine the corresponding distribution functions of the two types based on their equilibrium (expected) payoffs. To do so, we first determine the equilibrium payoffs for each type. Specifically, we show that when the players' types are sufficiently positively correlated, both types' equilibrium payoffs are zero; when the players' types are sufficiently negatively correlated, the l type's equilibrium payoff is zero, and the h type's equilibrium payoff is equal to the difference between their prize valuations. Finally, we derive the equilibrium cumulative distribution function (CDF) for each type by using the expected payoff equations that must hold in equilibrium.[25]

Proposition 3. *Under policy (C, C), the (unique) equilibrium mixed strategies of two players are as follows.*

[25] See Proof of Proposition 3 in the Appendix for details.

(i) In case (a) in which $\underline{\rho} < \rho < \bar{\rho}$, the corresponding CDFs of the two types are

$$F_h(x) = \frac{x - f(l|l)v_l}{f(h|h)v_h}, \quad \forall x \in [f(l|l)v_l, f(l|l)v_l + f(h|h)v_h],$$
$$F_l(x) = \frac{x}{f(l|l)v_l}, \quad \forall x \in [0, f(l|l)v_l].$$

The equilibrium expected payoff is $f(l|h)v_h - f(l|l)v_l$ for the h type and zero for the l type.

(ii) In case (b) in which $\bar{\rho} \leq \rho < 1$, the corresponding CDFs of the two types are

$$F_h(x) = \begin{cases} \frac{f(l|l)v_l - f(l|h)v_h}{(f(l|l)-f(l|h))v_h v_l}x, & \forall x \in [0, \bar{x}_l), \\ \frac{x - f(l|h)v_h}{f(h|h)v_h}, & \forall x \in [\bar{x}_l, v_h], \end{cases}$$
$$F_l(x) = \frac{f(h|h)v_h - f(h|l)v_l}{(f(l|l) - f(l|h))v_h v_l}x, \quad \forall x \in [0, \bar{x}_l],$$

where $\bar{x}_l = \frac{(f(l|l)-f(l|h))v_h v_l}{f(h|h)v_h - f(h|l)v_l}$. [26] *The equilibrium expected payoff of both types is zero.*

(iii) In case (c) in which $-1 < \rho \leq \underline{\rho}$, the corresponding CDFs of the two types are

$$F_h(x) = \frac{(f(l|h)v_h - f(l|l)v_l)x - f(l|l)v_l(v_h - v_l)}{(f(l|h) - f(l|l))v_h v_l}, \quad \forall x \in [\underline{x}_h, v_l],$$
$$F_l(x) = \begin{cases} \frac{x}{f(l|l)v_l}, & \forall x \in [0, \underline{x}_h), \\ \frac{(f(h|l)v_l - f(h|h)v_h)x + f(h|l)v_l(v_h - v_l)}{(f(l|h)-f(l|l))v_h v_l}, & \forall x \in [\underline{x}_h, v_l], \end{cases}$$

where $\underline{x}_h = \frac{f(l|l)v_l(v_h - v_l)}{f(l|h)v_h - f(l|l)v_l}$. [27] *The equilibrium expected payoff is $v_h - v_l$ for the h type and zero for the l type.*

Proof. See Appendix. □

The above results indicate that under no disclosure policy (C, C): (a) When players' types are mildly correlated in the sense that $\underline{\rho} < \rho < \bar{\rho}$, the maximal bidding value is $f(l|l)v_l + f(h|h)v_h$; the equilibrium expected payoff is $f(l|h)v_h - f(l|l)v_l$ for the h type and zero for the l type. (b) When players' types are sufficiently positively correlated in the sense that $\bar{\rho} \leq \rho < 1$, the maximal bidding value is v_h; the equilibrium expected payoff of both types is zero.(c) When players' types are sufficiently negatively corre-

[26] It can be verified that there is no mass point at $x = \bar{x}_l$.
[27] It can be verified that there is no mass point at $x = \bar{x}_h$.

lated in the sense that $-1 < \rho \leq \underline{\rho}$, the maximal bidding value is v_l; the equilibrium expected payoff is $v_h - v_l$ for the h type and zero for the l type.

The above results further imply that the intensity of competition between contestants becomes more fierce when players' types are more correlated—in the sense that (i) one's maximal bidding value increases with ρ, as evidenced by $v_l < f(l|l)v_l + f(h|h)v_h < v_h$, in which $f(l|l)v_l + f(h|h)v_h$ also increases with ρ; (ii) an h type's expected payoff decreases with ρ, as evidenced by $0 < f(l|h)v_h - f(l|l)v_l < v_h - v_l$, in which $f(l|h)v_h - f(l|l)v_l$ also decreases with ρ. Intuitively, if players' types are more positively correlated, with no information disclosure—i.e., when both players choose C—the playing field gets more balanced in the sense that each player faces an opponent who is more likely to be the same type. Thus, competition in the all-pay auction becomes relatively more fierce, which implies that players bid more aggressively on average and each player's expected payoff gets lower.

In addition, we also observe that in case (a), the h type bids in an upper interval and the l type bids in a lower interval, and the two intervals do not overlap; in contrast, in cases (b) and (c), the two types' bidding intervals overlap. More specifically, it is simple to show that the overlapping bidding interval in case (b) (resp. (c)) gets larger when players' types become more positively (resp. negatively) correlated. For instance, in case (b), both players bid on the overlapping interval $[0, \bar{x}_l]$—using $\bar{x}_l = \frac{(f(l|l)-f(l|h))v_h v_l}{f(h|h)v_h - f(h|l)v_l}$, we can see that $\bar{x}_l$ increases in ρ, which implies that the overlapping interval $[0, \bar{x}_l]$ gets larger for a larger ρ.[28] Therefore, intuitively, when ρ gets larger in case (b) or ρ gets smaller in case (c), an arbitrary player is able to expect his opponent's type with a higher probability, which makes their overlapping bidding interval relatively larger.

4 Disclosure policy choices (stage 1)

Let π denote a player's equilibrium expected payoff in an all-pay auction. By Propositions 1 to 3, the following results can be obtained.

[28] In case (c), it also can be shown that the overlapping (bidding) interval $[\underline{x}_h, v_l]$ becomes larger when ρ gets smaller, since $\underline{x}_h = \frac{f(l|l)v_l(v_h - v_l)}{f(l|h)v_h - f(l|l)v_l}$ increases in ρ.

Lemma 2. *(i) Under policy* (D, D)*, for each player,*

$$\pi \mid_{(D,D)} = f(h,l)(v_h - v_l). \tag{8}$$

(ii) Under policy (D, C)*, for the player who chooses* D*,*

$$\pi_D \mid_{(D,C)} = f(h,l)(v_h - v_l). \tag{9}$$

For the player who chooses C*,*

$$\pi_C \mid_{(D,C)} = \left(f(h,h)f(l|h) + f(h,l) + f(l,l)f(h|l)\frac{v_l}{v_h} \right)(v_h - v_l). \tag{10}$$

(iii) Under policy (C, C)*, it can be derived that*

$$\pi \mid_{(C,C)} = 0 \textit{ if } \bar{\rho} \leq \rho < 1, \tag{11}$$

$$\pi \mid_{(C,C)} = f(h)(f(l|h)v_h - f(l|l)v_l) \;\; \textit{if } \underline{\rho} < \rho < \bar{\rho}, \tag{12}$$

$$\pi \mid_{(C,C)} = f(h)(v_h - v_l) \textit{ if } -1 < \rho \leq \underline{\rho}. \tag{13}$$

Proof. See Appendix. □

It is straightforward to see that $\pi_C \mid_{(D,C)} > \pi \mid_{(D,D)}$, which implies that it is optimal for any player to choose C, given that his opponent chooses D.

We now compare $\pi \mid_{(C,C)}$ and $\pi_D \mid_{(D,C)}$ for ρ in different regions. If $\underline{\rho} < \rho < \bar{\rho}$, we derive that

$$\begin{aligned}
\pi \mid_{(C,C)} - \pi_D \mid_{(D,C)} &= f(h)(f(l|h)v_h - f(l|l)v_l) - f(h,l)(v_h - v_l) \\
&= (f(l|h) - f(l|l))f(h)v_l \\
&= -\rho f(h)v_l.
\end{aligned}$$

This implies that $\pi \mid_{(C,C)} < \pi_D \mid_{(D,C)}$ if $0 < \rho < \bar{\rho}$; $\pi \mid_{(C,C)} > \pi_D \mid_{(D,C)}$ if $\underline{\rho} < \rho < 0$; and $\pi \mid_{(C,C)} = \pi_D \mid_{(D,C)}$ if $\rho = 0$. Moreover, if $\bar{\rho} \leq \rho < 1$, we obtain that $\pi \mid_{(C,C)} < \pi_D \mid_{(D,C)}$; and if $-1 < \rho \leq \underline{\rho}$, we have $\pi \mid_{(C,C)} > \pi_D \mid_{(D,C)}$, since

$\pi\mid_{(C,C)} - \pi_D\mid_{(D,C)} = f(h,h)(v_h - v_l) > 0$. To sum up, the above results imply that (i) With a positive correlation (between the players' types), it is optimal for a player to choose D given that his opponent chooses C. (ii) With a negative correlation, it is optimal for a player to choose C given that his opponent chooses C. (iii) With no correlation, a player is indifferent between C and D given that his opponent chooses C.

When players choose their disclosure policies sequentially in stage 1, recall that player 1 is the leader and player 2 the follower. From player 1's perspective, if he chooses D, player 2 will choose C—since $\pi_C\mid_{(D,C)} > \pi\mid_{(D,D)}$—which implies that player 1 gets an expected payoff of $\pi_D\mid_{(D,C)}$ by choosing D. If player 1 chooses C, player 2's policy decision depends on the level of correlation: Player 2 will choose D (resp. C) if the players' types are positively (resp. negatively) correlated, and will be indifferent between D and C if there is no correlation. Thus, from player 1's perspective, he gets an expected payoff of $\pi_C\mid_{(D,C)}$ (resp. $\pi\mid_{(C,C)}$) by choosing C when there is a positive (resp. negative) correlation.

By comparing player 1's expected payoffs between choosing D and C, we obtain the following results.

Proposition 4. *When contestants move sequentially in stage 1: (i) If players' types are positively correlated, there is a unique subgame perfect Nash equilibrium (SPNE), denoted by $(C; D)$, in which player 1 chooses to conceal his type and player 2 chooses to disclose his type. (ii) If players' types are negatively correlated, there is a unique SPNE, denoted by $(C; C)$, in which both players choose to conceal their type. (iii) If players' types are independently distributed, there are two SPNE, denoted by $(C; C)$ and $(C; D)$, respectively, in which player 1 (the leader) always chooses to conceal his type, but player 2 (the follower) may choose either policy.*

Proposition 5. *When contestants move simultaneously in stage 1: (i) If players' types are positively correlated, one player chooses D and the other player chooses C; such an equilibrium is referred to as a "partial disclosure equilibrium." (ii) If players' types are negatively correlated, in equilibrium both players choose C; such an equilibrium is referred to as a "no disclosure equilibrium." (iii) If players' types are independently distributed, either a partial or a no disclosure equilibrium can occur.*

In summary, the above two propositions imply that in both cases in which players choose their disclosure policies sequentially or simultaneously, information sharing is possible if the players' types are positively correlated. However, if the players' types are negatively correlated, from an arbitrary player's perspective, sharing information is always dominated regardless of which policy his opponent chooses.

We now offer an intuitive explanation for the above results. If players' types are positively correlated, with no information disclosure—i.e., when both players choose C—the playing field is balanced in the sense that each player faces an opponent who is likely to be the same type. Thus, competition in the all-pay auction is relatively fierce, which implies that the expected total effort level is high and each player's expected payoff is low. In contrast, with partial information disclosure—i.e., when one player chooses D and the other chooses C, competition in the all-pay auction is less fierce. This is because there is a considerable chance that the player choosing C finds that his opponent choosing D is a different type—in this case, the playing field gets less balanced and the competition intensity decreases. The above explains why a partial disclosure equilibrium occurs in the case of positive correlation.

In contrast, if players' types are negatively correlated, with no information disclosure—i.e., when $(C; C)$ is chosen—the playing field is already imbalanced in the sense that each player faces an opponent who is likely to be a different type. Thus, relatively speaking, competition in the all-pay auction (in stage 2) is less fierce, which implies that the expected total effort level is low and each player's expected payoff is high. Intuitively, this explains why $(C; C)$ is chosen by the two players in equilibrium when there is a negative correlation between their types.

In the above analysis, in which contestants move simultaneously (resp. sequentially) in stage 1, we focus on pure strategy equilibria; it has been shown that if players types are positively correlated, it must be the case that one player (resp. the leader) chooses C and the other (resp. the follower) chooses D in a pure strategy equilibrium. Therefore, in a simultaneous-move game with positive correlation, from an arbitrary player's perspective, there are two pure strategy equilibria: First, player 1 chooses C and player 2 chooses D; and second, player 1 chooses D and player 2 chooses C. It can be further shown that, despite the fact that the above two equilibria yield the same level of expected payoff for the designer, they generate different expected payoffs for the players, in the sense that the player choosing C gets a strictly higher level of expected payoff than the player choosing D. In other words, the above partial disclosure equilibrium does not offer the same level of expected payoff for the two players ex ante in equilibrium.

Next, in the same simultaneous-move environment with correlated types, we consider a possible symmetric mixed strategy equilibrium, which is denoted by $(p, 1-p)$, where $p \in (0,1)$ is the probability of each player's choosing policy D. According to the property of such a symmetric mixed strategy equilibrium (SMSE), given an arbitrary player's equilibrium strategy—say, player 2's equilibrium strategy—player 1 should be indifferent between choosing C and D since choosing either policy yields the same level of expected payoff in equilibrium. We obtain the following proposition by analyzing an SMSE, as proposed above.

Proposition 6. *When contestants move simultaneously in stage 1: (i) If players' types are positively correlated, there exists a unique symmetric mixed strategy equilibrium (SMSE) denoted by* $(p, 1-p)$*, where* $p \in (0,1)$ *is the probability of each player's choosing policy D, such that*

$$p = \frac{(f(h)f(l|l) - f(h,l))v_l}{f(h,h)f(l|h)(v_h - v_l) + f(l,l)f(h|l)(v_h v_l - v_l^2)/v_h + (f(h)f(l|l) - f(h,l))v_l}$$

if players' types are mildly positively correlated with $0 < \rho < \bar{\rho}$*, and*

$$p = \frac{f(h,l)}{f(h,h)f(l|h) + f(h,l) + f(l,l)f(h|l)v_l/v_h}$$

if players' types are sufficiently positively correlated with $\bar{\rho} \leq \rho < 1$*. (ii) If the two players' types are negatively or independently distributed, such an SMSE does not exist. (iii) There does not exist an asymmetric mixed strategy equilibrium, in which the probability of choosing a policy differs across the two agents, regardless of players' types being positively, independently, or negatively distributed.*

Proof. See Appendix. □

Corollary 1. *When players' types are positively correlated, each player's ex ante expected payoff in an SMSE is* $\pi\mid_{(S,S)} = f(h,l)(v_h - v_l)$*.*

Combining the results obtained in Proposition 5 and Corollary 1, we derive that the expected payoff for an arbitrary player in an SMSE is weakly smaller than that of either player in a partial disclosure (pure strategy) equilibrium, since $\pi_C \mid_{(D,C)} > \pi_D \mid_{(D,C)} = \pi \mid_{(S,S)} = f(h,l)(v_h - v_l)$. This further implies that when the two players' types are positively correlated, from an arbitrary player's perspective, he prefers the partial disclosure (pure strategy) equilibrium to a corresponding SMSE ex ante, since either player (i.e., either a player is choosing C or a player is choosing D) in the partial disclosure equilibrium has a weakly larger expected payoff than that in the SMSE. In this sense, when the two players' types are positively correlated, a partial disclosure (pure strategy) equilibrium dominates a corresponding SMSE ex ante from each player's perspective.

Intuitively, in the SMSE, the battlefield is relatively balanced in the sense that each player adopts a symmetric (mixed strategy) strategy, while in the corresponding partial disclosure (pure strategy) equilibrium, the battlefield is relatively imbalanced in the sense that the two players adopt different strategies (C and D). The above implies that the competition is relatively more fierce in an SMSE compared with a corresponding partial disclosure equilibrium. This offers an intuitive explanation for the result of $\pi_C \mid_{(D,C)} > \pi_D \mid_{(D,C)} = \pi \mid_{(S,S)} = f(h,l)(v_h - v_l)$, which states that any player's equilibrium expected payoff is weakly lower in an SMSE than that in a corresponding partial disclosure equilibrium.

We have shown that there exists a unique SMSE for any $\rho \in (0,1)$. By analyzing the relation between each player's probability of choosing the "disclosure" policy, p, and the level of positive correlation, ρ, the following results are obtained.

Proposition 7. *In symmetric mixed strategy equilibria (SMSE), each player's disclosing probability p and the players' correlation coefficient ρ have the following relationship: p increases with ρ for $\rho \in (0,\bar{\rho})$ and decreases with ρ for $\rho \in [\bar{\rho},1)$. It can be shown that for all $\rho \in (0,1)$, p reaches its global maximum at $\rho = \bar{\rho}$; moreover, $p \to 0$ when $\rho \to 0$, $p \to \frac{1}{2+v_l/v_h} < \frac{1}{2}$ when $\rho \to 1$, and $p = \frac{1}{2}$ at $\rho = \bar{\rho}$.*

Proof. See Appendix. □

To illustrate the relationship between p and ρ, consider a specific example in which

$v_h = 2$, $v_l = 1$, $f(h) = \frac{1}{3}$ and $f(l) = \frac{2}{3}$. Figure 1 illustrates the nonmonotonic relationship between ρ and p in SMSE, which is consistent with the characterization of Proposition 7: As seen in Figure 1, each player's probability of information sharing (i.e., the disclosing probability), p, first increases and then decreases when ρ increases from 0 to 1; moreover, p reaches its global maximum $p = \frac{1}{2}$ at $\rho = \bar{\rho} = 0.4$.

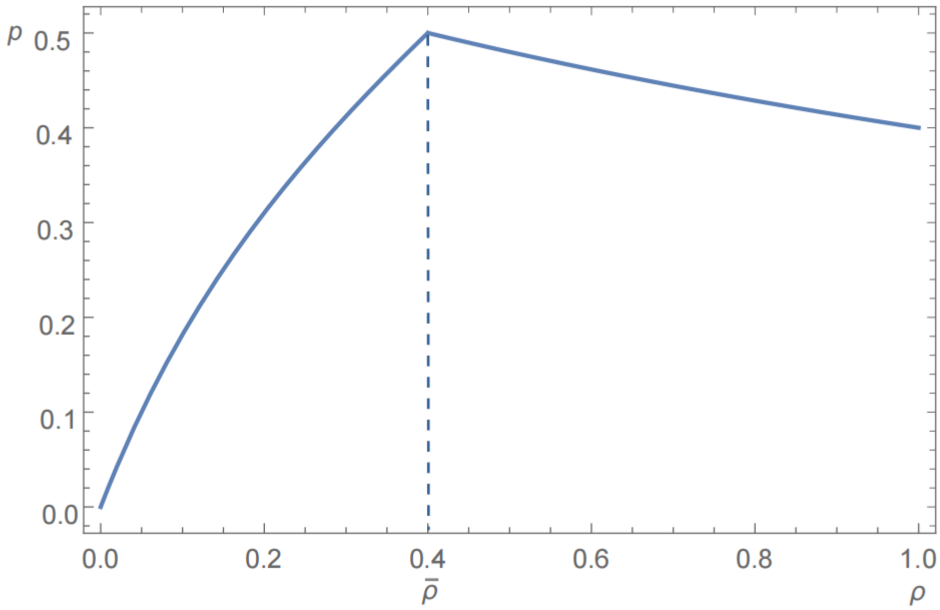

Figure 1.The nonmonotonic relation between ρ and p (in the specific example)

Notice that when $\rho = \bar{\rho}$, each player's disclosing probability reaches its global maximum $\frac{1}{2}$. This further implies that in such an SMSE with $\rho = \bar{\rho}$, the probability of information sharing (i.e., the probability of at least one player sharing information) in equilibrium is $\frac{3}{4}$, which equals the sum of the probability of exactly one player sharing information ($\frac{1}{2}$) and the probability of both players sharing information ($\frac{1}{4}$). Therefore, our analysis of mixed strategy equilibria reassures us of the robustness of our previous results obtained with pure strategy equilibria: Information sharing can indeed be observed when the players' types are positively correlated.[29]

[29] Information sharing occurs for certain in a pure strategy equilibrium, in which one player chooses D with probability one; it occurs with some positive probability in an SMSE, in which each player chooses D with probability $p > 0$ in equilibrium.

5 Concluding remarks

We analyze a two-player contest model with correlated type, in which contestants who observe their own types privately in the second stage (after entering the contest) choose their disclosure policies in the first stage (prior to the contest). In both cases in which contestants choose their disclosure policy sequentially and independently, similar results are obtained: If players' types are positively correlated, there is a partial disclosure equilibrium in which exactly one player chooses D (disclosure) and the other chooses C (concealment);[30] if players' types are negatively correlated, we will have a no-disclosure equilibrium in which both players choose C.[31]

The main results of this paper are obtained in a relatively restrictive model involving two players with binary correlated types. Considering the evident intuition behind these results (refer to the Introduction), it is plausible that the results of this paper could extend to more general settings. In conducting analysis within such broader settings, the main challenge lies in identifying a systematic approach to constructing a nonmonotonic equilibrium in cases where the monotonicity condition of Siegel (2014) does not hold. We leave this exploration for future research.

[30]In particular, when contestants move sequentially in stage 1, the leader chooses C and the follower chooses D in equilibrium. In a partial disclosure equilibrium, the player choosing C has a larger expected payoff than that of the player choosing D.

[31]After the analysis of pure strategy equilibria, we also investigate contestants' disclosure equilibria in mixed strategy, and our insights are robust to this generalization—information sharing can be observed in equilibrium when players' types are positively correlated.

Appendix

A.1 Proof of Proposition 3

For case (a), the equilibrium strategy can be established by using the constructive approach of Siegel (2014). Under policy (C, C), both types hold their belief about the opponent player's type as the prior distribution. We can thus obtain a symmetric equilibrium.

Note that in this case the h type must choose a strictly higher level of effort than the l type in equilibrium. Therefore, there are two intervals in the joint partition. In the top interval, both h types are the ones with the best responses, and in the bottom interval, both l types are the ones with the best responses.

We start with the top interval. For any two bids $0 < x < y$ in the top interval, the equilibrium expected payoffs of the h type are the same, which is shown as

$$f(h|h)v_h F_h(x) - x = f(h|h)v_h F_h(y) - y,$$
$$\frac{F_h(y) - F_h(x)}{y - x} = \frac{1}{f(h|h)v_h},$$

so by taking $y - x$ to 0, the density function of the h type is

$$f_h(\cdot) = \frac{1}{f(h|h)v_h}. \tag{14}$$

The length L_1 of the top interval is obtained from the fact that in the interval, either type h of one player exhausts the bidding probability of 1. Since the equilibrium is symmetric, we have

$$L_1 = \min\{\frac{1}{\frac{1}{f(h|h)v_h}}, \frac{1}{\frac{1}{f(h|h)v_h}}\} = f(h|h)v_h. \tag{15}$$

Next, for any two bids $0 < x < y$ in the bottom interval, the equilibrium expected payoffs of the l type are the same, which is shown as

$$\begin{aligned} f(l|l)v_l F_l(x) - x &= f(l|l)v_l F_l(y) - y, \\ \frac{F_l(y) - F_l(x)}{y - x} &= \frac{1}{f(l|l)v_l}, \end{aligned}$$

so the density function of the l type in the bottom interval is

$$f_l(\cdot) = \frac{1}{f(l|l)v_l}. \tag{16}$$

Similarly, we can obtain the length of the bottom interval L_2 as

$$L_2 = \min\{\frac{1}{\frac{1}{f(l|l)v_l}}, \frac{1}{\frac{1}{f(l|l)v_l}}\} = f(l|l)v_l. \tag{17}$$

All of the above results establish the equilibrium strategies. Furthermore, the conditions of the two-player auction under the full concealment policy also satisfy the monotonicity condition of Siegel (2014). Therefore, the symmetric equilibrium we pin down is a unique equilibrium.

The equilibrium expected payoff of the h type for $x \in [f(l|l)v_l, f(l|l)v_l + f(h|h)v_h]$ is thus

$$\begin{aligned} v_h(f(h|h) \cdot F_h(x) + f(l|h) \cdot 1) - x &= v_h \left(\frac{x - f(l|l)v_l}{v_h} + f(l|h) \right) - x \\ &= f(l|h)v_h - f(l|l)v_l, \end{aligned}$$

and the equilibrium expected payoff of the l type for $x \in [0, f(l|l)v_l]$ is

$$v_l \left(f(h|l) \cdot 0 + f(l|l)F_l(x) \right) - x = v_l \frac{x}{v_l} - x = 0.$$

Next, we characterize the equilibrium strategies for case (b) and case (c). Firstly, Lemma 1 of Siegel (2014) establishes that no holes exist within the support of each type, and that the equilibrium distributions of both types are continuous—this result is formally presented in Lemma 1. By Lemma 1, we obtain that if an equilibrium exists, even for a nonmonotonic one in cases (b) and (c), the l type bids continuously on the interval $[\underline{x}_l, \overline{x}_l]$, where $0 \leq \underline{x}_l < \overline{x}_l$, and the h type bids continuously on the interval $[\underline{x}_h, \overline{x}_h]$, where $0 \leq \underline{x}_h < \overline{x}_h$.

In both cases (b) and (c), it can be shown that in any possible equilibrium, the expected payoff for the l type is zero and $\underline{x}_l = 0$. We show the above results through proof by contradiction. If the equilibrium payoff for the l type is strictly positive, it can be further shown that the equilibrium payoff for the h type must also be strictly positive. This is because the h type's equilibrium payoff is always weakly greater than the l type's payoff in any possible equilibrium.[32] Additionally, it can be shown that $\underline{x}_l \leq \underline{x}_h$ in any possible equilibrium. For the l type, bidding $\underline{x}_l$ must leads to a zero or negative payoff since the winning probability of bidding $\underline{x}_l$ is always zero in equilibrium. This contradicts the initial assumption that the l type's equilibrium payoff is strictly positive.

Given that the l type's equilibrium (expected) payoff is zero, we further show that $\underline{x}_l = 0$ through proof by contradiction. Consider a case in which $\underline{x}_l > 0$. For the l type, bidding $\underline{x}_l > 0$ must be strictly dominated by bidding zero, since bidding $\underline{x}_l > 0$, which also leads to a zero probability of winning, is more costly than bidding zero. The above shows that it is impossible to have $\underline{x}_l > 0$ in equilibrium. Thus, we have $\underline{x}_l = 0$ in equilibrium.

Since the l type's equilibrium (expected) payoff is zero, we can show that $\underline{x}_l = 0$ through proof by contradiction. Let us consider a scenario where $\underline{x}_l > 0$. In this case, bidding $\underline{x}_l > 0$ would be strictly dominated by bidding zero for the l type. This is because bidding$\underline{x}_l > 0$, which also leads to a zero probability of winning, incurs a higher cost than bidding zero. Therefore, it is impossible for $\underline{x}_l$ to be greater than zero in equilibrium. As a result, we can conclude that $\underline{x}_l = 0$ in equilibrium.

Next, we further show that in case (b), the equilibrium payoff for the h type is also

[32]To see why, consider any equilibrium in which two players bid on their individual intervals. It is straightforward to show that: (i) When the two players' bidding intervals overlap, the h type's equilibrium expected payoff is greater than that of the l type. (ii) When the two players' bidding intervals do not overlap, it can be shown that the h (resp. l) type's interval is the top (resp. bottom) interval, and for the h type, bidding in the top interval must generate a weakly larger payoff than bidding in the bottom interval—because otherwise, the h type would have the incentive to bid in the bottom interval to get a strictly larger payoff.

zero—i.e., $\Pi_h = 0$, through proof by contradiction. Suppose $\Pi_h > 0$; we have

$$\begin{cases} \pi_h(x) = & v_h(f(h|h)F_h(x) + f(l|h)F_l(x)) - x = \Pi_h; \\ \pi_l(x) = & v_l(f(h|l)F_h(x) + f(l|l)F_l(x)) - x = 0. \end{cases} \tag{18}$$

We thus have that

$$F_l(x) = \frac{(f(h|h)v_h - f(h|l)v_l)x}{(f(l|l) - f(l|h))v_h v_l} - \frac{f(h|l)v_l\Pi_h}{(f(l|l) - f(l|h))v_l v_h}.$$

Notice that when $\rho \geq \bar{\rho}$, we obtain that $f(l|h)v_h < f(l|l)v_l$, which implies that $f(l|l) > f(l|h)$. In this case, $F_l(0) < 0$, which is a contradiction.

By (18) and $\Pi_h = \Pi_l = 0$, we obtain that

$$F_l(x) = \frac{f(h|h)v_h - f(h|l)v_l}{(f(l|l) - f(l|h))v_h v_l} x.$$

From $F_l(\bar{x}_l) = 1$, the upper bound of support is given by $\bar{x}_l = \frac{(f(l|l)-f(l|h))v_h v_l}{f(h|h)v_h - f(h|l)v_l}$. It is simple to verify that $\bar{x}_l < v_l$. On the other hand, for the h type, for any $x \in [0, \bar{x}_l)$,

$$F_h(x) = \frac{f(l|l)v_l - f(l|h)v_h}{(f(l|l) - f(l|h))v_h v_l} x.$$

Thus, we have $\underline{x}_h = 0$ from $F_h(\underline{x}_h) = 0$. Also, as $F_h(\bar{x}_l) = \frac{f(l|l)v_l - f(l|h)v_h}{f(h|h)v_h - f(h|l)v_l} < 1$, it must be the case that $\bar{x}_h > \bar{x}_l$. For the h type who bids $x > \bar{x}_l$,

$$\pi_h(x) = v_h(f(h|h)F_h(x) + f(l|h)) - x = 0,$$

which yields that $F_h(x) = \frac{x - f(l|h)v_h}{f(h|h)v_h}$. The upper bound of support of the h type is $\bar{x}_h = v_h$ since $F_h(v_h) = \frac{v_h - f(l|h)v_h}{f(h|h)v_h} = 1$.

Lastly, we show that the l type has no incentive to bid more than $\bar{x}_l$. If he bids $x >$

$\bar{x}_l$, his expected payoff is

$$\begin{aligned}\pi_l(x) &= v_l(f(l|l) + f(h|l)F_h(x)) - x \\ &= v_l(f(l|l) + f(h|l)\frac{x - f(l|h)v_h}{f(h|h)v_h}) - x \\ &= \frac{f(h|l)v_l - f(h|h)v_h}{f(h|h)v_h}x + \frac{f(l|l) - f(l|h)}{f(h|h)}v_l.\end{aligned}$$

We can obtain that $\pi_l(x)$ decreases with x for $x > \bar{x}_l$ and $\pi_l(\bar{x}_l) = 0$. We thus have $\pi_l(x) < 0$ for all $x > \bar{x}_l$. Hence, the l type has no incentive to bid more than $\bar{x}_l$.

In case (c), recall that we have shown that the l type's equilibrium expected payoff is zero and $\underline{x}_l = 0$. Next, we first show that in case (c), $\bar{x}_h = v_l$ and the equilibrium payoff of the h type is $\Pi_h = v_h - v_l$. Suppose $\bar{x}_h < v_l$, and for the l type, if he bids $\bar{x}_h < x < v_l$, we have

$$\pi_l(x) = v_l(f(h|l) + f(l|l)) - x = v_l - x > 0.$$

Next, we show that the h type has no incentive to bid more than v_l. If the h type bids $x > v_l$, we then have

$$\pi_h(x) = v_h(f(h|h)F_h(x) + f(l|h)) - x \leq v_h - x < v_h - v_l, \quad \forall x > v_l,$$

which means that $\bar{x}_h \leq v_l$. From the above analysis, we can obtain that $\bar{x}_h = v_l$ and the equilibrium payoff of the h type is $\Pi_h = \pi_h(v_l) = v_h - v_l$.

We then seek to pin down the equilibrium CDFs from the equilibrium payoffs. From the following equations

$$\begin{cases} \pi_h(x) = v_h(f(h|h)F_h(x) + f(l|h)F_l(x)) - x = v_h - v_l, \\ \pi_l(x) = v_l(f(h|l)F_h(x) + f(l|l)F_l(x)) - x = 0, \end{cases} \tag{19}$$

we can obtain that

$$F_h(x) = \frac{(f(l|h)v_h - f(l|l)v_l)x - f(l|l)v_l(v_h - v_l)}{(f(l|h) - f(l|l))v_h v_l}.$$

We can verify that $F_h(v_l) = 1$. Also, from $F_h(\underline{x}_h) = 0$, we get $\underline{x}_h = \frac{f(l|l)v_l(v_h - v_l)}{f(l|h)v_h - f(l|l)v_l}$.

Then, we first obtain $F_l(x)$ for $x \in [0, \underline{x}_h)$ by considering the l type's bid in this range. The l type's equilibrium payoff is zero. For any bid of the l type $x < \underline{x}_h$, his payoff is

$$\pi_l(x) = f(l|l)v_l F_l(x) - x = 0.$$

This implies that $\underline{x}_l = 0$. Thus, the equilibrium CDF of the l type for $x < \underline{x}_h$ is

$$F_l(x) = \frac{x}{f(l|l)v_l}, \quad \forall x \in [0, \underline{x}_h).$$

We now seek to derive $F_l(x)$ for $x \geq \underline{x}_h$. By (19), we obtain that

$$F_l(x) = \frac{(f(h|l)v_l - f(h|h)v_h)x + f(h|l)v_l(v_h - v_l)}{(f(l|h) - f(l|l))v_h v_l}.$$

Lastly, we only need to show that the h type has no incentive to bid lower than $\underline{x}_h$. If he bids $x < \underline{x}_h$, his expected payoff is

$$\begin{aligned} \pi_h(x) &= f(l|h)v_h F_l(x) - x \\ &= f(l|h)v_h \frac{x}{f(l|l)v_l} - x \\ &= \frac{f(l|h)v_h - f(l|l)v_l}{f(l|l)v_l} x, \end{aligned}$$

which increases with x for $x \in (0, \underline{x}_h)$ since $f(l|h)v_h - f(l|l)v_l > 0$. Note that $\pi_h(\underline{x}_h) = v_h - v_l$. We thus derive that for any $x \in [0, \underline{x}_h)$,

$$\pi_h(x) = \frac{f(l|h)v_h - f(l|l)v_l}{f(l|l)v_l} x < v_h - v_l.$$

Hence, the h type has no incentive to bid lower than $\underline{x}_h$. Q.E.D.

A.2 Proof of Lemma 2

Given the prior distribution of types, the expected payoff of each player, π, can be expressed as

$$\pi = f(h) \cdot \pi_h + f(l) \cdot \pi_l,$$

where π_h and π_l are the expected payoffs of the h type and the l type, respectively. For either type of each player, he competes against another h type with probability $f(h)$ or against another l type with probability $f(l)$.

Under policy (D, D), the expected payoffs of both types are

$$\begin{aligned} \pi_h \mid_{(D,D)} &= f(h|h) \cdot 0 + f(l|h) \cdot (v_h - v_l) = f(l|h)(v_h - v_l), \\ \pi_l \mid_{(D,D)} &= f(h|l) \cdot 0 + f(l|l) \cdot 0 = 0. \end{aligned}$$

So the expected payoff of each player is

$$\pi \mid_{(D,D)} = f(h) \cdot \pi_h \mid_{(D,D)} + f(l) \cdot \pi_l \mid_{(D,D)} = f(h,l)(v_h - v_l). \tag{20}$$

Under policy (D, C), for the player who chooses to disclose his types, the expected payoffs of both types are

$$\begin{aligned} \pi_h \mid_{(D,C)} &= f(l|h)(v_h - v_l), \\ \pi_l \mid_{(D,C)} &= 0. \end{aligned}$$

On the one hand, the expected payoff of the disclosed player is

$$\pi_D \mid_{(D,C)} = f(h) \cdot \pi_h \mid_{(D,C)} + f(l) \cdot \pi_l \mid_{(D,C)} = f(h,l)(v_h - v_l). \tag{21}$$

On the other hand, for the player who chooses C, the expected payoffs of both types are

$$\begin{aligned}
\pi_h \mid_{(D,C)} &= f(h|h) \cdot f(l|h)(v_h - v_l) + f(l|h) \cdot (v_h - v_l) \\
&= (1 + f(h|h))f(l|h)(v_h - v_l), \\
\pi_l \mid_{(D,C)} &= f(h|l) \cdot 0 + f(l|l) \cdot \frac{f(h|l)(v_h - v_l)v_l}{v_h} \\
&= \frac{f(l|l)f(h|l)(v_h - v_l)v_l}{v_h}.
\end{aligned}$$

Using $\pi_C \mid_{(D,C)} = f(h) \cdot \pi_h \mid_{(D,C)} + f(l) \cdot \pi_l \mid_{(D,C)}$, a player's (ex ante) expected payoff is

$$\pi_C \mid_{(D,C)} = \left(f(h,h)f(l|h) + f(h,l) + f(l,l)f(h|l)\frac{v_l}{v_h} \right)(v_h - v_l). \tag{22}$$

Under policy (C, C), there are three cases.

(i) If $\underline{\rho} < \rho < \bar{\rho}$, the expected payoffs of both types are

$$\begin{aligned}
\pi_h \mid_{(C,C)} &= f(l|h)v_h - f(l|l)v_l, \\
\pi_l \mid_{(C,C)} &= 0.
\end{aligned}$$

Using $\pi \mid_{(C,C)} = f(h) \cdot \pi_h \mid_{(C,C)} + f(l) \cdot \pi_l \mid_{(C,C)}$, a player's (ex ante) expected payoff is

$$\pi \mid_{(C,C)} = f(h)(f(l|h)v_h - f(l|l)v_l). \tag{23}$$

(ii) If $\bar{\rho} \leq \rho < 1$, the expected payoffs of both types are

$$\begin{aligned}
\pi_h \mid_{(C,C)} &= 0, \\
\pi_l \mid_{(C,C)} &= 0.
\end{aligned}$$

So the expected payoff of each player is

$$\pi \mid_{(C,C)} = f(h)\pi_h \mid_{(C,C)} + f(l)\pi_l \mid_{(C,C)} = 0.$$

(iii) If $-1 < \rho \leq \underline{\rho}$, the expected payoffs of both types are

$$\begin{aligned} \pi_h \mid_{(C,C)} &= v_h - v_l, \\ \pi_l \mid_{(C,C)} &= 0. \end{aligned}$$

So the expected payoff of each player is

$$\begin{aligned} \pi \mid_{(C,C)} &= f(h) \cdot \pi_h \mid_{(C,C)} + f(l) \cdot \pi_l \mid_{(C,C)} \\ &= f(h)(v_h - v_l). \end{aligned}$$

Q.E.D.

A.3 Proof of Proposition 6

Suppose there is a symmetric mixed strategy equilibrium denoted by $(p, 1-p)$, where $p \in (0,1)$ is the probability of each player's choosing policy D. According to the property of a mixed-strategy equilibrium, given an arbitrary player's strategy—say, player 2's strategy—player 1 should be indifferent between choosing strategies C and D, since choosing either policy yields the same level of expected payoff. We thus have

$$\left(\pi \mid_{(D,D)}, \pi_D \mid_{(D,C)}\right) \begin{pmatrix} p \\ 1-p \end{pmatrix} = \left(\pi_C \mid_{(D,C)}, \pi \mid_{(C,C)}\right) \begin{pmatrix} p \\ 1-p \end{pmatrix}. \tag{24}$$

Using (20) and (21), (24) implies that

$$\pi_C \mid_{(D,C)} p + \pi \mid_{(C,C)} (1-p) = f(h,l)(v_h - v_l). \tag{25}$$

(i) If $\underline{\rho} < \rho < \bar{\rho}$, using (22), (23), $\rho = f(l|l) - f(l|h)$, and (25), we can derive that

$$p = \frac{(f(h)f(l|l) - f(h,l))v_l}{f(h,h)f(l|h)(v_h - v_l) + f(l,l)f(h|l)(v_h v_l - v_l^2)/v_h + (f(h)f(l|l) - f(h,l))v_l} \tag{26}$$

for $0 < \rho < \bar{\rho}$; $p = 0$ for $\rho = 0$, since $f(h)f(l|l) - f(h,l) = 0$ when $\rho = f(l|l) - f(l|h) = 0$ (which implies that the numerator of (26) is zero); there exists no $p \in (0,1)$

for $\underline{\rho} < \rho < 0$, as $f(h)f(l|l) - f(h,l) < 0$ when $\rho = f(l|l) - f(l|h) < 0$.[33]

(ii) If $\bar{\rho} \leq \rho < 1$, $\pi \mid_{(C,C)} = 0$, using (22) and (25), we obtain that

$$p = \frac{f(h,l)}{f(h,h)f(l|h) + f(h,l) + f(l,l)f(h|l)v_l/v_h}. \tag{27}$$

(iii) If $-1 < \rho \leq \underline{\rho}$, $\pi \mid_{(C,C)} = f(h)(v_h - v_l)$, using (22) and (25), we obtain that

$$p = \frac{f(h,h)}{f(h,h)f(h|h) - f(l,l)f(h|l)v_l/v_h} > 1.$$

Using the above equation, it can be shown that there exists no such p where $p \in (0,1)$.

Next, we show that there does not exist any asymmetric mixed strategy equilibrium in which the probability of choosing a policy (either C or D) differs across the two players. Suppose there is an asymmetric mixed strategy equilibrium that is denoted by $(p_1, 1-p_1)$ for player 1 and $(p_2, 1-p_2)$ for player 2, where $p_i \in (0,1)$ is the probability of player i choosing policy D, $i \in \{1,2\}$, and $p_1 \neq p_2$. On the one hand, similar to that in the above proof for the nonexistence of a symmetric mixed strategy equilibrium, we show that there do not exist such values of p_1 and p_2, where $p_1 \in (0,1)$ and $p_2 \in (0,1)$, that can constitute an asymmetric mixed strategy equilibrium. On the other hand, consider a situation in which one player chooses a mixed strategy and the other player chooses a pure strategy. In this case, without loss of generality, assume that player 1 chooses $(p_1, 1-p_1)$ and player 2 chooses policy D with probability 1, where $p_1 \in (0,1)$ is the probability of player 1 choosing policy D. By the property of a mixed strategy equilibrium, given player 2's pure strategy, player 1 should be indifferent between choosing strategies C and D. Using (24), we should have that $\pi \mid_{(D,D)} = \pi_C \mid_{(D,C)}$, which is clearly not true by (20) and (21). Also, following a similar approach, it can be shown that there does not exist a mixed strategy equilibrium in which player 1 chooses $(p_1, 1-p_1)$ and player 2 chooses policy C with probability 1.

Q.E.D.

[33] By checking the expression of (26), we can see that there exists no p such that $p \in (0,1)$ if $\rho = f(l|l) - f(l|h) < 0$.

A.4 Proof of Proposition 7

Suppose $f(h) = m$, $f(l) = 1 - m$, and $f(h, l) = n$. Without loss of generality, assume that $0 < n < m < 1$.[34] We thus have that $f(h, h) = m - n$, $f(l, l) = 1 - m - n$, and $f(h|l) = \frac{n}{1-m}$, $f(l|h) = \frac{n}{m}$, $f(h|h) = \frac{m-n}{m}$, $f(l|l) = \frac{1-m-n}{1-m}$. Recall that $\rho = f(l|l) - f(l|h) = f(h|h) - f(h|l) = 1 - \frac{n}{m(1-m)}$, which implies that $n = (1-\rho)m(1-m)$. Using $n = (1-\rho)m(1-m)$, we can further derive that $f(h, h) = m(1-(1-\rho)(1-m))$, $f(l, l) = (1 - m)(1 - (1 - \rho)m)$, $f(h|l) = m(1 - \rho)$, $f(l|h) = (1 - m)(1 - \rho)$, $f(h|h) = 1 - (1 - m)(1 - \rho)$, and $f(l|l) = 1 - m(1 - \rho)$.

Recall that we have shown that the expression of p is given by (26) and (27) for $0 < \rho < \bar{\rho}$ and $\bar{\rho} \le \rho < 1$, respectively.

It can be derived that $0 < \rho < \bar{\rho}$ is equivalent to $0 < \rho < \frac{(1-m)(v_h - v_l)}{(1-m)v_h + mv_l}$, using $\rho = f(l|l) - f(l|h)$ and $\bar{\rho} = \frac{f(l|l)(v_h - v_l)}{v_h}$. Using (26), we derive that

$$\begin{aligned} p &= \frac{\rho v_l}{f(h|h)f(l|h)(v_h - v_l) + f(l|l)f(l|h)(v_h v_l - v_l^2)/v_h + \rho v_l} \\ &= \frac{\rho v_h v_l}{(1-\rho)(1-m)(v_h - v_l)((1-(1-\rho)(1-m))v_h + (1-m(1-\rho))v_l) + \rho v_h v_l}. \end{aligned} \tag{28}$$

It can be shown that $(1-(1-\rho)(1-m))v_h + (1-m(1-\rho))v_l$ increases with ρ, which means that $(1-\rho)(1-m)(v_h - v_l)((1-(1-\rho)(1-m))v_h + (1-m(1-\rho))v_l)$ decreases when ρ increases. Using (28), we obtain that when ρ increases, $\rho v_h v_l$ increases and $(1-\rho)(1-m)(v_h - v_l)((1-(1-\rho)(1-m))v_h + (1-m(1-\rho))v_l)$ decreases—thus, we derive that p increases in ρ for $0 < \rho < \frac{(1-m)(v_h - v_l)}{(1-m)v_h + mv_l}$. When $\rho \to 0$, $p \to 0$, and when $\rho \to \frac{(1-m)(v_h - v_l)}{(1-m)v_h + mv_l}$, $p \to \frac{1}{2}$ by (28).

Also, we show that $\bar{\rho} \le \rho < 1$ is equivalent to $\frac{(1-m)(v_h - v_l)}{(1-m)v_h + mv_l} \le \rho < 1$. Using (27), it can be derived that

$$\begin{aligned} p &= \frac{1}{1 + f(h|h) + f(l|l)v_l/v_h} \\ &= \frac{v_h}{v_h + (1-(1-\rho)(1-m))v_h + (1-m(1-\rho))v_l}. \end{aligned} \tag{29}$$

[34] Notice that $f(h) = f(h, h) + f(h, l)$; thus assuming $n < m$ is equivalent to assuming $f(h, h) > 0$.

Using the above expression, it is obvious that p decreases with ρ, since the denominator increases when ρ increases. In particular, substituting $\rho = \frac{(1-m)(v_h-v_l)}{(1-m)v_h+mv_l}$ into (29), we obtain that $p = \frac{1}{2}$ when $\rho = \bar{\rho}$; when $\rho \to 1$, we obtain that $p \to \frac{1}{2+v_l/v_h} < \frac{1}{2}$. Thus, we conclude that when ρ increases from $\bar{\rho}$ to 1, the value of p decreases from $\frac{1}{2}$ to $\frac{1}{2+v_l/v_h}$.

In summary, we have shown that p, which is the probability of each player's choosing policy D, increases with the correlation coefficient ρ when ρ increases on the interval $(0, \bar{\rho})$, but decreases when ρ increases on the interval $[\bar{\rho}, 1)$.

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