

Pairing Provision Price and Default Remedy: Optimal Two-Stage Procurement with Private R&D Efficiency*

Bin Liu[†]

Jingfeng Lu[‡]

March 2018

Abstract

This article studies cost-minimizing two-stage procurement with R&D. The principal wishes to procure a product from an agent. At the first stage, the agent can conduct R&D to discover a more cost-efficient production technology. First-stage R&D efficiency and effort and the realized second-stage production cost are the agent's private information. The optimal two-stage mechanism is implemented by a menu of single-stage contracts, each specifying a fixed provision price and remedy paid by a defaulting agent. A higher delivery price is paired with a higher default remedy, and a more efficient type opts for a higher price and higher remedy.

JEL Classification Numbers: D44, D82, H57, O32

Keywords: Adverse selection, Dynamic mechanism design, Favoritism, Fixed-price plus default-remedy contract, Moral hazard, Endogenous type, Endogenous impulse response term, Research and Development (R&D).

*We are grateful to the editor in charge, David Martimort, and two anonymous reviewers for insightful comments and suggestions, which significantly improved the quality of the article. We thank Masaki Aoyagi, Tilman Börgers, Jimmy Chan, Yi-Chun Chen, Jeffrey Ely, Robert Evans, Daniel Garrett, Yingni Guo, Chiu-Yu Ko, Daniel Krähmer, Laurent Lamy, Hao Li, Meng-Yu Liang, Hitoshi Matsushima, Vai-Lam Mui, Martin Osborne, Alessandro Pavan, Carolyn Pitchik, John Quah, Xianwen Shi, Roland Strausz, Balázs Szentes, Satoru Takahashi, Rakesh Vohra, John Wooders, Okan Yilankaya, Lixin Ye, Jun Yu, and participants in the 15th SAET Conference on Current Trends in Economics and the 11th World Congress of the Econometric Society for helpful comments and suggestions. Bin Liu gratefully acknowledges financial support from the President's Fund from The Chinese University of Hong Kong, Shenzhen (Project Code: PF.01.000322). Jingfeng Lu gratefully acknowledges financial support from MOE, Singapore (R122000252115). Any remaining errors are our own.

[†]The Chinese University of Hong Kong, Shenzhen; binliu@cuhk.edu.cn.

[‡]National University of Singapore; ecljlf@nus.edu.sg.

1 Introduction

Procurement is ubiquitously employed as a cost-efficient way to acquire goods, services, or work from an external source. On average, around 15% of yearly global domestic product is spent on public procurement alone, with military acquisitions as a significant component.¹ It has long been recognized that procurement of new goods/services often involves and stimulates private research and development (R&D) and/or innovation before production and delivery (see Rob, 1986; Hendricks, Porter, and Boudreau, 1987; Lichtenberg, 1988; Rogerson, 1989; and Tan, 1992; among others).² More recently, Nyiri et al. (2007) emphasize the role of public procurement in promoting R&D investment and innovation in information and communication technology (ICT) in EU member states. The United Nations Office for Project Services (UNOPS), in its 2013 report,³ also stresses the role of public procurement in fostering investment in new technology and research in both developed and developing countries.

Cost effectiveness has long been the central issue in procurement design.⁴ An established literature has been devoted to designing cost-minimizing acquisitions in a variety of environments. It is clear that contractors' pre-delivery R&D incentive for cost reduction should be used fully by procurers to lower their acquisition costs. To achieve the most cost reduction, an optimal procurement policy must appropriately balance extracting surplus *ex post* and providing the right R&D incentive *ex ante*.⁵

Typically, both R&D effort and the efficiency of goods/services delivery (i.e., production cost) are a contractor's private information. Moreover, situations abound in which a contractor's R&D efficiency (e.g., the marginal cost of R&D effort) is also his private information. R&D activities require both technical facilities and researchers with different expertise and specialties. The contractor's competence in organizing, coordinating, and carrying out a specific R&D task (e.g., the quality of its technical facilities, the abilities and experience of its researchers, and its efficiency in project management) is usually not observable by the procurer. An immediate implication is that the contractor's R&D incentive must respond to his R&D efficiency. Several interesting issues thus arise for procurement design. How does this additional dimension of the agent's private information affect his R&D incentive and, consequently, the optimal design for procurement? In particular, how

¹Refer to p. 1 in "Supplement to the 2013 Annual Statistical Report on United Nations Procurement: Procurement and Innovation," https://www.unops.org/SiteCollectionDocuments/ASR/ASR_Supplement_2013_WEB.pdf (accessed October 6, 2015).

²Hendricks, Porter, and Boudreau (1987) observe that in federal auctions for leases on the outer continental shelf, bidders make private investments to acquire more information before bidding. Abundant empirical evidence in defense procurement demonstrates that bidders make significant private investments in R&D prior to bidding. Tan (1992) provides an example of jet-fighter procurement by the U.S. Air Force. Lichtenberg (1988) provides more examples of private R&D investment in public defense procurements.

³"Supplement to the 2013 Annual Statistical Report on United Nations Procurement: Procurement and Innovation," 2014, www.unops.org.

⁴Rob (1986) points out that "the importance of the cost effectiveness in the acquisition process cannot be overemphasized."

⁵Notably, contractors' pre-delivery R&D incentive has been incorporated into the analysis in many pioneer studies, including Rob (1986), Dasgupta (1990), Tan (1992), Piccione and Tan (1996), and Arozamena and Cantillon (2004), among others.

should the optimal design incorporate this new element in the information flow into the natural dynamics of the procurement process? Which form does the optimal contract take?

The fixed-price contract is among the most widely adopted procurement contracts. According to the “Performance of the Defense Acquisition System, 2014 Annual Report,”⁶ fixed-price contracts constitute about half of all Department of Defense (DoD) obligated contracts. They are most common in production contracts, and are often used for goods and services acquisitions. According to the “DoD SBIR Desk Reference,”⁷ fixed-price contracts are almost always used for Phase I awards. The National Aeronautics and Space Administration (NASA) awarded fixed-price contracts worth \$6.8 billion to Boeing and SpaceX in 2014 to ferry astronauts to the International Space Station. More recently, the National Defense Authorization Act for Fiscal Year 2017 establishes a preference for the DoD to use fixed-price contracts.

A fixed-price contract has several apparent advantages from the procurer’s perspective, which could explain its popularity. It shifts most or all the risk to the contractor and is easier to design and implement, as it imposes a minimal regulatory and administrative burden, and fewer financial/cost reports are required. Although these advantages are well understood, it would be interesting to investigate whether there are deeper and subtler economic justifications for the wide adoption of fixed-price contracts. In this article, we address this issue and provide a plausible justification from a dynamic mechanism design perspective by establishing fixed-price contracts as the cost-minimizing procurement design in a two-stage environment.

We consider a two-stage contract between a procurer (she, the principal) and a supplier (he, the agent) in the following environment. The principal wishes to procure a product from the agent that she can acquire from an alternative source at cost c_0 . The agent can invest in R&D that improves his endowed production technology and generates a production cost potentially lower than c_0 . The agent’s ability to conduct R&D is his private information; higher (lower) R&D ability is referred to as a more (less) efficient type. At the first stage, the agent is offered the contract. If he accepts, then he exerts an unobservable effort in R&D. Each effort level leads to a distribution of production cost. Higher effort generates better distribution, in the sense of first-order stochastic dominance. But higher effort also costs more, given the type of the agent. In the second stage, the production cost is realized, and it is again the agent’s private information. The contract has to sequentially elicit the agent’s private information and provide the right incentive for the agent to exert effort in R&D at the same time. The principal’s goal is to design the optimal contract to minimize her expected procurement cost.

We focus on deterministic mechanisms in searching for the optimal design, and provide a set of sufficient conditions for the optimality of deterministic mechanisms. A necessary and sufficient condition for a deterministic mechanism to be incentive compatible is provided, which greatly facilitates the analysis. We find that the optimal two-stage mechanism is implemented by a menu

⁶<http://www.defense.gov/Portals/1/Documents/pubs/Performance-of-Defense-Acquisition-System-2014.pdf> (accessed September 29, 2015).

⁷<http://www.acq.osd.mil/osbp/sbir/sb/resources/deskreference/deskreference.pdf> (accessed September 29, 2015).

of single-stage contracts, each specifying a fixed provision price and remedy paid by a defaulting agent. That is, each contract specifies the (fixed) provision price for the product if the agent delivers it and the default remedy the agent has to pay to the procurer if the agent fails to deliver the product because of his realized high delivery cost. The contract with higher provision price is paired with a higher default remedy. At equilibrium, a more efficient type (i.e., higher R&D ability) chooses the contract with higher price and higher default remedy. This feature of the optimal contract is largely consistent with the commonly observed practice in reality, by which a more reliable supplier (who is reputed to have better capacity) usually demands a higher price but offers a higher default remedy.

In practice, a default clause is often inserted into fixed-price procurement contracts, which states what will happen if one of the parties fails to live up to the agreement. In our procurement scenario, the agent defaults if he fails (is not willing) to provide the product because his delivery cost is too high. By law, the procurer (the principal) has the right to claim a remedy from the supplier (the agent) with the amount of (up to) the difference between the market price (i.e., c_0) and the contract price (i.e., the provision price) if the supplier defaults.⁸ Another important feature of the optimal contract is that the sum of the contract price and remedy is always smaller than or equal to the market price c_0 . This feature notably coincides with widely observed practice—which is regulated, for example, by the Uniform Commercial Code and Federal Acquisition Regulation. In addition to the aforementioned advantages of fixed-price contracts, the established optimality of the fixed-price plus default-remedy contract in our setting provides an in-depth economic rationale for the use of a fixed-price plus default-remedy contract in reality: It is indeed optimal from the procurer’s perspective. Given that procurement often involves a natural dynamic flow of information, unobservable R&D investment, and private R&D efficiency and delivery cost, it is reassuring to know that this convenient contract form is optimal in these quite common environments.

Before discussing other features of the optimal contract, it is helpful to introduce two benchmark scenarios: the first-best environment and the pure adverse selection setting. In the first-best environment, the agent’s R&D effort is observable and his private information at both stages is public. It is straightforward that the second-stage allocation must be *ex post* efficient. In the dynamic pure adverse selection benchmark setting, R&D ability and production efficiency are the agent’s private information, but his R&D effort is contractible. It turns out that at the optimum, the second-stage allocation is *ex post* efficient for all first-stage types; this diverges from insights in the dynamic adverse selection literature (e.g., Courty and Li, 2000; Eső and Szentes, 2007), in which the second-stage allocation is discriminatory.

Now we are ready to discuss another noteworthy feature of the optimal contract. Unlike the first-best benchmark and the pure adverse selection benchmark, the second-stage trading cutoff depends on the first-stage type. The sum of the provision price and the default remedy is no larger than c_0 (the principal’s outside option). Moreover, except for the most efficient type agent, who

⁸For example, the Uniform Commercial Code (U.C.C) Article 2-712 and Article 2-713. The Federal Acquisition Regulation (FAR) is another example; see, for example, FAR Article 52.249-8 on Default (Fixed-Price Supply and Service) and FAR 52.249-9 Default (Fixed-Price Research and Development).

chooses the contract with the sum being exactly c_0 , the contract selected in equilibrium for any other type has a sum that is strictly smaller than c_0 , which leads to a downward distortion for acquisition because the agent delivers the product if and only if his delivery cost is smaller than the sum of the provision price and the remedy (i.e., the sum is the acquisition cutoff).⁹ This *ex post* efficiency loss results from the fact that when moral hazard is present, the agent has an informational advantage about his second-stage type distribution due to the privacy of effort, as effort determines the distribution of the second-stage type. Baron and Besanko (1984) coined the term “impulse response” to measure the impact of the first-stage type on the second-stage allocation rule when analyzing pure dynamic adverse selection problems. In a dynamic environment in which the agent’s action is introduced, our analysis shows that “moral hazard” restores the usual function of an “impulse response” term: The principal has to sacrifice second-stage efficiency to overcome her informational disadvantage. Recalling the no-distortion result in our pure adverse selection benchmark, one can infer that it is the combination of moral hazard and adverse selection that leads to allocative distortion at the optimum. Moreover, in our model, the agent’s endogenously chosen hidden action bridges his first-stage type and the distribution of his second-stage type. In this sense, one can view the “impulse response” term as endogenous in relation to Baron and Besanko (1984).

Our article belongs to the literature on procurement design with R&D, which includes Baron and Myerson (1982), Dasgupta (1990), Tan (1992), Piccione and Tan (1996), Bag (1997), and Arozamena and Cantillon (2004), among others. Our article complements these studies by accommodating the supplier’s private R&D efficiency and explicitly examining optimal two-stage procurement in a framework of dynamic mechanism design with sequential private information.

Our study is also related to the literature on default in auctions and procurements, which includes Waehrer (1995), Zheng (2001), Parlane (2003), Board (2007), Burguet, Ganuza, and Hauk (2012), and Lewis and Bajari (2011, 2014), among others. In our article, the optimal contract features a remedy paid by a defaulting agent. We differentiate from these studies in that we do not explicitly assume what would happen in case of default in the setting; rather, the default remedy comes from a necessary component of implementation of the optimal mechanism.

Methodologically, our article contributes to the growing literature on dynamic mechanism design, which originates from the seminal work of Baron and Besanko (1984) in a two-period regulation environment in which a regulated firm’s private costs evolve over time. Courty and Li (2000) and Esó and Szentes (2007) demonstrate, in different environments of sequential screening with pure adverse selection, that at the optimum the second-stage mechanism is discriminatory across first-stage types. Pavan, Segal, and Toikka (2014) provide a general treatment of optimal dynamic mechanism design with pure adverse selection.¹⁰ Other articles address dynamic contracting with

⁹To see this, suppose that the agent’s second-stage type is c . If he delivers the product, his net payoff is the provision price minus c ; if he defaults, his net payoff is the negative of the default remedy. Therefore, the agent will deliver the product if and only if c is smaller than the sum of the provision price and the remedy.

¹⁰Another strand of literature focuses on the implementation of efficient mechanisms, e.g., Bergemann and Välimäki (2010) and Athey and Segal (2013). Although we focus on cost-minimizing procurement design, the efficient mecha-

mixed adverse selection and moral hazard, among which Krämer and Strausz (2011), Gershkov and Perry (2012), Asseger (2015), Esó and Szentes (2017), and Halac, Kartik, and Liu (2016) are closely related to our article.

We differentiate from these previous studies on dynamic mechanism design by fully endogenizing distribution of the second-stage type by introducing moral hazard, and identify a different source (i.e., moral hazard) for allocative discrimination at the optimum. In our model, because there is no distortion in the pure adverse selection benchmark, the discrimination does not arise from dynamic pure adverse selection, as identified in the literature. It rather results from the combination of adverse selection and moral hazard: There is an allocative distortion if and only if both adverse selection and moral hazard are present. This observation from a different setting echoes that of Halac, Kartik, and Liu (2016), who also find that in their environment, absent either moral hazard or adverse selection, there is no efficiency loss.¹¹

The rest of the article is organized as follows. Section 2 introduces the model. We study the optimal mechanism in the benchmark case in Section 3. Section 4 analyzes the optimal mechanism for the mixed problem. We study the pure adverse selection benchmark in Section 5. Section 6 discusses the properties of the optimal contract. Section 7 provides some concluding remarks, and the appendix collects some technical proofs.

2 Model Setup

A risk-neutral buyer (she, the principal) wants to procure a product from a risk-neutral supplier (he, the agent). The agent can carry out R&D to improve his production technology. The agent's R&D efficiency, θ , is his private information, which, from the principal's perspective, is a random variable following a cumulative distribution function $G(\cdot)$ with density function $g(\cdot) > 0$ over the support $[\underline{\theta}, \bar{\theta}]$, where $\underline{\theta} > 0$. Exerting R&D effort $\alpha \geq 0$ costs him $\theta\alpha$. Thus, both cost and marginal cost increase when θ increases, which means that a lower θ is more efficient for conducting R&D. Thus, we call type $\underline{\theta}$ the most efficient type.

The agent's cost c of delivering the product when he exerts R&D effort α is a random variable with a cumulative distribution function $H(\cdot, \alpha)$ defined on support $[\underline{c}, \bar{c}]$ with $0 \leq \underline{c} < \bar{c} \leq \infty$. After R&D, the delivery cost is privately observed by the agent. Production cost c is incurred by the agent if and only if the trade occurs between the procurer and the supplier. The principal, however, can exercise an outside option of acquiring the product at cost c_0 with $\underline{c} < c_0 < \bar{c}$. Equivalently, c_0 can be treated as the procurer's value of the product.

The timing of the game is as follows.

nism can be analyzed analogously.

¹¹Halac, Kartik, and Liu (2016) consider a setting in which the agent's private type is persistent over time, but there is moral hazard (work or shirk) in each period. Their model also involves the agent's private learning about the quality of the project. In our article, we have adverse selection in both stages and moral hazard in the first stage, so that types evolve over time.

Time 0: $G(\cdot)$, $H(\cdot, \cdot)$, and c_0 are revealed by nature as public information. Nature draws θ for the agent. The agent is privately informed about his type θ .

Time 1: The principal offers a contract and she commits to it. If the agent rejects, then the game ends and he obtains the reservation utility, which is normalized as zero. If the agent accepts, he reports his type θ and the first-stage contract is executed. The agent decides on his R&D effort α , which is unobservable,¹² and then his delivery cost c is realized according to $H(\cdot, \alpha)$. His realized delivery cost c is also his private information.

Time 2: The agent decides whether to quit. If he quits, the game is over. If he continues, he reports his delivery cost c . The contract is executed.

The principal's objective is to minimize the expected procurement cost. We use t to denote the gross transfer (the sum of first-stage and second-stage payments) from the principal to the agent. Suppose the agent with type θ exerts effort level α , and the realized cost is c . If the principal acquires the product from the agent, her procurement cost is t and the agent's payoff is $t - \theta\alpha - c$. If the principal does not acquire the product from the agent, her procurement cost is $t + c_0$ and the agent's payoff is $t - \theta\alpha$.

We assume that the density function $h(c, \alpha)$ (i.e., $H_1(c, \alpha)$) is continuously differentiable in $(c, \alpha) \in [\underline{c}, \bar{c}] \times [0, +\infty)$, and for any c , $H(c, \alpha)$ is a thrice continuously differentiable function with respect to α .¹³ In addition,¹⁴

$$H_2(c, \alpha) > 0, H_{22}(c, \alpha) < 0, \forall c \in (\underline{c}, \bar{c}).$$

Positive $H_2(c, \alpha)$ means that higher effort leads to a better cost distribution in the sense of first-order stochastic dominance. Negative $H_{22}(c, \alpha)$ means that the marginal effect of α decreases.¹⁵ Because $H(c, \alpha) \in [0, 1]$, these conditions mean that when α goes to infinity, the marginal effect $H_2(c, \alpha)$ diminishes to zero.

Our formulation of $H(c, \alpha)$ covers the following widely adopted form as a special case:

$$H(c, \alpha) = 1 - (1 - F(c))^{\alpha + \beta_0},$$

where $F(\cdot)$ is a CDF with strictly positive density function everywhere over the support $[\underline{c}, \bar{c}]$, and $\beta_0 \geq 0$ is the agent's initial technology endowment.¹⁶ The case $\beta_0 = 0$ corresponds to the R&D technology used in Tan (1992).

¹²In Section 5, we consider a pure adverse selection benchmark in which the agent's effort α is observable and verifiable. In this case, the agent's effort α is specified by the contract.

¹³For a two-variable function, we use subscripts 1 and 2 to represent the partial derivative with respect to the first and second argument, respectively.

¹⁴Note that because the family of CDFs $H(c, \alpha)$ has common support, $H(\underline{c}, \alpha) = 0$, $H(\bar{c}, \alpha) = 1$ for any $\alpha \geq 0$. Therefore, $H_2(\underline{c}, \alpha) = H_2(\bar{c}, \alpha) = 0$ for any $\alpha \geq 0$.

¹⁵This assumption validates the "first-order approach," which replaces the agent's effort choice with the first-order condition.

¹⁶Thus, if the agent does not invest in R&D, his delivery cost is randomly drawn according to CDF $H(c, 0)$.

3 The First-Best: A Benchmark

We first analyze a full-information benchmark in which the agent’s action and the agent’s types are public information. Obviously, the first-best outcome coincides with social efficiency. At the first best, the mechanism is *ex post* efficient at the second stage; each type θ of agent has a zero expected payoff in the first period; and $\alpha^{FB}(\theta)$ —the first-best effort required for type θ —decreases with θ and strictly decreases whenever $\alpha^{FB}(\theta) > 0$. (The proof of these properties is relegated to Appendix A.)

Hereafter, we assume that $\alpha^{FB}(\bar{\theta}) > 0$, i.e., it is socially efficient to induce the least efficient type agent to exert strictly positive effort, which is satisfied if and only if the following assumption holds:

Assumption 0

$$\bar{\theta} < \int_{\underline{c}}^{c_0} H_2(c, 0) dc.$$

In Section 4, we will analyze the central question of this article: How should the principal design the contract when facing a dynamic private information flow and unobservable R&D effort? After that, we examine the role of moral hazard, as a comparison, by assuming observable R&D effort in Section 5. We will show that such a pure adverse selection environment, at the optimum, distorts R&D investment but still retains *ex post* efficiency in the second stage. Therefore, it is the unobservability of R&D effort that yields discriminatory provision rules among R&D efficiency types.

4 Analysis of the Optimal Contract

We have a mixed adverse selection and moral hazard problem in a dynamic setting. As usual, we restrict our attention to truthful direct mechanisms according to Myerson (1986). In the first stage, the mechanism is a mapping $\rho : [\underline{\theta}, \bar{\theta}] \rightarrow \mathbb{R} \times \mathbb{R}_+$ such that when the agent reports θ , he receives from the principal a payment $x(\theta)$ and an R&D effort recommendation $\alpha(\theta) \geq 0$. The agent decides on his R&D effort α after reporting θ , and his delivery cost c is realized according to $H(\cdot, \alpha)$. In the second stage, the agent further reports his cost realization c , after which the payment rule $y(\theta, c)$ and acquisition probability $p(\theta, c)$ are executed.¹⁷

Stage Two

For the second stage, we ignore the individual rationality (IR) constraint for the time being and consider only the incentive compatibility (IC) constraint. We will verify later that at the optimum,

¹⁷Because the agent has a quasi-linear preference, there is no loss of generality to restrict attention to mechanisms in which the transfers x and y are deterministic.

the agent's second-stage IR is satisfied for the proposed optimal mechanism. Assuming truthfully reported θ in stage one, suppose that the agent's true provision cost is c , but he reports \hat{c} . Let $\tilde{\pi}(\theta, \hat{c}, c)$ be his expected payoff in stage two. Then

$$\tilde{\pi}(\theta, \hat{c}, c) = y(\theta, \hat{c}) - p(\theta, \hat{c})c. \quad (1)$$

The envelope theorem yields

$$\frac{d\tilde{\pi}(\theta, c, c)}{dc} = \left. \frac{\partial \tilde{\pi}(\theta, \hat{c}, c)}{\partial c} \right|_{\hat{c}=c} = -p(\theta, c),$$

which leads to

$$\tilde{\pi}(\theta, c, c) = \tilde{\pi}(\theta, \bar{c}, \bar{c}) + \int_c^{\bar{c}} p(\theta, s) ds. \quad (2)$$

It is clear that the second-stage IC is equivalent to that (2) holds and that $p(\theta, c)$ is decreasing in c for any fixed θ . Note that the agent will still truthfully report his second-stage type c on the off-equilibrium path. That is, if the agent misreported his type in stage one as $\hat{\theta}$, he will still truthfully report c in stage two.¹⁸

Stage One

We consider both IC and IR in stage one. The IC requires that the agent will report his type truthfully and follow the principal's recommendation for R&D effort supply (truthful and obedient). This can be decomposed into two requirements: First, if the agent truthfully reports his type θ , then it is optimal for him to follow the principal's recommendation. Second, the agent will truthfully report his type, given that he will accordingly choose the optimal effort level conditional on his report (truthful or not) and the principal's recommendation. Note that when the agent reports his type and then receives the recommendation (which depends on the report), he always chooses a unique optimal effort level regardless of the recommendation he receives.¹⁹ Such an effort level only depends on his true type and the type he reported to the principal.

If the agent with type θ reports $\hat{\theta}$ and exerts effort α , his expected payoff is

$$\hat{\pi}(\alpha, \hat{\theta}, \theta) = x(\hat{\theta}) - \theta\alpha + \int_{\underline{c}}^{\bar{c}} \tilde{\pi}(\hat{\theta}, c, c) h(c, \alpha) dc.$$

The first term on the right-hand side of the above equation is the payment, the second term is the agent's investment cost, and the last term is his expected profit from the second stage.

¹⁸The argument is as follows. Suppose the reported type is $\hat{\theta}$ in stage one and the realized cost is c , and he instead reports \hat{c} in stage two. Then his payoff (1) becomes $\tilde{\pi}(\hat{\theta}, \hat{c}, c) = y(\hat{\theta}, \hat{c}) - p(\hat{\theta}, \hat{c})c$. Note that if $\hat{\theta}$ is the true type, then $\hat{c} = c$ maximizes $\tilde{\pi}(\hat{\theta}, \hat{c}, c)$ by the optimality of truthful reporting. However, because $\tilde{\pi}(\hat{\theta}, \hat{c}, c)$ does not depend on the true type θ , the optimality of truthful reporting at the second stage holds regardless of the first-stage report. By the same argument, (2) holds for any reported type $\hat{\theta}$.

¹⁹First, the belief of the agent is not affected by the recommendation. Second, as we will show later, the agent's payoff is concave in effort so that he will not randomize his effort level.

Let $\pi(\hat{\theta}, \theta) = \max_{\alpha \geq 0} \hat{\pi}(\alpha, \hat{\theta}, \theta)$, which is the agent's expected utility when his true type is θ but he reports $\hat{\theta}$, given that he will respond optimally when receiving the recommendation $\alpha(\hat{\theta})$. The first-stage IC then requires that

$$\pi(\theta, \theta) \geq \pi(\hat{\theta}, \theta), \forall \theta, \hat{\theta}, \quad (3)$$

which we call IC_1 constraint.

When the type θ agent reports truthfully, the principal's recommendation $\alpha(\theta)$ must coincide with the agent's optimal effort choice (obedience). This is characterized by the following first-order condition:²⁰

$$-\theta + \int_{\underline{c}}^{\bar{c}} p(\theta, c) H_2(c, \alpha(\theta)) dc \leq 0, \text{ with equality if } \alpha(\theta) > 0, \forall \theta, \quad (4)$$

which we call the moral hazard constraint MHC . IC_1 and MHC constitute the first-stage IC constraints. The first-stage IC implies the following envelope condition.

Lemma 1 *The first-stage IC implies that the type θ agent's expected payoff can be expressed as*

$$\pi(\theta, \theta) = \pi(\bar{\theta}, \bar{\theta}) + \int_{\theta}^{\bar{\theta}} \alpha(s) ds. \quad (5)$$

Note that the above lemma is only a necessary condition for the first-stage IC; in general, it is hard to obtain an equivalent characterization in a dynamic and mixed environment.

The Optimal Mechanism

The Principal's Problem

According to Lemma 1, the principal's total cost can be expressed as

$$TC = \int_{\underline{\theta}}^{\bar{\theta}} \left(\left(\theta + \frac{G(\theta)}{g(\theta)} \right) \alpha(\theta) + \int_{\underline{c}}^{\bar{c}} p(\theta, c) (c - c_0) h(c, \alpha(\theta)) dc \right) g(\theta) d\theta + \pi(\bar{\theta}, \bar{\theta}) + c_0.$$

Now we are ready to state the principal's original problem, which we call Problem (O):

$$\min_{\{\alpha(\theta) \geq 0, x(\theta), p(\theta, c), y(\theta, c)\}} TC$$

s.t.:

$$\text{Constraints } IC_1 \text{ (3) and } MHC \text{ (4);} \quad (6)$$

$$\tilde{\pi}(\theta, c, c) = \int_c^{\bar{c}} p(\theta, s) ds, \forall c, \forall \theta; \quad (7)$$

$$p(\theta, c) \text{ is decreasing in } c, \forall \theta; \quad (8)$$

²⁰The derivation of (4) is in Appendix A.

$$0 \leq p(\theta, c) \leq 1, \forall c, \forall \theta; \quad (9)$$

$$\pi(\theta, \theta) \geq 0, \forall \theta. \quad (10)$$

Here (6) contains the first-stage IC constraints; (7) and (8) jointly constitute the equivalent conditions for the second-stage IC;²¹ (9) is the constraint imposed on the provision probability; and (10) is the first-stage IR constraint.

Deterministic Mechanisms

Before we tackle Problem (O), in which we allow stochastic mechanisms, let us first focus on deterministic mechanisms. As is standard, a mechanism is called *deterministic* if $p(\theta, c) \in \{0, 1\}$, $\forall \theta, \forall c$. Obviously, a deterministic mechanism is feasible in the second stage if and only if it is a *cutoff mechanism*, which is defined as follows.

Definition 1 (Cutoff Mechanism): *A mechanism is called a cutoff mechanism if the second-stage allocation rule satisfies the following conditions: For each θ , there exists a cutoff $c(\theta) \in [\underline{c}, \bar{c}]$ such that $p(\theta, c) = \begin{cases} 1, & \text{if } c \leq c(\theta), \\ 0, & \text{if } c > c(\theta). \end{cases}$*

Given a second-stage cutoff $c(\theta)$ for type θ , the moral hazard constraint *MHC* (4) simplifies to

$$-\theta + \int_{\underline{c}}^{c(\theta)} H_2(c, \alpha(\theta)) dc \leq 0, \text{ with equality if } \alpha(\theta) > 0,$$

which determines the effort $\alpha(\theta)$ for type θ . Denote a cutoff mechanism as $\{\alpha(\theta), c(\theta)\}_\theta$, where $(\alpha(\theta), c(\theta))$ satisfies the above simplified condition.

The central difficulty of solving the optimal mechanism is the characterization of IC, which, in our setting, is the first-stage IC constraint (6), as cutoff mechanisms automatically satisfy the second-stage IC. In the following theorem, we provide a *necessary and sufficient* condition for a cutoff mechanism to be incentive compatible. To our knowledge, this is new in the dynamic mechanism design literature with mixed adverse selection and moral hazard.

Theorem 1 *A cutoff mechanism $\{\alpha(\theta), c(\theta)\}_\theta$ is incentive compatible if and only if the following three conditions hold:*

i) The envelope condition: The expected payoff of the agent with first-stage type θ can be expressed as

$$\pi(\theta, \theta) = \pi(\bar{\theta}, \bar{\theta}) + \int_{\theta}^{\bar{\theta}} \alpha(s) ds;$$

ii) The monotonicity condition: The second-stage cutoff is decreasing in θ , i.e., $c(\theta)$ is decreasing

²¹At the optimum, we have $\tilde{\pi}(\theta, \bar{c}, \bar{c}) = 0, \forall \theta$.

in θ .

iii) The recommendation $\alpha(\theta)$ satisfies the moral hazard constraint *MHC* (4):

$$-\theta + \int_{\underline{c}}^{c(\theta)} H_2(c, \alpha(\theta)) dc \leq 0, \text{ with equality if } \alpha(\theta) > 0, \forall \theta.$$

Note that the above theorem does not need any assumptions. In particular, it implies that within the class of deterministic mechanisms, the second-stage cutoff is decreasing in the first-stage type, as required by IC; and more importantly, together with the standard envelope condition and the first-order condition, such monotonicity condition is also sufficient for IC. Theorem 1 is also informative in terms of our main insights of provision price and default remedy, if restricting to deterministic mechanisms. As will be clear in Lemma 5, the monotonicity of cutoff implies the monotonicity in provision price and default remedy.

Theorem 1 also suggests that obtaining the monotonicity of cutoff while restricting to deterministic mechanisms in Problem (O) is in general unlikely to be informative for the optimality of deterministic mechanisms, as the monotonicity is necessary for IC. Essentially, one needs to relax the original problem that accommodates stochastic mechanisms, and show that the relaxed problem leads to a monotone cutoff, which our analysis will demonstrate later.

When restricting to deterministic mechanisms in Problem (O), as long as the pointwise optimization leads to a monotone cutoff, one can conclude that this must be the optimal deterministic mechanism due to Theorem 1. This then solves the optimal deterministic mechanism. However, in general, the solution obtained by pointwise optimization may violate IC when the resulting cutoff is not monotone. Our analysis of stochastic mechanisms will serve two purposes: 1) when is a deterministic mechanism optimal? 2) the characterization of the explicit solution—i.e., when does pointwise optimization lead to a monotone cutoff? Note that only for optimality of the deterministic mechanism do we need extra assumptions. Also note that the monotonicity of cutoff also does not require any assumptions; assumptions are needed for validating pointwise optimization.

A Roadmap of Analysis

It is difficult to solve Problem (O) directly, mainly because of the IC constraint (6). We will identify the optimal mechanism by considering relaxed problems, and show that the solution of the relaxed problems remains the solution of the original problem. We next sketch the major steps for solving Problem (O). We first drop constraint IC_1 (3) from the first-stage IC constraint in Problem (O), leaving only the *MHC* constraint (4). After simplifying other constraints in Problem (O), this will lead to a relaxed problem we call Problem (O-R).²² Notice that in Problem (O-R), the only constraint linking different θ types of agent—constraint IC_1 (3)—has been dropped. Thus Problem (O-R) is essentially a “pointwise” optimization problem for each θ .

²²Detailed derivations for solving Problem (O) are relegated to Appendix A, including the formulations of relaxed problems.

Problem (O-R) allows for stochastic mechanisms, which creates a twofold difficulty: It may be difficult to solve the pointwise optimization problem; furthermore, and more importantly, it can be complicated to check whether the pointwise solution satisfies the first-stage IC—the constraint IC_1 (3)—so that the solution indeed solves the original problem Problem (O). Focusing on deterministic mechanisms would greatly avoid this difficulty: One only needs to solve the optimal second-stage cutoff, which means that Problem (O-R) can be easily solved. More importantly, Theorem 1 provides a *necessary and sufficient* condition for a deterministic mechanism to be incentive compatible, which would help us check whether the solution of Problem (O-R) is indeed incentive compatible, and hence solves Problem (O).

To validate the restriction to deterministic mechanisms, we characterize a set of sufficient conditions—Assumptions 1–4 introduced below—under which a deterministic mechanism is optimal. Under Assumption 1, there is no loss of generality to consider cutoff mechanisms in Problem (O-R), which leads to an equivalent problem Problem (O-R-D). Assumptions 2–4 guarantee that the solution to Problem (O-R-D) is incentive compatible. Finally, we characterize the optimal incentive compatible *deterministic mechanism* by solving Problem (O-R-D), which must also be the solution to Problem (O) as deterministic mechanisms are optimal under Assumptions 1–4. As mentioned above, solving Problem (O-R-D) only requires solving the optimal cutoff by pointwise optimization, which is routine.

Single Crossing and Deterministic Mechanisms

Focusing on cutoff mechanisms, as mentioned above, can greatly simplify the analysis. Nevertheless, one concern is whether this restriction introduces loss of generality.²³ We address this concern by providing sufficient conditions under which cutoff mechanisms are optimal. To proceed, let us start with the following assumption.

Assumption 1 (*Single Crossing*): For any given $\alpha \geq 0$ and any given $\lambda \in \mathbb{R}$, the following function $\varphi(c; \alpha, \lambda) : [\underline{c}, \bar{c}] \rightarrow \mathbb{R}$ has exactly one root, i.e., $\varphi(c; \alpha, \lambda) = 0$ has only one solution, where

$$\varphi(c; \alpha, \lambda) = c - c_0 + \lambda \frac{H_2(c, \alpha)}{h(c, \alpha)}.$$

$\varphi(c; \alpha, \lambda)$ is a family of functions satisfying $\varphi(\underline{c}; \alpha, \lambda) = \underline{c} - c_0 < 0$ and $\varphi(\bar{c}; \alpha, \lambda) = \bar{c} - c_0 > 0$ for any α and λ . Assumption 1 requires that each function in this family cross zero only once.²⁴ This single-crossing condition guarantees that the “virtual cost” that will be introduced in Section 6 would cross c_0 only once. The condition is satisfied if, for example, when $H_2(c, \alpha)/h(c, \alpha)$ is strictly concave, strictly convex, or linear in c . When $H(c, \alpha) = 1 - (1 - F(c))^{\alpha + \beta_0}$ —the widely adopted form in the R&D literature—Assumption 1 is satisfied when $\frac{1 - F(c)}{f(c)} \ln(1 - F(c))$ is strictly concave,

²³As pointed out by Laffont and Martimort (2002), the commitment issue is more involved with stochastic mechanisms.

²⁴Because $\varphi(\underline{c}; \alpha, \lambda) < 0$ and $\varphi(\bar{c}; \alpha, \lambda) > 0$, $\varphi(c; \alpha, \lambda) = 0$ has at least one solution.

strictly convex, or linear.²⁵ The single-crossing condition leads to the following results.

Lemma 2 *Under Assumption 1,*

i) There is no loss of generality to assume that MHC constraint (4) is always binding in Problem (O-R);

ii) There is no loss of generality to focus on cutoff mechanisms to solve Problem (O-R).

The Optimality of Cutoff Mechanisms

When restricting to cutoff mechanisms, Problem (O-R) further simplifies to Problem (O-R-D), whose solution is expressed in terms of $(\alpha(\theta), c(\theta))$. Denote the optimal solution to Problem (O-R-D) by $\{(\alpha^*(\theta), c^*(\theta))\}_\theta$. The following lemma establishes one basic property of it.

Lemma 3 $c^*(\theta) < c_0$ for all $\theta \in (\underline{\theta}, \bar{\theta}]$, and $c^*(\underline{\theta}) = c_0$.

The optimal solution to Problem (O-R-D) may not necessarily be the optimal solution to the original problem, Problem (O), as it may violate the IC constraint (6) there—the constraint that is almost dropped from Problem (O) to obtain the relaxed problem Problem (O-R). Therefore, it is important to establish conditions for the incentive compatibility of cutoff mechanisms. This issue has been addressed by Theorem 1. Assumptions 2–4 below are to guarantee the monotonicity of $c^*(\theta)$. Denote $\hat{c}(\bar{\theta})$ as the cutoff such that $\int_{\underline{c}}^{\hat{c}(\bar{\theta})} H_2(c, 0)dc = \bar{\theta}$. By Assumption 0, $\hat{c}(\bar{\theta}) \in (\underline{c}, c_0)$. $\hat{c}(\bar{\theta})$ is thus the lowest possible cutoff such that all first-stage types would exert positive effort. Now we are ready to introduce Assumptions 2–4.²⁶

Assumption 2 $\frac{G(\theta)}{g(\theta)}$ is strictly increasing in θ .

Assumption 3 For all $\alpha \in [0, \alpha^{FB}(\underline{\theta})]$ and all $\hat{c} \in [\hat{c}(\bar{\theta}), c_0)$,

$$\frac{H_{22}(\hat{c}, \alpha)}{H_2(\hat{c}, \alpha)} \leq \frac{\int_{\underline{c}}^{\hat{c}} H_{222}(c, \alpha)dc}{\int_{\underline{c}}^{\hat{c}} H_{22}(c, \alpha)dc}.$$

Assumption 4 For all $\alpha \in [0, \alpha^{FB}(\underline{\theta})]$ and all $\hat{c} \in [\hat{c}(\bar{\theta}), c_0)$,

$$H_{12}(\hat{c}, \alpha) - \frac{H_{22}(\hat{c}, \alpha) \cdot H_2(\hat{c}, \alpha)}{\int_{\underline{c}}^{\hat{c}} H_{22}(c, \alpha)dc} \geq -\frac{MN}{c_0 - \underline{c}},$$

where M and N are nonnegative constants, with $M = \inf_{\hat{c} \in [\hat{c}(\bar{\theta}), c_0)} H_2(\hat{c}, \alpha^{FB}(\underline{\theta}))$ and $N = \inf_{\theta \in [\underline{\theta}, \bar{\theta}]} \left(\frac{G(\theta)}{g(\theta)} \right)'$,

where $\left(\frac{G(\theta)}{g(\theta)} \right)'$ denotes the derivative of $\frac{G(\theta)}{g(\theta)}$.

²⁵ For example, $\frac{1-F(c)}{f(c)} \ln(1-F(c))$ is strictly convex in c when F is the uniform distribution. As another example, $\frac{1-F(c)}{f(c)} \ln(1-F(c))$ is linear in c when F is the exponential distribution, i.e., $F(c) = 1 - e^{-\beta c}$, where $\beta > 0$ and the support is $[0, +\infty)$.

²⁶ Assumption 3 and Assumption 4 can be weakened; see Remark 1 in Appendix A.

To understand these assumptions, it is helpful to define the *marginal productivity of effort*. To this end, notice that the expected payoff (denoted as $\Pi(\hat{c}, \alpha)$) of the agent with type θ from the second stage, when he exerts effort α and the second-stage cutoff is \hat{c} , can be expressed as $\Pi(\hat{c}, \alpha) = \int_{\underline{c}}^{\hat{c}} H(c, \alpha) dc$.²⁷ $\Pi(\hat{c}, \alpha)$ can be viewed as a production function in which the second-stage cutoff and the agent's effort are the inputs. By a rewrite of the *MHC* constraint (4) when restricting to cutoff mechanisms,²⁸ the agent's optimal choice of effort level satisfies $\Pi_2(\hat{c}, \alpha) = \theta$, which means that the marginal productivity of effort is always a constant. We use $\tilde{\Pi}(\hat{c}, \alpha)$ to denote $\Pi_2(\hat{c}, \alpha)$ for notational convenience.

Now we are ready to interpret the above three assumptions. Assumption 2 is the standard regularity assumption, which requires that the CDF $G(\cdot)$ is log-concave—or, equivalently, the reverse hazard rate of $G(\cdot)$ is decreasing. Note that Assumption 2 holds for $G(\theta) = \left(\frac{\theta-\underline{\theta}}{\bar{\theta}-\underline{\theta}}\right)^r$, where $r > 0$ is a constant. Assumption 3 and Assumption 4 are similar to the regularity condition of Assumption 2 in Eső and Szentes (2007), which is interpreted as a monotonicity condition in their article.

Assumption 3 implies that: An *increase* in α , holding the marginal productivity of effort constant, weakly *decreases* the marginal effect of α on the marginal productivity of effort.²⁹ Assumption 4 implies that: An *increase* in \hat{c} , holding the marginal productivity of effort constant, weakly *increases* or *does not decrease too much* the marginal effect of \hat{c} on the marginal productivity of effort (call this “the marginal effect of \hat{c} ” for short).³⁰ More precisely, Assumption 4 implies that the rate of change of the marginal effect of \hat{c} is bounded below by $-\frac{MN}{c_0 - \underline{c}}$.

Assumption 3 and Assumption 4 are satisfied for a large family of functions. For example, Assumption 3 is satisfied for $H(c, \alpha) = 1 - (1 - F(c))^{\alpha + \beta_0}$.³¹ Assumption 4 is satisfied when $M > 0$

²⁷See equation (14) in the appendix.

²⁸See equation (26) in the appendix.

²⁹To see this interpretation, keeping $\tilde{\Pi}$ constant implies $d\hat{c} = -\frac{\tilde{\Pi}_2}{\tilde{\Pi}_1} d\alpha$. The total differential of $\tilde{\Pi}_2$ is $\tilde{\Pi}_{12} d\hat{c} + \tilde{\Pi}_{22} d\alpha = (\tilde{\Pi}_{22} - \tilde{\Pi}_{12} \cdot \frac{\tilde{\Pi}_2}{\tilde{\Pi}_1}) d\alpha$, which is negative when $d\alpha > 0$ if and only if $\tilde{\Pi}_{22} - \tilde{\Pi}_{12} \cdot \frac{\tilde{\Pi}_2}{\tilde{\Pi}_1} \leq 0$. Note that $\tilde{\Pi}_1 = H_2(\hat{c}, \alpha)$, $\tilde{\Pi}_2 = \int_{\underline{c}}^{\hat{c}} H_{22}(c, \alpha) dc < 0$, $\tilde{\Pi}_{22} = \int_{\underline{c}}^{\hat{c}} H_{222}(c, \alpha) dc > 0$ so that

$$\tilde{\Pi}_{22} - \tilde{\Pi}_{12} \cdot \frac{\tilde{\Pi}_2}{\tilde{\Pi}_1} \leq 0 \Leftrightarrow \frac{\tilde{\Pi}_{12}}{\tilde{\Pi}_1} \leq \frac{\tilde{\Pi}_{22}}{\tilde{\Pi}_2} \Leftrightarrow \frac{H_{22}(\hat{c}, \alpha)}{H_2(\hat{c}, \alpha)} \leq \frac{\int_{\underline{c}}^{\hat{c}} H_{222}(c, \alpha) dc}{\int_{\underline{c}}^{\hat{c}} H_{22}(c, \alpha) dc}.$$

³⁰Similar to the interpretation of Assumption 3, keeping $\tilde{\Pi}$ constant implies $d\alpha = -\frac{\tilde{\Pi}_1}{\tilde{\Pi}_2} d\hat{c}$. The total differential of $\tilde{\Pi}_1$ is $\tilde{\Pi}_{11} d\hat{c} + \tilde{\Pi}_{12} d\alpha = (\tilde{\Pi}_{11} - \tilde{\Pi}_{12} \cdot \frac{\tilde{\Pi}_1}{\tilde{\Pi}_2}) d\hat{c}$, so the rate of change of the marginal effect of \hat{c} is

$$\tilde{\Pi}_{11} - \tilde{\Pi}_{12} \cdot \frac{\tilde{\Pi}_1}{\tilde{\Pi}_2} = H_{12}(\hat{c}, \alpha) - \frac{H_{22}(\hat{c}, \alpha) \cdot H_2(\hat{c}, \alpha)}{\int_{\underline{c}}^{\hat{c}} H_{22}(c, \alpha) dc}.$$

³¹To see this, notice that $H_{222}(c, \alpha) = H_{22}(c, \alpha) \ln(1 - F(c))$ and $H_{22} \leq 0$, so that

$$\int_{\underline{c}}^{\hat{c}} H_{222}(c, \alpha) dc = \int_{\underline{c}}^{\hat{c}} H_{22}(c, \alpha) \ln(1 - F(c)) dc \leq \ln(1 - F(\hat{c})) \int_{\underline{c}}^{\hat{c}} H_{22}(c, \alpha) dc.$$

Therefore,

$$\frac{\int_{\underline{c}}^{\hat{c}} H_{222}(c, \alpha) dc}{\int_{\underline{c}}^{\hat{c}} H_{22}(c, \alpha) dc} \geq \frac{\ln(1 - F(\hat{c})) \int_{\underline{c}}^{\hat{c}} H_{22}(c, \alpha) dc}{\int_{\underline{c}}^{\hat{c}} H_{22}(c, \alpha) dc} = \ln(1 - F(\hat{c})) = \frac{H_{22}(\hat{c}, \alpha)}{H_2(\hat{c}, \alpha)}.$$

and N is large enough—for example, $M > 0$ for $H(c, \alpha) = 1 - (1 - F(c))^{\alpha + \beta_0}$, and N is large enough when $\bar{\theta} - \underline{\theta}$ is small enough or r is close to 0 for $G(\theta) = \left(\frac{\theta - \underline{\theta}}{\bar{\theta} - \underline{\theta}}\right)^r$, where $r > 0$ is a constant.³²

We are now ready to present the following comparative statics result.

Lemma 4 *Under Assumptions 2, 3, and 4, for Problem (O-R-D), $c^*(\theta)$ is strictly decreasing in θ . As a consequence, $\alpha^*(\theta)$ is also strictly decreasing in θ .*

Combining Theorem 1 and Lemmas 2 and 4, we have the following proposition.

Proposition 1 *Under Assumptions 1–4, the optimal mechanism for Problem (O) is deterministic, which is given by the solution to Problem (O-R-D). The optimal mechanism assigns higher cutoff $c^*(\theta)$ to a more efficient type (i.e., lower θ), and a more efficient type exerts more R&D effort $\alpha^*(\theta)$.*

As revealed by Lemma 3, there is *ex post* efficiency loss in the second stage at the optimum, except for the most efficient type $\underline{\theta}$. We highlight this property in the following corollary.

Corollary 1 *Under Assumptions 1–4, for the optimal mechanism for Problem (O), the second-stage cutoff $c^*(\theta) \leq c_0$ with equality only for $\theta = \underline{\theta}$. Therefore, in general there is *ex post* efficiency loss in the second stage.*

In Section 5, we will study the scenario with observable R&D effort, in which we will show that there is no *ex post* efficiency loss. This contrast will illustrate the role moral hazard plays in our environment with dynamic information flow.

Optimal Mechanism and Implementation

We first describe the second-stage optimal allocation rule and payments. From (1), (7), and Proposition 1, it is straightforward to obtain the second-stage optimal allocation rule and payment rule, which is summarized in the proposition below.

Proposition 2 *Under Assumptions 1–4, at the optimum, the second-stage allocation rule is:*

$$p^*(\theta, c) = \begin{cases} 1, & \text{if } c \leq c^*(\theta), \\ 0, & \text{if } c > c^*(\theta). \end{cases}$$

³²To see this, notice that

$$J = \inf_{\hat{c} \in [\hat{c}(\bar{\theta}), c_0], \alpha \in [0, \alpha^{FB}(\underline{\theta})]} \left[H_{12}(\hat{c}, \alpha) - \frac{H_{22}(\hat{c}, \alpha) \cdot H_2(\hat{c}, \alpha)}{\int_{\underline{c}}^{\hat{c}} H_{22}(c, \alpha) dc} \right] > -\infty.$$

That is, J is finite. Thus, Assumption 4 is satisfied if MN is large enough, which is the case when, for example, $M > 0$ and N is large enough. When $H(c, \alpha) = 1 - (1 - F(c))^{\alpha + \beta_0}$, $M > 0$ because $\hat{c} > \underline{c}$. When $G(\theta) = \left(\frac{\theta - \underline{\theta}}{\bar{\theta} - \underline{\theta}}\right)^r$, $\left(\frac{G(\theta)}{g(\theta)}\right)' = \frac{1}{r(\bar{\theta} - \underline{\theta})}$, which approaches $+\infty$ when $\bar{\theta} - \underline{\theta} \rightarrow 0^+$ or $r \rightarrow 0^+$.

The second-stage payment rule is:

$$y^*(\theta, c) = \begin{cases} c^*(\theta), & \text{if } c \leq c^*(\theta), \\ 0, & \text{if } c > c^*(\theta). \end{cases}$$

Because the agent bears the production cost only when the principal buys the product from him, the agent's second-stage *ex post* payoff must be nonnegative. As a result, the second-stage IR is satisfied.

The first-stage payment rule is summarized in the following proposition. The specific expression is provided in the proof.

Proposition 3 *Under Assumptions 1–4, at the optimum, all θ types pay the principal in the first stage, and a more efficient type pays more. That is, $x^*(\theta_1) < x^*(\theta_2) < 0$ for any θ_1, θ_2 with $\theta_1 < \theta_2$.*

Implementation: Fixed-Price Plus Default-Remedy Contract The optimal two-stage mechanism $\{x^*(\theta), \alpha^*(\theta), p^*(\theta, c), y^*(\theta, c)\}_\theta$ can be implemented by a menu of single-stage fixed provision price plus default-remedy contracts. Each contract specifies a different fixed provision price when the agent delivers the product and a different remedy paid to the procurer by a defaulting agent.

We use $r^*(\theta)$ to denote the default remedies, and $p^*(\theta)$ to denote the provision prices. Let $r^*(\theta) = -x^*(\theta)$ and $p^*(\theta) = c^*(\theta) - r^*(\theta)$ for all θ . The following lemma summarizes the property of $r^*(\theta)$ and $p^*(\theta)$, which implies that a more efficient type agent enjoys a higher provision price, but the default remedy is also higher when he defaults.

Lemma 5 *i) $r^*(\theta) > 0$ and $r^*(\theta)$ is strictly decreasing in θ .
ii) $p^*(\theta) \in (0, c_0)$ and $p^*(\theta)$ is strictly decreasing in θ .*

For contract $\{p^*(\theta), r^*(\theta)\}$, the agent with type θ is willing to deliver the product when and only when his delivery cost is smaller than $p^*(\theta) + r^*(\theta) = c^*(\theta)$. Therefore, $c^*(\theta)$ is the prevailing acquisition cutoff,³³ which means that the contract implements the same allocation rule as the optimal mechanism. Then it is easy to see that, from the agent's perspective at the first stage, this contract is equivalent to the one that first asks the agent to pay $|x^*(\theta)|$ at the first stage, and then the second stage is a take-it-or-leave-it offer at price $c^*(\theta) = p^*(\theta) + r^*(\theta)$. The ICs then follow from the payment rules. Contracts $\{p^*(\theta), r^*(\theta)\}_\theta$ can be viewed as standard single-stage contracts, as the menu only relies on first-stage types.

The implementation result is formally presented in the following proposition.

Proposition 4 *The optimal mechanism of Problem (O) can be implemented by a menu of (single-stage) fixed-price plus default-remedy contracts: $\{p^*(\theta), r^*(\theta)\}_\theta$, where $r^*(\theta) > 0$ and $p^*(\theta) \in (0, c_0)$,*

³³Suppose the agent's second-stage type is c (his delivery cost). If he delivers the product, his net payoff is $p^*(\theta) - c$; if he defaults, his net payoff is $-r^*(\theta)$. Therefore, he is willing to produce it when and only when $c < p^*(\theta) + r^*(\theta)$.

and both $r^*(\theta)$ and $p^*(\theta)$ are strictly decreasing in θ . $p^*(\theta)$ is the provision price for the agent if he delivers the product, and $r^*(\theta)$ is the default remedy paid by the agent to the principal if the agent fails to deliver the product. At equilibrium, the type θ agent opts for contract $\{p^*(\theta), r^*(\theta)\}$.

The agent is offered a menu of fixed-price plus default-remedy contracts $\{p^*(\theta), r^*(\theta)\}_\theta$. If the agent selects contract $\{p^*(\theta), r^*(\theta)\}$, then the price for his product is $p^*(\theta)$ if he delivers it, but he must compensate the principal by $r^*(\theta)$ if he defaults. Thus, if the agent wishes to have a higher selling price for the product, the potential default remedy is also higher. A more efficient type would choose the high-price-but-high-remedy contract, because he can easily boost his R&D effort for a higher chance of selling his product at a higher price and avoid the high remedy. A less efficient type opts for the contract with low price but low remedy, as his high R&D effort cost keeps him from effectively avoiding the penalty of a default remedy. He thus prefers a lower default remedy and, at the same time, has to tolerate a lower provision price.

Limited Liability

As shown above, in the optimal contract, the default remedy is bounded by the difference between the market price c_0 and the provision price. If the agent has limited liability, i.e., the default remedy cannot exceed $K > 0$, then depending on the magnitude of K , the optimal deterministic contract may involve bunching for sufficiently efficient types, which is summarized in the following proposition.

Proposition 5 *Suppose that the default remedy cannot exceed $K (> 0)$. Then there is a unique corresponding first-stage efficiency cutoff $\tilde{\theta}$, decreasing in K , such that the optimal deterministic contract is a pooling one when $\theta \leq \tilde{\theta}$. More precisely, for all $\theta \leq \tilde{\theta}$, the optimal contract always takes the form $\{p^*(\tilde{\theta}), r^*(\tilde{\theta})\}$; for all $\theta > \tilde{\theta}$, the optimal deterministic contract is the same as the one without limited liability.*

5 Observable R&D Effort: Pure Adverse Selection Benchmark

Our article differs from classical dynamic screening articles in that the first stage involves moral hazard in addition to adverse selection. It is thus interesting to know the role moral hazard plays in the optimal design. To this end, in this section, we study a pure adverse selection setting: The agent's R&D efficiency θ and provision cost c are his private information, but his R&D investment (i.e., α) is observable. According to the revelation principle (Myerson, 1986), there is no loss of generality to restrict to direct mechanisms, which is truthful on the equilibrium path. The mechanism specifies the first-stage payment to the agent $\tilde{x}(\theta)$ and the effort required $\tilde{\alpha}(\theta)$ after receiving the agent's report θ . The mechanism also specifies the acquisition probability $\tilde{p}(\theta, c)$ and the payment to the agent $\tilde{y}(\theta, c)$ in the second stage, which depend on both the first-stage report

θ and the second-stage report c .^{34,35}

The Optimal Mechanism

Analysis and derivation of the optimal mechanism in this case is relegated to Appendix A. We present the results here. Denote the optimal mechanism as $\{\tilde{x}^*(\theta), \tilde{\alpha}^*(\theta), \tilde{p}^*(\theta, c), \tilde{y}^*(\theta, c)\}_\theta$. Then

$$\tilde{p}^*(\theta, c) = \begin{cases} 1, & \text{if } c \leq c_0, \\ 0, & \text{if } c > c_0, \end{cases}$$

$$\text{and } \tilde{y}^*(\theta, c) = \begin{cases} c_0, & \text{if } c \leq c_0, \\ 0, & \text{if } c > c_0. \end{cases}$$

In addition, $\tilde{\alpha}^*(\theta)$ is decreasing in θ and $\tilde{\alpha}^*(\theta) \leq \alpha^{FB}(\theta)$ with equality only when $\theta = \underline{\theta}$; and $\tilde{x}^*(\theta) \leq 0$ and is increasing in θ . Notice that at the optimum, the second-stage mechanism is independent of the first-stage type and is *ex post* efficient—i.e., the cutoff is c_0 . The following proposition summarizes the main findings.

Proposition 6 *In the pure adverse selection benchmark setting with observable R&D investment, at the optimum,*

- i) The second-stage mechanism is always efficient and thus is independent of the agent's first-stage type;*
- ii) A more efficient type is required to exert more effort;*
- iii) There is a downward distortion of effort provision, except for the most efficient type.*

Implementation: Fixed-Price Plus Default-Remedy Contract with Effort Requirement

The optimal mechanism $\{\tilde{x}^*(\theta), \tilde{\alpha}^*(\theta), \tilde{p}^*(\theta, c), \tilde{y}^*(\theta, c)\}_\theta$ described above can be implemented by a menu of single-stage fixed-price plus default-remedy contracts with effort requirement. Let $\tilde{r}^*(\theta) = -\tilde{x}^*(\theta)$ and $\tilde{p}^*(\theta) = c_0 - \tilde{r}^*(\theta)$, where $\tilde{p}^*(\theta) > 0$ is the provision price for the product if the agent delivers it,³⁶ and $\tilde{r}^*(\theta) \geq 0$ is the default remedy paid by the agent to the principal if the agent fails to deliver the product because his delivery cost is too high. It is easy to see that $\tilde{p}^*(\theta)$ is increasing in θ and $\tilde{r}^*(\theta)$ is decreasing in θ .

For contract $\{\tilde{p}^*(\theta), \tilde{r}^*(\theta), \tilde{\alpha}^*(\theta)\}$, the agent is willing to deliver the product when and only when

³⁴Because the agent's first-stage and second-stage payoffs are linear in payment, there is no loss of generality to focus on deterministic payment rules $\tilde{x}(\theta)$ and $\tilde{y}(\theta, c)$. In addition, as can be seen from the derivation in the appendix, the principal's objective function is strictly convex in effort, so that there is no benefit of randomizing the effort requirement.

³⁵It is clear that allowing the mechanism to depend on investment cannot improve generality, as investment is a function of the first-stage report.

³⁶To see that $\tilde{p}^*(\theta) > 0$, please refer to "Derivation 3" in Appendix A.

his delivery cost is smaller than $\tilde{p}^*(\theta) + \tilde{r}^*(\theta) = c_0$. Therefore, c_0 is the acquisition cutoff,³⁷ which means that the contract implements the same allocation rule as the optimal mechanism. Then it is easy to see that, from the agent's perspective at the first stage, this contract is equivalent to the one that first asks the agent to pay $|\tilde{x}^*(\theta)|$ and exert effort $\tilde{\alpha}^*(\theta)$ at the first stage, and the second stage is a take-it-or-leave-it offer at price c_0 . The ICs then follow from the payment rules. Contracts $\{\tilde{p}^*(\theta), \tilde{r}^*(\theta), \tilde{\alpha}^*(\theta)\}_\theta$ can be viewed as standard single-stage contracts, as the menu only relies on first-stage types.

The implementation result is formally presented in the following proposition.

Proposition 7 *In the pure adverse selection setting with observable R&D investment, the optimal mechanism is implemented by a menu of (single-stage) fixed-price plus default-remedy contracts with effort level requirement: $\{\tilde{p}^*(\theta), \tilde{r}^*(\theta), \tilde{\alpha}^*(\theta)\}_\theta$, where $\tilde{r}^*(\theta) \geq 0$ and is decreasing in θ , $\tilde{p}^*(\theta) \in (0, c_0]$ and is increasing in θ , and $\tilde{\alpha}^*(\theta)$ is decreasing in θ . For contract $\{\tilde{p}^*(\theta), \tilde{r}^*(\theta), \tilde{\alpha}^*(\theta)\}$, $\tilde{p}^*(\theta)$ is the provision price for the agent if he delivers the product, $\tilde{r}^*(\theta)$ is the default remedy paid by the agent to the principal if the agent fails to deliver the product, and $\tilde{\alpha}^*(\theta)$ is the effort required. At equilibrium, type θ agent opts for contract $\{\tilde{p}^*(\theta), \tilde{r}^*(\theta), \tilde{\alpha}^*(\theta)\}$.*

6 Discussions

Ex Post Efficiency

As shown in Corollary 1 and Proposition 6, there is *ex post* efficiency loss (distortive allocation) in the mixed problem, whereas it is always *ex post* efficient in the pure adverse selection benchmark. For the latter scenario, the *ex post* efficiency in the second stage is rather surprising, given that the optimal mechanism is typically discriminatory in the dynamic screening literature (e.g., Courty and Li, 2000; Esó and Szentes, 2007). After all, at first glance, the second-stage type distribution depends on the effort required by the principal, which in turn depends on the agent's first-stage type. Therefore, the second-stage type distribution indirectly depends on the first-stage type. It seems to suggest that the agent, in the beginning of the first stage, has an informational advantage about his second-stage type distribution, which entails distortive second-stage allocation. Nevertheless, this is contradicted by our finding.

To understand these results, let us first go back to the seminal article of Baron and Besanko (1984), who coined the informativeness of the first-stage type. When the first-stage type and second-stage type are independent, knowing his first-stage type does not give the agent any information about his second-stage type distribution, so that he and the principal have symmetric beliefs about the second-stage type distribution. Therefore, the principal does not need to sacrifice any *ex post* efficiency. When the first-stage type and the second-stage type are correlated, knowing his first-stage type gives the agent informational advantage about his second-stage type distribution.

³⁷The reason is similar to the one explained in footnote 33.

This leads to the well-established impulse response term (or “informativeness”) in the dynamic mechanism design literature. The principal then has to sacrifice *ex post* efficiency to overcome her informational disadvantage in the optimal mechanism.

In our pure adverse selection benchmark, though the second-stage type distribution indirectly depends on the first-stage type, it is actually fully pinned down by the effort requirement from the principal. Before signing the contract with the principal, knowing his first-stage type at that point does not give the agent any informational advantage about his second-stage type distribution, because he and the principal both understand that it is the effort that the principal requires from him that determines his second-stage type distribution, not his first-stage type. Therefore, when signing the contract, the principal does not need to distort the second-stage allocation, as the informativeness of the first-stage type about the second-stage type degenerates and the impulse response term disappears. However, the downward distortion of effort in Proposition 6 implies that the principal has to sacrifice effort efficiency to concede information rent to the agent because of the agent’s private information of his first-stage type.

Let us now turn to interpretation of the second-stage allocation distortion result in the mixed problem. In Problem (O-R-D), let λ be the Lagrangian multiplier associated with the binding moral hazard constraint *MHC* (4), and write the Lagrangian as

$$L = \left(\theta + \frac{G(\theta)}{g(\theta)} \right) \alpha + \int_{\underline{c}}^{\hat{c}} \left(c - c_0 + \lambda \frac{H_2(c, \alpha)}{h(c, \alpha)} \right) h(c, \alpha) dc - \lambda \theta.$$

Suppose the multiplier associated with the optimal solution $(\alpha^*(\theta), c^*(\theta))$ is $\lambda^*(\theta)$. In the proof of Lemma 3, we show that $\lambda^*(\theta) \geq 0$ with equality only when $\theta = \underline{\theta}$. In the first stage, the optimal contract induces type θ agent to exert effort $\alpha^*(\theta)$. Now the Lagrangian becomes

$$L = \int_{\underline{c}}^{\hat{c}} \left(c - c_0 + \lambda^*(\theta) \frac{H_2(c, \alpha^*(\theta))}{h(c, \alpha^*(\theta))} \right) h(c, \alpha^*(\theta)) dc + \left(\theta + \frac{G(\theta)}{g(\theta)} \right) \alpha^*(\theta) - \lambda^*(\theta) \theta.$$

The first-order condition with respect to \hat{c} , by Assumption 1, gives rise to a unique solution $c^*(\theta)$ —the optimal cutoff for type θ —satisfying

$$[c^*(\theta) - c_0] + \lambda^*(\theta) \frac{H_2(c^*(\theta), \alpha^*(\theta))}{h(c^*(\theta), \alpha^*(\theta))} = 0.$$

The distortion from the first-best cutoff c_0 is then $\lambda^*(\theta) \frac{H_2(c^*(\theta), \alpha^*(\theta))}{h(c^*(\theta), \alpha^*(\theta))} \geq 0$, with equality only when $\theta = \underline{\theta}$.

Notice that it is the effort level α that fully determines the distribution of the second-stage type, and α is dependent on θ . Therefore, the term $\frac{H_2(c, \alpha^*(\theta))}{h(c, \alpha^*(\theta))}$ then characterizes the “informativeness” of the effort $\alpha^*(\theta)$ (or impulse response), which is first introduced in Baron and Besanko (1984). Note that the impulse response term does not depend on the first-stage type, but rather on the effort level. The multiplier $\lambda^*(\theta)$, which is associated with the moral hazard constraint (the first-

order condition), reflects the cost of incentivizing the agent to follow the effort recommendation. In other words, it measures the cost of moral hazard. On the other hand, if there is no moral hazard issue—i.e., the moral hazard constraint is irrelevant, so that $\lambda^*(\theta) = 0$ —then we have that there is no distortion by our unified approach here, which is consistent with our analysis of the pure adverse selection benchmark. Therefore, $c + \lambda^*(\theta) \frac{H_2(c, \alpha^*(\theta))}{h(c, \alpha^*(\theta))}$ can be interpreted as the “virtual cost,” whose second term is the first-stage adjustment (i.e., $\lambda^*(\theta)$) due to moral hazard multiplied by the informativeness of the effort (i.e., $\frac{H_2(c, \alpha^*(\theta))}{h(c, \alpha^*(\theta))}$). The optimal cutoff is then determined by equating the virtual cost with the outside option c_0 . It is thus clear that: 1) Assumption 1 implies that the virtual cost crosses c_0 only once; 2) Assumptions 2–4 are to ensure that the endogenous informativeness term, $\lambda^*(\theta) \frac{H_2(c^*(\theta), \alpha^*(\theta))}{h(c^*(\theta), \alpha^*(\theta))}$, is increasing in θ .

Note that the privacy of effort here, or equivalently moral hazard, is crucial, as it restores the agent’s informational advantage about the second-stage type, as it is the effort that determines the distribution of the second-stage type, not the first-stage type θ . As discussed above, if effort is observable, there is no distortion in the second stage. Therefore, moral hazard is a key element that drives the impulse response term.

Another benchmark is pure moral hazard in the first stage and pure adverse selection in the second stage. That is, the agent’s R&D ability θ is public, whereas his effort level and the second-stage type are his private information, which is a standard “moral hazard followed by adverse selection.” Because the agent is risk neutral, the principal can achieve the first-best by making the agent the residual claimant for the profit. Thus, there is *ex post* efficiency loss (or discriminatory allocation) *if and only if* both moral hazard and adverse selection are present in the first stage. One insight from the dynamic screening literature (e.g., Courty and Li, 2000; Eső and Szentes, 2007) is that pure adverse selection could lead to discriminatory allocation in the second stage. In our setting, the distortion does not come from pure dynamic adverse selection, but rather from the combination of moral hazard and adverse selection. Thus, we identify another source of discriminatory allocation that extends the insight in the dynamic screening literature to a new environment.

Comparison of Optimal Contracts

With Propositions 4 and 7, we can compare the optimal mechanisms across the two scenarios with observable and unobservable R&D effort. The comparison would highlight the impact of the unobservability of R&D effort—i.e., moral hazard—on the optimal design of the procurement contract. Corollary 2 highlights how provision price and default remedy are paired differently across the two scenarios.

Corollary 2 *In the pure adverse selection setting, the provision price and the default remedy are negatively correlated: Lower provision price is paired with higher default remedy. In the mixed problem, the provision price and the default remedy are positively correlated: Higher provision price is paired with higher default remedy.*

The next corollary further illustrates how the sum of provision price and default remedy differs across the two scenarios, and its implications for *ex post* allocation efficiency and discrimination in product supply across different R&D efficiency types of agent.

Corollary 3 *In the pure adverse selection setting, the sum of provision price and default remedy equals the principal’s outside option for all contracts in the menu. In the mixed problem, this is only true for the contract of the most efficient type $\underline{\theta}$; for the contract of the other types, the sum of provision price and default remedy is strictly lower than the principal’s outside option. Therefore, *ex post* efficiency loss and discrimination in product provision across different R&D efficiency types of agent arise if and only if R&D effort is unobservable.*

Fixed-Price Plus Default-Remedy Contracts

The optimal mechanism for the mixed problem can be implemented by a menu of (single-stage) fixed-price plus default-remedy contracts $\{p^*(\theta), r^*(\theta)\}_\theta$ by Proposition 4. Several features of the optimal fixed-price contract in this case are worth noting. First, unlike the contract in the pure adverse selection case, a higher provision price $p^*(\theta)$ is paired with a higher default remedy $r^*(\theta)$. Therefore, if the agent wishes to have a higher selling price for his product, he also bears the risk of paying a higher remedy if his production cost is so high that he prefers nondelivery.

Another key feature is that the sum of the provision price and the default remedy for the most efficient type is exactly c_0 , whereas it is strictly lower than c_0 for all other types (it equals $c^*(\theta)$). This results from the discriminatory acquisition cutoff against these types and leads to *ex post* efficiency loss. To see this, recall that in the contract $\{p^*(\theta), r^*(\theta)\}_\theta$, the agent is willing to deliver the product when and only when his delivery cost is smaller than $p^*(\theta) + r^*(\theta)$. Therefore, if $c^*(\theta)$ is smaller than c_0 , which is the case for the optimal mechanism, the agent prefers defaulting when his cost $c \in (c^*(\theta), c_0)$ so that the principal has to turn to the outside option, which costs her c_0 . Therefore, there is efficiency loss *ex post*.

Third, the default remedy plays a dual role in the optimal contract. On the one hand, the remedy (as the “stick”) and the provision price (as the “carrot”) work together to incentivize the agent to exert effort when moral hazard arises. On the other hand, the remedy paid by the agent if he defaults also serves to extract surplus. Notice that the default remedy for the principal is higher for a more efficient type. This is rather intuitive, because with a uniform default remedy for different types, a more efficient type definitely has no incentive to mimic a less efficient type due to the discriminatory provision price. Therefore, the principal is able to extract more surplus from a more efficient type by setting a higher default remedy.

Finally, the magnitudes of the provision price and the remedy in the mixed problem are also worth mentioning. Notice that the provision price $p^*(\theta)$ is always smaller than c_0 . Therefore, the

principal will never default, because the provision price is always lower than the market price c_0 .³⁸ By law, the procurer (the principal) has the right to claim a remedy from the supplier (the agent) with the amount of (up to) the difference between the market price (i.e., c_0) and the contract price (i.e., provision price) if the supplier defaults.³⁹ The derived fixed-price plus default-remedy contract in our setting is consistent with the regulations in reality. For the mixed problem, the sum of the price and the remedy for the most efficient type is exactly c_0 , whereas it is strictly lower than c_0 for all other types. In other words, for the most efficient type, the remedy is exactly the difference between the market price c_0 and the provision price $p^*(\theta)$, whereas for the other types, the remedy is “partial” in the sense that it is strictly smaller than the difference.⁴⁰ For the purpose of screening different R&D efficiency supplier types, the procurer only claims a partial remedy in the contract—although she is allowed by law to demand that the contractor fully cover the damage due to his default.

7 Conclusion

This article studies the optimal procurement design in a two-stage environment in which the R&D efficiency of the supplier is his private information, and the supplier can exert R&D effort to improve his chance of discovering a more cost-efficient way to provide the product. We analyze this design problem from a dynamic mechanism design perspective. To our best knowledge, this is the first time in the procurement design literature that the contractor’s private R&D ability and endogenized R&D effort (observable or unobservable) are jointly integrated into an analytical framework of dynamic mechanism design.

We find that observable effort segregates the impact of the agent’s first-stage private information on the second-stage product-provision rule. Regardless of R&D efficiency, the second-stage product-provision rule is always *ex post* efficient. The principal relies solely on the first-stage mechanism (effort provision and transfer rules) to elicit the agent’s private information on R&D efficiency.

When the agent’s R&D effort is unobservable, the principal must also rely on a discriminatory second-stage product-provision rule to optimally elicit first-stage private information and induce the desired R&D effort level. Moral hazard restores the impulse response term, leaving the second-stage allocation distortive. Note that the second-stage product-provision rule is discriminatory if and only if both moral hazard and adverse selection are present in the first stage. In other words, the distortion does not come solely from adverse selection in our environment.

The optimal two-stage mechanism can be implemented by a menu of (single-stage) contracts of fixed price plus default remedy. A higher provision price is paired with a higher default remedy. The remedy in the contract plays a dual role: It serves as a “stick” to motivate the agent to

³⁸This observation also applies to the pure adverse selection contract.

³⁹Please refer to footnote 8 for several examples.

⁴⁰In the pure adverse selection contract, because the sum of the provision price and the remedy is always c_0 , this means that the remedy for the principal is exactly the difference between the market price and the provision price.

exert effort and, at the same time, it extracts the agent’s surplus. The optimality of this family of contracts and its simple form rationalize its wide adoption in reality. Moreover, the theoretically predicted positive correlation between provision price and default remedy suggests the possibility of an empirical work that tests the validity and implication of our analytical framework using real data from procurement contracts.

The insights obtained in our article extend to other environments. For example, the principal’s problem with observable total R&D cost is equivalent to that with observable R&D effort.⁴¹ When total R&D cost is contractible, once the agent reports his type, the principal can impose an R&D cost that equals the total effort cost induced by the optimal mechanism with observable R&D effort. Clearly, the resulting mechanism duplicates the optimal mechanism with observable R&D effort.

Our analysis illustrates the subtle impacts of the agent’s private information about R&D efficiency on optimal procurement design for both observable and unobservable R&D effort. Moreover, we find that the optimal mechanism crucially depends on the observability of R&D effort. These observations provide useful guidelines by which the designer can appropriately take these factors into account when considering the optimal procurement design that targets acquisition cost effectiveness. For example, in practice, sometimes the principal may be able to monitor the agent’s R&D activity and the monitoring is not costly; this, then, corresponds to the pure adverse selection benchmark. However, when it is too costly to monitor R&D activity, or the accounting error of measuring the total R&D cost is too large, the principal then faces the mixed adverse selection and moral hazard setting.

We have focused on an environment with a single agent. Although we expect that the main insights can be extended to a multi-agent setting, new issues related to information disclosure and belief updating would arise and create additional challenges for analysis. We leave these interesting issues to future work.

8 Appendix A

Derivation of the First-Best Benchmark

Suppose that in the second stage, for type θ and realized cost c , the contract specifies the payment to the agent $y^{FB}(\theta, c)$ and the acquisition probability $p^{FB}(\theta, c)$. In the first stage, the contract prescribes the agent’s effort level $\alpha^{FB}(\theta)$ and the payment to the agent $x^{FB}(\theta)$. First notice that there is no loss of generality to focus on deterministic mechanisms. As we will see later, the total procurement cost is convex in effort when the second-stage mechanism is efficient (which is optimal given any effort level), so there is no loss of generality to assume that the required effort $\alpha^{FB}(\theta)$ is deterministic. Also note that the agent’s payoffs are linear in transfers $x^{FB}(\theta)$ and $y^{FB}(\theta, c)$, so we can focus on deterministic transfers.

⁴¹In our article, the R&D cost is completely determined by the agent’s type and effort level. If an additional noise kicks in, then we have a mixed problem (cf. Laffont and Tirole, 1986).

The agent's second-period payoff when his type is θ and the cost realization is c can be expressed as⁴²

$$\tilde{\pi}^{FB}(\theta, c) = y^{FB}(\theta, c) - p^{FB}(\theta, c)c.$$

The first-period expected payoff for the agent with type θ is

$$\pi^{FB}(\theta) = x^{FB}(\theta) - \theta\alpha^{FB}(\theta) + \int_{\underline{c}}^{\bar{c}} \tilde{\pi}^{FB}(\theta, c)h(c, \alpha^{FB}(\theta))dc.$$

As a result, the expected total procurement cost when the agent's type is θ is

$$\begin{aligned} x^{FB}(\theta) + \int_{\underline{c}}^{\bar{c}} [y^{FB}(\theta, c) + (1 - p^{FB}(\theta, c))c_0]h(c, \alpha^{FB}(\theta))dc \\ = \theta\alpha^{FB}(\theta) + \int_{\underline{c}}^{\bar{c}} p^{FB}(\theta, c)(c - c_0)h(c, \alpha^{FB}(\theta))dc + \pi^{FB}(\theta) + c_0, \end{aligned} \quad (11)$$

which is the sum of social cost and the agent's first-period expected utility.

Obviously, to minimize (11), we should set $\pi^{FB}(\theta) = 0$. Then notice that for any fixed α , to minimize the total cost, we should set $p^{FB}(\theta, c) = 1$ when $c \leq c_0$, and $p^{FB}(\theta, c) = 0$ when $c > c_0$. Thus the optimization problem amounts to minimizing

$$\theta\alpha + \int_{\underline{c}}^{c_0} (c - c_0)h(c, \alpha)dc = \theta\alpha - \int_{\underline{c}}^{c_0} H(c, \alpha)dc. \quad (12)$$

Note that this function has increasing difference in (α, θ) . The first-order partial derivative with respect to α is

$$\theta - \int_{\underline{c}}^{c_0} H_2(c, \alpha)dc.$$

The second-order partial derivative with respect to α is

$$- \int_{\underline{c}}^{c_0} H_{22}(c, \alpha)dc > 0.$$

Note that the cutoff is fixed at c_0 , regardless of the effort level, and the objective function (12) is convex in effort. Thus, the principal would implement a deterministic effort.

The unique optimal solution $\alpha^{FB}(\theta)$ satisfies

$$\theta - \int_{\underline{c}}^{c_0} H_2(c, \alpha^{FB}(\theta))dc \geq 0, \text{ with equality when } \alpha^{FB}(\theta) > 0. \quad (13)$$

Note that because the function (12) has increasing difference in (α, θ) , we have $\alpha^{FB}(\theta)$ is decreasing in θ and is strictly decreasing whenever $\alpha^{FB}(\theta) > 0$.

⁴²For the time being, we ignore individual rationality constraints. We will verify that the agent's payoff is nonnegative for both stages under the first-best contract.

Payment $y^{FB}(\theta, c)$ is set at c_0 when $p^{FB}(\theta, c) = 1$; otherwise, $y^{FB}(\theta, c) = 0$. Payment $x^{FB}(\theta)$ equals $\theta\alpha^{FB}(\theta) - \int_{\underline{c}}^{c_0} H(c, \alpha^{FB}(\theta))dc$ to extract all the surplus. It is easy to verify that the agent's payoff is nonnegative for both stages, so that the individual rationality constraints are satisfied. \square

Proofs of constraint (4), Lemma 1, and Theorem 1

Derivation of the moral hazard constraint (4): Recall that if the agent with type θ reports $\hat{\theta}$ and exerts effort α , his expected payoff is

$$\hat{\pi}(\alpha, \hat{\theta}, \theta) = x(\hat{\theta}) - \theta\alpha + \int_{\underline{c}}^{\bar{c}} \tilde{\pi}(\hat{\theta}, c, c)h(c, \alpha)dc.$$

There is no loss of generality to assume that $\tilde{\pi}(\hat{\theta}, \bar{c}, \bar{c}) = 0$ for all $\hat{\theta} \in [\underline{\theta}, \bar{\theta}]$. This is true because we can always define $\bar{x}(\hat{\theta}) = x(\hat{\theta}) + \tilde{\pi}(\hat{\theta}, \bar{c}, \bar{c})$, and such change does not affect the first-stage expected payoff $\hat{\pi}(\alpha, \hat{\theta}, \theta)$; thus the agent's type reporting and R&D incentive remain the same. Based on this observation and the fact that (2) holds for any reported type $\hat{\theta}$, we can rewrite the expected payoff as

$$\hat{\pi}(\alpha, \hat{\theta}, \theta) = x(\hat{\theta}) - \theta\alpha + \int_{\underline{c}}^{\bar{c}} \left(\int_{\underline{c}}^{\bar{c}} p(\hat{\theta}, s)ds \right) h(c, \alpha)dc = x(\hat{\theta}) - \theta\alpha + \int_{\underline{c}}^{\bar{c}} p(\hat{\theta}, c)H(c, \alpha)dc. \quad (14)$$

Taking derivative with respect to α yields

$$\frac{\partial \hat{\pi}(\alpha, \hat{\theta}, \theta)}{\partial \alpha} = -\theta + \int_{\underline{c}}^{\bar{c}} p(\hat{\theta}, c)H_2(c, \alpha)dc. \quad (15)$$

The second-order derivative

$$\frac{\partial^2 \hat{\pi}(\alpha, \hat{\theta}, \theta)}{\partial \alpha^2} = \int_{\underline{c}}^{\bar{c}} p(\hat{\theta}, c)H_{22}(c, \alpha)dc < 0,$$

when $p(\hat{\theta}, c) > 0$ on a positive measure set. Because the agent's expected payoff $\hat{\pi}(\alpha, \hat{\theta}, \theta)$ is strictly concave in α , the optimal α —denoted as $\alpha(\hat{\theta}, \theta)$ —is unique.⁴³ Thus, the agent with type θ who reports $\hat{\theta}$ will choose the optimal effort level $\alpha(\hat{\theta}, \theta)$.

Note that

$$\pi(\hat{\theta}, \theta) = \max_{\alpha \geq 0} \hat{\pi}(\alpha, \hat{\theta}, \theta) = \hat{\pi}(\alpha(\hat{\theta}, \theta), \hat{\theta}, \theta) = x(\hat{\theta}) - \theta\alpha(\hat{\theta}, \theta) + \int_{\underline{c}}^{\bar{c}} p(\hat{\theta}, c)H(c, \alpha(\hat{\theta}, \theta))dc. \quad (16)$$

This is the agent's expected utility when his true type is θ but he reports $\hat{\theta}$, given that he will respond optimally when receiving the recommendation. Therefore, the first-stage truth-telling can

⁴³ $\frac{\partial^2 \hat{\pi}(\alpha, \hat{\theta}, \theta)}{\partial \alpha^2} = 0$ only when $p(\hat{\theta}, c) = 0$ for almost all c except on a zero measure set. However, in this case, from the first-order condition we know that the agent will optimally choose $\alpha = 0$. Therefore, in all cases the optimal α is unique.

be written as

$$\pi(\theta, \theta) \geq \pi(\hat{\theta}, \theta), \forall \theta, \hat{\theta},$$

where $\alpha(\hat{\theta}, \theta) \geq 0$ satisfies

$$-\theta + \int_{\underline{c}}^{\bar{c}} p(\hat{\theta}, c) H_2(c, \alpha(\hat{\theta}, \theta)) dc \leq 0, \text{ with equality if } \alpha(\hat{\theta}, \theta) > 0, \forall \theta, \hat{\theta}. \quad (17)$$

The recommendation $\alpha(\theta)$ for type θ agent must coincide with $\alpha(\hat{\theta}, \theta)$, which is the moral hazard constraint *MHC* (4):

$$-\theta + \int_{\underline{c}}^{\bar{c}} p(\theta, c) H_2(c, \alpha(\theta)) dc \leq 0, \text{ with equality if } \alpha(\theta) > 0, \forall \theta.$$

Recall that the agent's utility function $\hat{\pi}(\alpha, \hat{\theta}, \theta)$ is strictly concave in α (> 0) (refer to footnote 43); thus the “first-order approach” is valid. We can replace the original incentive compatibility constraint for moral hazard with the above first-order condition. \square

Proof of Lemma 1: Recall that, by (17), the optimal effort level $\alpha(\hat{\theta}, \theta)$ a type θ agent would choose when he reports $\hat{\theta}$ satisfies

$$-\theta + \int_{\underline{c}}^{\bar{c}} p(\hat{\theta}, c) H_2(c, \alpha(\hat{\theta}, \theta)) dc \leq 0, \text{ with equality if } \alpha(\hat{\theta}, \theta) > 0. \quad (18)$$

Note that $\alpha(\hat{\theta}, \theta)$ must be bounded, as $\lim_{\alpha \rightarrow +\infty} \hat{\pi}(\alpha, \hat{\theta}, \theta) = -\infty$ from (14). The following lemma establishes some properties of $\alpha(\hat{\theta}, \theta)$. (All of the proofs of lemmas in Appendix A, unless specified, are relegated to Appendix B.)

Lemma A1 *For any $\hat{\theta}$, $\alpha(\hat{\theta}, \cdot)$ is continuous. Moreover, it is differentiable everywhere except possibly at one point.*

Now recall that, from (16), the highest expected payoff a type θ agent can have when reporting $\hat{\theta}$ is

$$\pi(\hat{\theta}, \theta) = \hat{\pi}(\alpha(\hat{\theta}, \theta), \hat{\theta}, \theta) = x(\hat{\theta}) - \theta \alpha(\hat{\theta}, \theta) + \int_{\underline{c}}^{\bar{c}} p(\hat{\theta}, c) H(c, \alpha(\hat{\theta}, \theta)) dc. \quad (19)$$

The following lemma states some properties of $\pi(\hat{\theta}, \theta)$.

Lemma A2 *For any $\hat{\theta}$, $\pi(\hat{\theta}, \cdot)$ is Lipschitz continuous, and differentiable everywhere except possibly at one point. Moreover, $\frac{\partial \pi(\hat{\theta}, \theta)}{\partial \theta} = \alpha(\hat{\theta}, \theta)$ when the derivative exists.*

Because $\pi(\hat{\theta}, \theta)$ is Lipschitz continuous in θ , it is absolutely continuous. Also note that it is not differentiable only possibly at one point. The proof of Lemma A2 also shows that $\frac{\partial \pi(\hat{\theta}, \theta)}{\partial \theta}$ is bounded. By the envelope theorem (cf. Milgrom and Segal, 2002), the “value function” $\pi(\theta, \theta)$ is

absolutely continuous, and when the derivative exists,

$$\frac{d\pi(\theta, \theta)}{d\theta} = \frac{\partial\pi(\hat{\theta}, \theta)}{\partial\theta} \Big|_{\hat{\theta}=\theta} = -\alpha(\theta, \theta) = -\alpha(\theta).$$

Thus we have the following envelope condition:

$$\pi(\theta, \theta) = \pi(\bar{\theta}, \bar{\theta}) + \int_{\theta}^{\bar{\theta}} \alpha(s) ds,$$

which completes the proof of Lemma 1. \square

Proof of Theorem 1: 1) Necessity: Suppose that $\{\alpha(\theta), c(\theta)\}_{\theta}$ is incentive compatible. Then by Lemma 1, the envelope condition follows directly. Also, it must satisfy the *MHC* constraint (4). Thus, we just need to prove the monotonicity condition. The following lemma is helpful for establishing this.

Lemma A3 *Assume that $\theta_l, \theta_h \in [\underline{\theta}, \bar{\theta}]$ and $\theta_l < \theta_h$, and that $c_h \in [\underline{c}, \bar{c}]$. Define α_{hh} and α_{hl} by*

$$-\theta_h + \int_{\underline{c}}^{c_h} H_2(c, \alpha_{hh}) dc \leq 0, \text{ with equality if } \alpha_{hh} > 0,$$

and

$$-\theta_l + \int_{\underline{c}}^{c_h} H_2(c, \alpha_{hl}) dc \leq 0, \text{ with equality if } \alpha_{hl} > 0,$$

respectively. Then the function $\phi(c_h) := \theta_h \alpha_{hh} - \theta_l \alpha_{hl} + \int_{\underline{c}}^{c_h} [H(c, \alpha_{hl}) - H(c, \alpha_{hh})] dc$ is increasing in c_h .

Pick any θ and $\hat{\theta}$ such that $\theta \leq \hat{\theta}$. Our goal is to show that $c(\theta) \geq c(\hat{\theta})$. Note that by (16), we have

$$\begin{aligned} \pi(\hat{\theta}, \theta) &= x(\hat{\theta}) - \theta \alpha(\hat{\theta}, \theta) + \int_{\underline{c}}^{c(\hat{\theta})} H(c, \alpha(\hat{\theta}, \theta)) dc \\ &= x(\hat{\theta}) - \hat{\theta} \alpha(\hat{\theta}, \hat{\theta}) + \int_{\underline{c}}^{c(\hat{\theta})} H(c, \alpha(\hat{\theta}, \hat{\theta})) dc + \hat{\theta} \alpha(\hat{\theta}, \hat{\theta}) - \int_{\underline{c}}^{c(\hat{\theta})} H(c, \alpha(\hat{\theta}, \hat{\theta})) dc \\ &\quad - \theta \alpha(\hat{\theta}, \theta) + \int_{\underline{c}}^{c(\hat{\theta})} H(c, \alpha(\hat{\theta}, \theta)) dc \\ &= \pi(\hat{\theta}, \hat{\theta}) + \hat{\theta} \alpha(\hat{\theta}, \hat{\theta}) - \theta \alpha(\hat{\theta}, \theta) + \int_{\underline{c}}^{c(\hat{\theta})} \left(H(c, \alpha(\hat{\theta}, \theta)) - H(c, \alpha(\hat{\theta}, \hat{\theta})) \right) dc \\ &\leq \pi(\theta, \theta). \end{aligned} \tag{20}$$

Similarly,

$$\begin{aligned}
\pi(\theta, \hat{\theta}) &= x(\theta) - \hat{\theta}\alpha(\theta, \hat{\theta}) + \int_{\underline{c}}^{c(\theta)} H(c, \alpha(\theta, \hat{\theta}))dc \\
&= x(\theta) - \theta\alpha(\theta, \theta) + \int_{\underline{c}}^{c(\theta)} H(c, \alpha(\theta, \theta))dc + \theta\alpha(\theta, \theta) - \int_{\underline{c}}^{c(\theta)} H(c, \alpha(\theta, \theta))dc \\
&\quad - \hat{\theta}\alpha(\theta, \hat{\theta}) + \int_{\underline{c}}^{c(\theta)} H(c, \alpha(\theta, \hat{\theta}))dc \\
&= \pi(\theta, \theta) + \theta\alpha(\theta, \theta) - \hat{\theta}\alpha(\theta, \hat{\theta}) + \int_{\underline{c}}^{c(\theta)} \left(H(c, \alpha(\theta, \hat{\theta})) - H(c, \alpha(\theta, \theta)) \right) dc \\
&\leq \pi(\hat{\theta}, \hat{\theta}). \tag{21}
\end{aligned}$$

Combining the last two lines of (20) and (21), we have

$$\begin{aligned}
&\hat{\theta}\alpha(\hat{\theta}, \hat{\theta}) - \theta\alpha(\hat{\theta}, \theta) + \int_{\underline{c}}^{c(\hat{\theta})} \left(H(c, \alpha(\hat{\theta}, \theta)) - H(c, \alpha(\hat{\theta}, \hat{\theta})) \right) dc \\
&\leq \hat{\theta}\alpha(\theta, \hat{\theta}) - \theta\alpha(\theta, \theta) + \int_{\underline{c}}^{c(\theta)} \left(H(c, \alpha(\theta, \theta)) - H(c, \alpha(\theta, \hat{\theta})) \right) dc.
\end{aligned}$$

In Lemma A3, letting $\theta = \theta_l$, $\hat{\theta} = \theta_h$, the above inequality is simply $g(c(\hat{\theta})) \leq g(c(\theta))$. By Lemma A3, $g(\cdot)$ is increasing. Thus, we must have $c(\hat{\theta}) \leq c(\theta)$. This completes the proof for the necessity.

2) Sufficiency: First note that condition *iii*) in the theorem is exactly the moral hazard constraint *MHC* (4). What we need to show is that: Picking any first-stage type θ , we always have $\pi(\hat{\theta}, \theta) \leq \pi(\theta, \theta)$ for any $\hat{\theta}$, where $\pi(\hat{\theta}, \theta)$ and $\alpha(\hat{\theta}, \theta)$ are pinned down by (16) and (17). In other words, we only need to show constraint *IC*₁ (3). Lemma A1 still holds, as it does not rely on *IC*₁ (3). Similar to (20), $\pi(\hat{\theta}, \theta) \leq \pi(\theta, \theta)$ is equivalent to showing that

$$\hat{\theta}\alpha(\hat{\theta}, \hat{\theta}) - \theta\alpha(\hat{\theta}, \theta) + \int_{\underline{c}}^{c(\hat{\theta})} \left(H(c, \alpha(\hat{\theta}, \theta)) - H(c, \alpha(\hat{\theta}, \hat{\theta})) \right) dc \leq \pi(\theta, \theta) - \pi(\hat{\theta}, \hat{\theta}). \tag{22}$$

Our goal is to show that the above inequality (22) holds if $c(\theta)$ is decreasing and the envelope condition holds. (Note that by condition *iii*), $\alpha(\theta) = \alpha(\theta, \theta)$, $\forall \theta$.)

We show this for the case when $\hat{\theta} \geq \theta$; the case when $\hat{\theta} < \theta$ is similar. Notice that by direct calculation, we have

$$\hat{\theta}\alpha(\hat{\theta}, \hat{\theta}) - \theta\alpha(\hat{\theta}, \theta) = \int_{\theta}^{\hat{\theta}} \left(s \cdot \frac{\partial \alpha(\hat{\theta}, s)}{\partial s} + \alpha(\hat{\theta}, s) \right) ds.$$

Here $s\alpha(\hat{\theta}, s)$ is differentiable everywhere except possibly at one point (a similar reason as in Lemma A1). Similarly, we can verify

$$\int_{\underline{c}}^{c(\hat{\theta})} \left(H(c, \alpha(\hat{\theta}, \theta)) - H(c, \alpha(\hat{\theta}, \hat{\theta})) \right) dc = - \int_{\theta}^{\hat{\theta}} \int_{\underline{c}}^{c(\hat{\theta})} \frac{\partial \alpha(\hat{\theta}, s)}{\partial s} \cdot H_2(c, \alpha(\hat{\theta}, s)) dc ds.$$

Therefore, the left-hand side of (22) can be written as

$$\int_{\theta}^{\hat{\theta}} \left(\frac{\partial \alpha(\hat{\theta}, s)}{\partial s} \left(s - \int_{\underline{c}}^{c(\hat{\theta})} H_2(c, \alpha(\hat{\theta}, s)) dc \right) + \alpha(\hat{\theta}, s) \right) ds.$$

Note that (60) in the proof of Lemma A2 does not depend on IC_1 , so the above expression can be simplified as $\int_{\theta}^{\hat{\theta}} \alpha(\hat{\theta}, s) ds$. By the envelope condition, the right-hand side of (22) is⁴⁴ $\int_{\theta}^{\hat{\theta}} \alpha(s, s) ds$.

Therefore, to show IC_1 , it suffices to show that $\alpha(s, s) \geq \alpha(\hat{\theta}, s)$ for any $s \in [\theta, \hat{\theta}]$. This is implied by the fact that $c(\theta)$ is decreasing in θ . To see this, note that by (17)

$$\begin{aligned} -s + \int_{\underline{c}}^{c(\hat{\theta})} H_2(c, \alpha(\hat{\theta}, s)) dc &\leq 0, \text{ with equality if } \alpha(\hat{\theta}, s) > 0, \\ -s + \int_{\underline{c}}^{c(s)} H_2(c, \alpha(s, s)) dc &\leq 0, \text{ with equality if } \alpha(s, s) > 0. \end{aligned}$$

Because $s \leq \hat{\theta}$, $c(\hat{\theta}) \leq c(s)$. This further implies that $\alpha(s, s) \geq \alpha(\hat{\theta}, s)$ because $H_{22} \leq 0$. This completes the proof of sufficiency. \square

Detailed Derivation of the Analysis of Problem (O)

Problem (O-R)

It is difficult to solve Problem (O) directly, mainly because of the IC constraint (6). We first relax Problem (O) by simplifying its constraints and relaxing constraint (6). To this end, recall that the first-stage IC implies that (5) holds by Lemma 1, where $\alpha(\theta)$ satisfies *MHC* (4), and in fact, the expression of the total cost TC has employed (5). In addition, by (5) and the fact that $\alpha(\theta)$ is nonnegative, we have $\pi(\theta, \theta) \geq 0, \forall \theta$, if and only if $\pi(\bar{\theta}, \bar{\theta}) \geq 0$. Also note that (7) has also been applied for simplifying the expression of TC . Therefore, if we drop (7), replace (10) with $\pi(\bar{\theta}, \bar{\theta}) \geq 0$, and drop transfers x, y in the choice set, we will have an equivalent optimization problem as Problem (O). Now, if we further drop constraint IC_1 (3) and consider only constraint *MHC* (4), then we will have a relaxed problem Problem (O-R), as follows.

$$\min_{\alpha(\theta) \geq 0, p(\theta, c)} TC$$

⁴⁴Note that by condition iii), $\alpha(\theta)$ is the equilibrium effort provision, which satisfies $\alpha(\theta) = \alpha(\theta, \theta)$.

subject to

$$(4), (8), (9), \text{ and } \pi(\bar{\theta}, \bar{\theta}) \geq 0.$$

If the optimal solution to Problem (O-R) satisfies the first-stage IC, then such a solution must be feasible in Problem (O), and it must be the optimal solution to Problem (O).

Obviously, for Problem (O-R), at the optimum, $\pi(\bar{\theta}, \bar{\theta}) = 0$. Notice that there is no constraint in Problem (O-R) linking different first-stage types θ ; thus we can minimize the objective function in Problem (O-R) pointwisely, i.e., minimize the objective function TC for each fixed θ . Then Problem (O-R) is equivalent to the following pointwise problem for each fixed θ :

$$\min_{\alpha \geq 0, p(\theta, c)} \left(\theta + \frac{G(\theta)}{g(\theta)} \right) \alpha + \int_{\underline{c}}^{\bar{c}} p(\theta, c)(c - c_0)h(c, \alpha)dc$$

subject to

$$-\theta + \int_{\underline{c}}^{\bar{c}} p(\theta, c)H_2(c, \alpha)dc \leq 0, \text{ with equality if } \alpha > 0; \quad (23)$$

$$p(\theta, c) \text{ is decreasing in } c; \quad (24)$$

$$0 \leq p(\theta, c) \leq 1, \forall c. \quad (25)$$

With a little abuse of notation, we also call this problem Problem (O-R).

We have Lemma 2 in the main text for the above Problem (O-R). The proof is immediately after the next subsection, entitled ‘‘Problem (O-R-D).’’

Problem (O-R-D)

Obviously, a cutoff mechanism automatically satisfies (24) and (25). Therefore, by Lemma 2, Problem (O-R) is equivalent to the following Problem (O-R-D):

$$\min_{\alpha \geq 0, \hat{c} \in [\underline{c}, \bar{c}]} TC(\alpha, \hat{c}; \theta) = \left(\theta + \frac{G(\theta)}{g(\theta)} \right) \alpha + \int_{\underline{c}}^{\hat{c}} (c - c_0)h(c, \alpha)dc$$

subject to

$$\int_{\underline{c}}^{\hat{c}} H_2(c, \alpha)dc = \theta. \quad (26)$$

Here, for each θ , we seek effort level $\alpha \geq 0$ and the cutoff in the second stage \hat{c} jointly satisfying (26) to minimize $TC(\alpha, \hat{c}; \theta)$. Denote the optimal solution as $(\alpha^*(\theta), c^*(\theta))$.⁴⁵ Solving Problem (O-R-D) for each θ , then $\{(\alpha^*(\theta), c^*(\theta))\}_\theta$ is the optimal solution to Problem (O-R-D), which must be the optimal solution to Problem (O-R) under Assumption 1. Lemma 3 has established one basic property of the optimal solution to Problem (O-R-D). The proof is immediately after this

⁴⁵For simplicity, we assume that the optimal solution is unique and that every type exerts positive R&D effort at the optimum. We focus on such environments to avoid the issue of optimum selection and the complication of dealing with corner solutions, which significantly hinders the flow of the analysis but does not deliver additional insights.

subsection.

As mentioned in the main text, the optimal solution to Problem (O-R-D)— $\{(\alpha^*(\theta), c^*(\theta))\}_\theta$, which can be solved pointwisely—may not necessarily be the optimal solution to the original problem Problem (O), as it may violate the IC constraint (6) there. According to Theorem 1, as long as the cutoff $c^*(\theta)$ is decreasing, then such a solution must also be feasible in Problem (O). As Problem (O-R-D) is a relaxed problem of Problem (O), this further implies that this solution must also be the solution to Problem (O). Therefore, analysis of the optimal contract in Problem (O) boils down to a comparative statics problem, in which we need to establish that the optimal cutoff $c^*(\theta)$ in Problem (O-R-D) is decreasing in the parameter θ . Assumptions 2–4 serve this purpose.

Remark 1 *Assumption 3 and Assumption 4 are imposed on all $\alpha \in [0, \alpha^{FB}(\underline{\theta})]$ and all $\hat{c} \in [\hat{c}(\bar{\theta}), c_0]$. In fact, from the proof of Lemma 4, these two assumptions can be weakened to requiring that they hold only for specific $\alpha \in [0, \alpha^{FB}(\underline{\theta})]$ and $\hat{c} \in [\hat{c}(\bar{\theta}), c_0]$ with $\int_{\underline{c}}^{\hat{c}} H_2(c, \alpha) dc = \theta$, $\theta \in (\underline{\theta}, \bar{\theta}]$. Furthermore, as can be easily seen from the proof of Observation 3 in Appendix B, Assumption 4 can be further weakened to: For all $\theta \in (\underline{\theta}, \bar{\theta}]$,*

$$H_{12}(\hat{c}, \alpha) - \frac{H_{22}(\hat{c}, \alpha) \cdot H_2(\hat{c}, \alpha)}{\int_{\underline{c}}^{\hat{c}} H_{22}(c, \alpha) dc} \geq -\frac{H_2(\hat{c}, \alpha)}{c_0 - \hat{c}} \left(\frac{G(\theta)}{g(\theta)} \right)',$$

where $\alpha \in [0, \alpha^{FB}(\underline{\theta})]$ and $\hat{c} \in [\hat{c}(\bar{\theta}), c_0]$ satisfy $\int_{\underline{c}}^{\hat{c}} H_2(c, \alpha) dc = \theta$.

Proofs of Lemmas 2, 3, and 4

Proof of Lemma 2: Let us first prove that (23) must be binding at the optimum. Suppose, to the contrary, that (23) is slack at the optimum.⁴⁶ This means that we need to fix $\alpha = 0$; otherwise, (23) must be binding. Now Problem (O-R) can be rewritten as

$$\min_{p(\theta, c)} \int_{\underline{c}}^{\bar{c}} p(\theta, c)(c - c_0)h(c, 0)dc.$$

s.t.:

$$\int_{\underline{c}}^{\bar{c}} p(\theta, c)H_2(c, 0)dc < \theta; \tag{27}$$

$$p(\theta, c) \text{ is decreasing in } c; \tag{28}$$

$$0 \leq p(\theta, c) \leq 1, \forall c. \tag{29}$$

For convenience, call this Problem (S). We will show that Problem (S) has no solutions.

In fact, the fact that (27) is slack implies that if Problem (S) has a solution, then it must be

$$\hat{p}_I(\theta, c) = \begin{cases} 1, & \text{when } c \leq c_0; \\ 0, & \text{when } c > c_0. \end{cases}$$

⁴⁶Note that when the induced $\alpha = 0$, it is still possible that (23) is binding.

To see this, suppose, to the contrary, that the optimal solution to Problem (S) is $p_I(\theta, c)$ and $p_I(\theta, c) \neq \hat{p}_I(\theta, c)$. Denote $c_1 = \sup\{c : p_I(\theta, c) = 1\}$ and $c_2 = \inf\{c : p_I(\theta, c) = 0\}$.

Case 1: $c_1 > c_0$. In this case, because $p_I(\theta, c)$ satisfies (27), $\hat{p}_I(\theta, c)$ also satisfies (27). Because $c - c_0 > 0$ when $c > c_0$, it is obvious that

$$\int_{\underline{c}}^{\bar{c}} \hat{p}_I(\theta, c)(c - c_0)h(c, 0)dc < \int_{\underline{c}}^{\bar{c}} p_I(\theta, c)(c - c_0)h(c, 0)dc,$$

which is a contradiction to $p_I(\theta, c)$ being the optimal solution.

Case 2: $c_1 < c_0$. Consider

$$p'_I(\theta, c) = \begin{cases} 1, & \text{when } c \leq c_1 + \varepsilon, \\ p_I(\theta, c), & \text{when } c > c_1 + \varepsilon, \end{cases}$$

where $\varepsilon > 0$ is small enough such that $c_1 + \varepsilon < c_0$ and $\int_{\underline{c}}^{\bar{c}} p'_I(\theta, c)H_2(c, 0)dc < \theta$. This is possible, because $\int_{\underline{c}}^{\bar{c}} p_I(\theta, c)H_2(c, 0)dc < \theta$. However, because $c - c_0 < 0$ when $c < c_0$, it is obvious that

$$\int_{\underline{c}}^{\bar{c}} p'_I(\theta, c)(c - c_0)h(c, 0)dc < \int_{\underline{c}}^{\bar{c}} p_I(\theta, c)(c - c_0)h(c, 0)dc,$$

which is a contradiction to $p_I(\theta, c)$ being the optimal solution.

The above discussion implies that $c_1 = c_0$. Given this, it is easy to obtain that $c_2 = c_1 = c_0$. This is because if $c_2 > c_0$, then such a solution must be strictly worse than $\hat{p}_I(\theta, c)$. (Note that because $c_2 > c_0$, it is easy to see that $\hat{p}_I(\theta, c)$ must be feasible.) This again leads to a contradiction.

In sum, when (23) is slack in Problem (O-R), we must have $c_1 = c_2 = c_0$ so that $\hat{p}_I(\theta, c)$ is the only possible solution. However, $\hat{p}_I(\theta, c)$ does not satisfy (27), because $\int_{\underline{c}}^{\bar{c}} \hat{p}_I(\theta, c)H_2(c, 0)dc < \theta$ implies that $\int_{\underline{c}}^{c_0} H_2(c, 0)dc < \theta$, which contradicts Assumption 0: $\bar{\theta} < \int_{\underline{c}}^{c_0} H_2(c, 0)dc$. The contradiction means that at the optimum, (23) must be binding.

Now we turn to the second part of Lemma 2—the optimality of cutoff mechanisms in Problem (O-R). It suffices to show that for any fixed first-stage type θ and each induced effort level α , the cutoff mechanism is the optimum. More precisely, consider the following minimization problem,

Problem I:

$$\min_{p(\theta, c)} \left(\theta + \frac{G(\theta)}{g(\theta)} \right) \alpha + \int_{\underline{c}}^{\bar{c}} p(\theta, c)(c - c_0)h(c, \alpha)dc,$$

s.t.:

$$-\theta + \int_{\underline{c}}^{\bar{c}} p(\theta, c)H_2(c, \alpha)dc \leq 0, \text{ with equality if } \alpha > 0; \quad (30)$$

$$p(\theta, c) \text{ is decreasing in } c; \quad (31)$$

$$0 \leq p(\theta, c) \leq 1, \forall c. \quad (32)$$

In this minimization problem, θ and $\alpha \geq 0$ are given. If we can show that an optimal solution to

the above problem is a cutoff mechanism, then we are done.

By our previous argument, we only need to consider the case in which constraint (30) is binding at the optimum. Then, for induced effort level α , there exists some $p(\theta, c)$ satisfying (31) and (32) such that

$$\int_{\underline{c}}^{\bar{c}} p(\theta, c) H_2(c, \alpha) dc = \theta. \quad (33)$$

This implies that there exists a cutoff $\hat{c}(\theta, \alpha)$ such that

$$\int_{\underline{c}}^{\hat{c}(\theta, \alpha)} H_2(c, \alpha) dc = \theta. \quad (34)$$

In other words, there is a cutoff mechanism with the cutoff being $\hat{c}(\theta, \alpha)$, which induces effort level α . Our goal is to show that this cutoff mechanism solves Problem I.

Consider the following problem:

$$\min_{p(\theta, c)} \int_{\underline{c}}^{\bar{c}} p(\theta, c) \left(c - c_0 + \tilde{\lambda} \frac{H_2(c, \alpha)}{h(c, \alpha)} \right) h(c, \alpha) dc, \quad \text{s.t. constraints (31) and (32),}$$

where $\tilde{\lambda} = \frac{(c_0 - \hat{c}(\theta, \alpha))h(\hat{c}(\theta, \alpha), \alpha)}{H_2(\hat{c}(\theta, \alpha), \alpha)}$, which is a constant. Call this **Problem II**. Obviously, the optimal solution to Problem II cannot be worse than the solution to the following **Problem III**:

$$\min_{p(\theta, c)} \int_{\underline{c}}^{\bar{c}} p(\theta, c) \left(c - c_0 + \tilde{\lambda} \frac{H_2(c, \alpha)}{h(c, \alpha)} \right) h(c, \alpha) dc, \quad \text{s.t. constraints (31)–(33),}$$

because Problem III has one more constraint, i.e., constraint (33), than Problem II.

It is easy to see that Problem I and Problem III are equivalent. This is because by (30), the objective function of Problem III can be written as

$$\begin{aligned} & \int_{\underline{c}}^{\bar{c}} p(\theta, c)(c - c_0)h(c, \alpha)dc + \tilde{\lambda} \int_{\underline{c}}^{\bar{c}} p(\theta, c)H_2(c, \alpha)dc \\ &= \int_{\underline{c}}^{\bar{c}} p(\theta, c)(c - c_0)h(c, \alpha)dc + \tilde{\lambda}\theta. \end{aligned}$$

But because θ is fixed, minimizing $\int_{\underline{c}}^{\bar{c}} p(\theta, c)(c - c_0)h(c, \alpha)dc + \tilde{\lambda}\theta$ is the same as minimizing $\int_{\underline{c}}^{\bar{c}} p(\theta, c)(c - c_0)h(c, \alpha)dc$. Also notice that the constraints in Problem I and Problem III are exactly the same, as (30) is binding. Therefore, Problem I and Problem III are equivalent.

Because the solution to Problem II cannot be worse than that to Problem III, we obtain that the solution to Problem II cannot be worse than that to Problem I. Nevertheless, we show that the optimal solution to Problem II is feasible in Problem I. In Problem II, let

$$\varphi(c; \theta, \alpha, \tilde{\lambda}) = c - c_0 + \tilde{\lambda} \frac{H_2(c, \alpha)}{h(c, \alpha)}.$$

Then the objective function of Problem II can be rewritten as

$$\int_{\underline{c}}^{\bar{c}} p(\theta, c) \varphi(c; \theta, \alpha, \tilde{\lambda}) h(c, \alpha) dc.$$

Notice that $\varphi(\underline{c}; \theta, \alpha, \tilde{\lambda}) = \underline{c} - c_0 < 0$, $\varphi(\hat{c}(\theta, \alpha); \theta, \alpha, \tilde{\lambda}) = 0$ by the definition of $\tilde{\lambda}$, and $\varphi(\bar{c}; \theta, \alpha, \tilde{\lambda}) = \bar{c} - c_0 > 0$. Assumption 1 (Single Crossing) then implies that $\varphi(c; \theta, \alpha, \tilde{\lambda}) < 0$ when $c < \hat{c}(\theta, \alpha)$; $\varphi(c; \theta, \alpha, \tilde{\lambda}) = 0$ when $c = \hat{c}(\theta, \alpha)$; $\varphi(c; \theta, \alpha, \tilde{\lambda}) > 0$ when $c > \hat{c}(\theta, \alpha)$. It is then obvious that under constraints (31) and (32), the optimal solution to Problem II is

$$\hat{p}(\theta, c) = \begin{cases} 1, & \text{when } c \leq \hat{c}(\theta, \alpha); \\ 0, & \text{when } c > \hat{c}(\theta, \alpha). \end{cases}$$

In other words, the optimal solution to Problem II is a cutoff mechanism.

By the definition of $\hat{c}(\theta, \alpha)$ in equation (34), $\hat{p}(\theta, c)$ satisfies constraint (30). Thus the optimal solution to Problem II is also feasible in Problem I. Because the optimal solution to Problem I cannot be better than that to Problem II, $\hat{p}(\theta, c)$ must be the optimal solution to Problem I. Therefore, we have established that an optimal solution to Problem (O-R) is a cutoff mechanism. The proof completes. \square

Proof of Lemma 3: Problem (O-R-D) is a standard constrained optimization problem. It can be equivalently written as

$$\min_{\hat{c} \in [\underline{c}, \bar{c}]} TC(\alpha, \hat{c}; \theta) = \left(\theta + \frac{G(\theta)}{g(\theta)} \right) \alpha + \int_{\underline{c}}^{\hat{c}} (c - c_0) h(c, \alpha) dc$$

subject to

$$\int_{\underline{c}}^{\hat{c}} H_2(c, \alpha) dc = \theta; \tag{35}$$

$$\alpha \geq 0. \tag{36}$$

Construct the Lagrangian as

$$L = \left(\theta + \frac{G(\theta)}{g(\theta)} \right) \alpha + \int_{\underline{c}}^{\hat{c}} (c - c_0) h(c, \alpha) dc + \lambda(\theta) \left(\int_{\underline{c}}^{\hat{c}} H_2(c, \alpha) dc - \theta \right) - \xi(\theta)(\bar{c} - \hat{c}) - \mu(\theta)\alpha.$$

The Kuhn-Tucker conditions are⁴⁷

$$\frac{\partial L}{\partial \hat{c}} = (\hat{c} - c_0) h(\hat{c}, \alpha) + \lambda(\theta) H_2(\hat{c}, \alpha) + \xi(\theta) = 0, \tag{37}$$

⁴⁷Note that there is no need to worry about corner solutions regarding $\hat{c} = \underline{c}$, because $\hat{c} = \underline{c}$ can never satisfy (35).

$$\begin{aligned}
\frac{\partial L}{\partial \alpha} &= \theta + \frac{G(\theta)}{g(\theta)} + \int_{\underline{c}}^{\hat{c}} (c - c_0) H_{12}(c, \alpha) dc + \lambda(\theta) \int_{\underline{c}}^{\hat{c}} H_{22}(c, \alpha) dc - \mu(\theta) \\
&= \theta + \frac{G(\theta)}{g(\theta)} + \int_{\underline{c}}^{\hat{c}} (c - c_0) dH_2(c, \alpha) + \lambda(\theta) \int_{\underline{c}}^{\hat{c}} H_{22}(c, \alpha) dc - \mu(\theta) \\
&= \theta + \frac{G(\theta)}{g(\theta)} + (c - c_0) H_2(c, \alpha)|_{c=\underline{c}}^{\hat{c}} - \int_{\underline{c}}^{\hat{c}} H_2(c, \alpha) dc + \lambda(\theta) \int_{\underline{c}}^{\hat{c}} H_{22}(c, \alpha) dc - \mu(\theta) \\
&= \frac{G(\theta)}{g(\theta)} + (\hat{c} - c_0) H_2(\hat{c}, \alpha) + \lambda(\theta) \int_{\underline{c}}^{\hat{c}} H_{22}(c, \alpha) dc - \mu(\theta) = 0, \tag{38}
\end{aligned}$$

$$\alpha \geq 0, \mu(\theta) \geq 0, \text{ and } \mu(\theta)\alpha = 0, \tag{39}$$

$$\bar{c} - \hat{c} \geq 0, \xi(\theta) \geq 0, \text{ and } \xi(\theta)(\bar{c} - \hat{c}) = 0, \tag{40}$$

and

$$\int_{\underline{c}}^{\hat{c}} H_2(c, \alpha) dc = \theta. \tag{41}$$

We first show that $c^*(\theta) \leq c_0$ for all $\theta \in [\underline{\theta}, \bar{\theta}]$. Suppose, to the contrary, that there exists some $\tilde{\theta}$ such that $c^*(\tilde{\theta}) > c_0$. Then (37) implies that $\lambda(\tilde{\theta}) < 0$. Because $\frac{G(\theta)}{g(\theta)} \geq 0$ and $H_{22}(c, \alpha) < 0$ when $c \in (\underline{c}, \bar{c})$, (38) leads to $\mu(\tilde{\theta}) > 0$. This then implies that $\alpha = 0$. However, by (41) we have $\int_{\underline{c}}^{\hat{c}} H_2(c, 0) dc = \tilde{\theta} \leq \bar{\theta}$ with $\hat{c} > c_0$, which violates Assumption 0. Thus, $c^*(\theta) \leq c_0$ for all $\theta \in [\underline{\theta}, \bar{\theta}]$.

We next show that $c^*(\theta) < c_0$ for all $\theta \in (\underline{\theta}, \bar{\theta}]$. Suppose, to the contrary, that there exists some $\tilde{\theta}$ such that $c^*(\tilde{\theta}) = c_0$. Then (37) implies that $\lambda(\tilde{\theta}) = 0$; (38) then implies that $\mu(\tilde{\theta}) = \frac{G(\tilde{\theta})}{g(\tilde{\theta})} > 0$ so that $\alpha = 0$. However, (41) then implies that $\int_{\underline{c}}^{c_0} H_2(c, 0) dc = \tilde{\theta}$, which violates Assumption 0. Thus, $c^*(\theta) < c_0$ for all $\theta \in (\underline{\theta}, \bar{\theta}]$.

Finally, we show that $c^*(\underline{\theta}) = c_0$. It suffices to show that $c^*(\underline{\theta}) < c_0$ is not possible. Suppose, to the contrary, that this is the case; we must have $\xi(\underline{\theta}) = 0$ by slackness condition (40). Then (37) implies that $\lambda(\underline{\theta}) > 0$. Given $\frac{G(\underline{\theta})}{g(\underline{\theta})} = 0$, (38) further implies that $\mu(\underline{\theta}) < 0$, which violates (39).

One useful fact from the proof of Lemma 3 is that $\lambda(\theta) \geq 0$ with equality only when $\theta = \underline{\theta}$. This is obvious from equation (37) and the fact that $c^*(\theta) \leq c_0$ for all θ with equality only when $\theta = \underline{\theta}$. The proof completes. \square

Proof of Lemma 4: Notice that if $c^*(\theta)$ is strictly decreasing in θ , then because $\int_{\underline{c}}^{c^*(\theta)} H_2(c, \alpha^*(\theta)) dc = \theta$ and $H_{22}(c, \alpha) < 0$ when $c \in (\underline{c}, \bar{c})$, when θ increases, $\alpha^*(\theta)$ must strictly decrease. Therefore, we only need to show that $c^*(\theta)$ is strictly decreasing in θ .

Note that Lemma 3 implies that we only need to show Lemma 4 for $\theta > \underline{\theta}$. Then this means that we can restrict the choice of \hat{c} in Problem (O-R-D) from $[\underline{c}, \bar{c}]$ to $[\underline{c}, c_0)$. For each first-stage type θ , define $\hat{c}(\theta)$ as the cutoff satisfying $\int_{\underline{c}}^{\hat{c}(\theta)} H_2(c, 0) dc = \theta$. It is easy to see that (26) can hold only for $\hat{c} \geq \hat{c}(\theta)$. Therefore, to prove Lemma 4, we can restrict to $\theta > \underline{\theta}$ and $\hat{c} \in [\hat{c}(\theta), c_0)$. Obviously, $\hat{c}(\theta)$ is strictly increasing in θ . It is also straightforward that the induced effort $\alpha < \alpha^{FB}(\underline{\theta})$ when $\theta > \underline{\theta}$.

This is why Assumption 3 and Assumption 4 are imposed on $\alpha \in [0, \alpha^{FB}(\theta)]$ and $\hat{c} \in [\hat{c}(\bar{\theta}), c_0]$.

Equation (26) defines α as a function of \hat{c} and θ —denoted as $\alpha(\hat{c}, \theta)$ —which is twice continuously differentiable by the implicit function theorem. Substituting the function $\alpha(\hat{c}, \theta)$ into the objective function of Problem (O-R-D) would lead to an unconstrained optimization problem

$$\min_{\hat{c} \in [\hat{c}(\theta), c_0]} \widetilde{TC}(\hat{c}, \theta) = \left(\theta + \frac{G(\theta)}{g(\theta)} \right) \alpha(\hat{c}, \theta) + \int_{\underline{c}}^{\hat{c}} (c - c_0) h(c, \alpha(\hat{c}, \theta)) dc,$$

parameterized by θ .

The following lemma is helpful to show Lemma 4. The proof is in Appendix B.

Lemma A4 $\frac{\partial^2 \widetilde{TC}(c^*(\theta), \theta)}{\partial \theta \partial \hat{c}} > 0$ implies Lemma 4.

According to Lemma A4, we only need to show $\frac{\partial^2 \widetilde{TC}(c^*(\theta), \theta)}{\partial \theta \partial \hat{c}} > 0$. Notice that

$$\begin{aligned} \widetilde{TC} &= \left(\theta + \frac{G(\theta)}{g(\theta)} \right) \alpha(\hat{c}, \theta) + \int_{\underline{c}}^{\hat{c}} (c - c_0) dH(c, \alpha(\hat{c}, \theta)) \\ &= \left(\theta + \frac{G(\theta)}{g(\theta)} \right) \alpha(\hat{c}, \theta) + (\hat{c} - c_0) H(\hat{c}, \alpha(\hat{c}, \theta)) - \int_{\underline{c}}^{\hat{c}} H(c, \alpha(\hat{c}, \theta)) dc. \end{aligned}$$

Thus,

$$\begin{aligned} \frac{\partial \widetilde{TC}}{\partial \hat{c}} &= \left(\theta + \frac{G(\theta)}{g(\theta)} \right) \frac{\partial \alpha}{\partial \hat{c}} + H(\hat{c}, \alpha) + (\hat{c} - c_0) \left(h(\hat{c}, \alpha) + H_2(\hat{c}, \alpha) \frac{\partial \alpha}{\partial \hat{c}} \right) - H(\hat{c}, \alpha) - \frac{\partial \alpha}{\partial \hat{c}} \int_{\underline{c}}^{\hat{c}} H_2(c, \alpha(\hat{c}, \theta)) dc \\ &= \frac{G(\theta)}{g(\theta)} \frac{\partial \alpha}{\partial \hat{c}} + (\hat{c} - c_0) \left(h(\hat{c}, \alpha) + H_2(\hat{c}, \alpha) \frac{\partial \alpha}{\partial \hat{c}} \right), \end{aligned}$$

where the second equality follows from (26). Then

$$\begin{aligned} \frac{\partial^2 \widetilde{TC}}{\partial \theta \partial \hat{c}} &= \left(\frac{G(\theta)}{g(\theta)} \right)' \frac{\partial \alpha}{\partial \hat{c}} + \frac{G(\theta)}{g(\theta)} \frac{\partial^2 \alpha}{\partial \theta \partial \hat{c}} + (\hat{c} - c_0) \left(H_{12}(\hat{c}, \alpha) \frac{\partial \alpha}{\partial \theta} + H_{22}(\hat{c}, \alpha) \frac{\partial \alpha}{\partial \theta} \frac{\partial \alpha}{\partial \hat{c}} + H_2(\hat{c}, \alpha) \frac{\partial^2 \alpha}{\partial \theta \partial \hat{c}} \right) \\ &= \frac{\partial^2 \alpha}{\partial \theta \partial \hat{c}} \left(\frac{G(\theta)}{g(\theta)} + (\hat{c} - c_0) H_2(\hat{c}, \alpha) \right) + \left(\frac{G(\theta)}{g(\theta)} \right)' \frac{\partial \alpha}{\partial \hat{c}} + (\hat{c} - c_0) \left(H_{12}(\hat{c}, \alpha) + H_{22}(\hat{c}, \alpha) \frac{\partial \alpha}{\partial \hat{c}} \right) \frac{\partial \alpha}{\partial \theta}, \end{aligned}$$

where $\left(\frac{G(\theta)}{g(\theta)} \right)'$ denotes $\frac{d}{d\theta} \frac{G(\theta)}{g(\theta)}$. Therefore,

$$\begin{aligned} \frac{\partial^2 \widetilde{TC}(c^*(\theta), \theta)}{\partial \theta \partial \hat{c}} &= \underbrace{\frac{\partial^2 \alpha(c^*, \theta)}{\partial \theta \partial \hat{c}} \left(\frac{G(\theta)}{g(\theta)} + (c^* - c_0) H_2(c^*, \alpha^*) \right)}_A \\ &\quad + \underbrace{\left(\frac{G(\theta)}{g(\theta)} \right)' \frac{\partial \alpha(c^*, \theta)}{\partial \hat{c}} + (c^* - c_0) \left(H_{12}(c^*, \alpha^*) + H_{22}(c^*, \alpha^*) \frac{\partial \alpha(c^*, \theta)}{\partial \hat{c}} \right) \frac{\partial \alpha(c^*, \theta)}{\partial \theta}}_B, \end{aligned}$$

where c^* denotes $c^*(\theta)$ and α^* denotes $\alpha(c^*(\theta), \theta)$.

To show that $\frac{\partial^2 \widetilde{TC}(c^*(\theta), \theta)}{\partial \theta \partial \hat{c}} > 0$, it suffices to show that $A \geq 0$ and $B > 0$. Let us start with showing $A \geq 0$.

Step 1: $A \geq 0$. Recall that moral hazard constraint (26) defines function $\alpha(\hat{c}, \theta)$. Taking partial derivative with respect to \hat{c} on both sides of (26) leads to

$$H_2(\hat{c}, \alpha) + \int_{\underline{c}}^{\hat{c}} H_{22}(c, \alpha) \frac{\partial \alpha}{\partial \hat{c}} dc = 0, \quad (42)$$

so that

$$\frac{\partial \alpha}{\partial \hat{c}} = -\frac{H_2(\hat{c}, \alpha)}{\int_{\underline{c}}^{\hat{c}} H_{22}(c, \alpha) dc}. \quad (43)$$

Taking partial derivative with respect to θ on both sides of (26) leads to $\int_{\underline{c}}^{\hat{c}} H_{22}(c, \alpha) \frac{\partial \alpha}{\partial \theta} dc = 1$, so that

$$\frac{\partial \alpha}{\partial \theta} = \frac{1}{\int_{\underline{c}}^{\hat{c}} H_{22}(c, \alpha) dc}. \quad (44)$$

Therefore, we obtain

$$\frac{\partial \alpha}{\partial \hat{c}} = -H_2(\hat{c}, \alpha) \frac{\partial \alpha}{\partial \theta}. \quad (45)$$

It is easy to see that $\frac{\partial \alpha}{\partial \hat{c}} > 0$ and $\frac{\partial \alpha}{\partial \theta} < 0$. Now, taking partial derivative with respect to θ on both sides of equation (42) leads to

$$H_{22}(\hat{c}, \alpha) \frac{\partial \alpha}{\partial \theta} + \int_{\underline{c}}^{\hat{c}} H_{222}(c, \alpha) \frac{\partial \alpha}{\partial \theta} \frac{\partial \alpha}{\partial \hat{c}} dc + \int_{\underline{c}}^{\hat{c}} H_{22}(c, \alpha) \frac{\partial^2 \alpha}{\partial \theta \partial \hat{c}} dc = 0,$$

so that

$$\frac{\partial^2 \alpha}{\partial \theta \partial \hat{c}} = -\frac{H_{22}(\hat{c}, \alpha) \frac{\partial \alpha}{\partial \theta} + \frac{\partial \alpha}{\partial \theta} \frac{\partial \alpha}{\partial \hat{c}} \int_{\underline{c}}^{\hat{c}} H_{222}(c, \alpha) dc}{\int_{\underline{c}}^{\hat{c}} H_{22}(c, \alpha) dc}. \quad (46)$$

We have the following observation regarding the sign of $\frac{\partial^2 \alpha(c^*, \theta)}{\partial \theta \partial \hat{c}}$. (The proof is in Appendix B.)

Observation 1: Under Assumption 3, $\frac{\partial^2 \alpha(c^*, \theta)}{\partial \theta \partial \hat{c}} \geq 0$.

In addition, the proof of Lemma 3 leads to the following observation.

Observation 2:

$$\frac{G(\theta)}{g(\theta)} + (c^* - c_0)H_2(c^*, \alpha^*) \geq 0.$$

To see this, recall that in the proof of Lemma 3 we established that $\lambda(\theta) \geq 0$. Substituting this into (38) leads to

$$\frac{G(\theta)}{g(\theta)} + (c^* - c_0)H_2(c^*, \alpha^*) = \mu(\theta) - \lambda(\theta) \int_{\underline{c}}^{c^*} H_{22}(c, \alpha^*) dc \geq 0,$$

because $\mu(\theta) \geq 0$ and $\int_{\underline{c}}^{c^*} H_{22}(c, \alpha^*) dc < 0$.

Combining Observations 1 and 2, it is obvious that $A \geq 0$. Now we turn to $B > 0$.

Step 2: $B > 0$. From (43)–(45),

$$\begin{aligned}
B &= - \left(\frac{G(\theta)}{g(\theta)} \right)' H_2(c^*, \alpha^*) \frac{\partial \alpha(c^*, \theta)}{\partial \theta} + (c^* - c_0) \left(H_{12}(c^*, \alpha^*) + H_{22}(c^*, \alpha^*) \frac{\partial \alpha(c^*, \theta)}{\partial \hat{c}} \right) \frac{\partial \alpha(c^*, \theta)}{\partial \theta} \\
&= \frac{\partial \alpha(c^*, \theta)}{\partial \theta} \cdot \left\{ - \left(\frac{G(\theta)}{g(\theta)} \right)' H_2(c^*, \alpha^*) + (c^* - c_0) \left(H_{12}(c^*, \alpha^*) + H_{22}(c^*, \alpha^*) \frac{\partial \alpha(c^*, \theta)}{\partial \hat{c}} \right) \right\} \\
&= \frac{\partial \alpha(c^*, \theta)}{\partial \theta} \cdot \underbrace{\left\{ - \left(\frac{G(\theta)}{g(\theta)} \right)' H_2(c^*, \alpha^*) + (c^* - c_0) \left(H_{12}(c^*, \alpha^*) - \frac{H_{22}(c^*, \alpha^*) \cdot H_2(c^*, \alpha^*)}{\int_{\underline{c}}^{c^*} H_{22}(c, \alpha^*) dc} \right) \right\}}_{B_1},
\end{aligned}$$

where the first equality follows from (45) and the last equality follows from (43).

Recall that $\frac{\partial \alpha}{\partial \theta} < 0$. Thus, $B > 0$ is equivalent to $B_1 < 0$. The following observation is easy to see. (The proof is relegated to Appendix B.)

Observation 3: Under Assumptions 2 and 4, $B_1 < 0$.

The proof of Lemma 4 completes. \square

Proofs of Proposition 3, Lemma 5, and Proposition 5

Proof of Proposition 3: Recall the envelope condition that

$$\pi(\theta, \theta) = \pi(\bar{\theta}, \bar{\theta}) + \int_{\theta}^{\bar{\theta}} \alpha^*(s) ds,$$

$\pi(\bar{\theta}, \bar{\theta}) = 0$, and equation (16) that

$$\pi(\theta, \theta) = x^*(\theta) - \theta \alpha^*(\theta) + \int_{\underline{c}}^{c^*(\theta)} H(c, \alpha^*(\theta)) dc.$$

It is then easy to obtain

$$x^*(\theta) = \theta \alpha^*(\theta) + \int_{\theta}^{\bar{\theta}} \alpha^*(s) ds - \int_{\underline{c}}^{c^*(\theta)} H(c, \alpha^*(\theta)) dc.$$

To prove this proposition, it suffices to show that $x^*(\bar{\theta}) < 0$ and $x^{*\prime}(\theta) > 0$. Notice that

$$x^*(\bar{\theta}) = \bar{\theta} \alpha^*(\bar{\theta}) - \int_{\underline{c}}^{c^*(\bar{\theta})} H(c, \alpha^*(\bar{\theta})) dc.$$

By definition, $\alpha^*(\bar{\theta}) > 0$ is the unique maximizer of the following function (e.g., see (14))

$$\psi(\alpha) = -\bar{\theta}\alpha + \int_{\underline{c}}^{c^*(\bar{\theta})} H(c, \alpha)dc, \alpha \geq 0.$$

Therefore,

$$x^*(\bar{\theta}) = \bar{\theta}\alpha^*(\bar{\theta}) - \int_{\underline{c}}^{c^*(\bar{\theta})} H(c, \alpha^*(\bar{\theta}))dc < \bar{\theta} \cdot 0 - \int_{\underline{c}}^{c^*(\bar{\theta})} H(c, 0)dc \leq 0.$$

Now we proceed to show $x^{*'}(\theta) > 0$. In fact,

$$\begin{aligned} x^{*'}(\theta) &= \alpha^*(\theta) + \theta\alpha^{*'}(\theta) - \alpha^*(\theta) - H(c^*(\theta), \alpha^*(\theta))c^{*'}(\theta) - \alpha^{*'}(\theta) \int_{\underline{c}}^{c^*(\theta)} H_2(c, \alpha^*(\theta))dc \\ &= \alpha^{*'}(\theta) \left(\theta - \int_{\underline{c}}^{c^*(\theta)} H_2(c, \alpha^*(\theta))dc \right) - H(c^*(\theta), \alpha^*(\theta))c^{*'}(\theta) \\ &= -H(c^*(\theta), \alpha^*(\theta))c^{*'}(\theta) > 0, \end{aligned} \tag{47}$$

where the third equality uses the fact that

$$\alpha^{*'}(\theta) \left(\theta - \int_{\underline{c}}^{c^*(\theta)} H_2(c, \alpha^*(\theta))dc \right) = 0,$$

as $\int_{\underline{c}}^{c^*(\theta)} H_2(c, \alpha^*(\theta))dc = \theta$. The proof completes. \square

Proof of Lemma 5: The first part of Lemma 5 follows directly from Proposition 3.

For the second part, recall that by definition, $p^*(\theta) = c^*(\theta) - r^*(\theta) = c^*(\theta) + x^*(\theta)$. From Proposition 3, $x^*(\theta) < 0$. Therefore, $p^*(\theta) < c^*(\theta) \leq c_0$ for all θ by Lemma 3. To prove the rest of the claims, it suffices to show that $p^*(\bar{\theta}) > 0$ and $p^*(\theta)$ is strictly decreasing.

In fact,

$$\begin{aligned} p^*(\bar{\theta}) &= c^*(\bar{\theta}) + x^*(\bar{\theta}) = c^*(\bar{\theta}) + \bar{\theta}\alpha^*(\bar{\theta}) - \int_{\underline{c}}^{c^*(\bar{\theta})} H(c, \alpha^*(\bar{\theta}))dc \\ &> c^*(\bar{\theta}) + \bar{\theta}\alpha^*(\bar{\theta}) - (c^*(\bar{\theta}) - \underline{c}) = \bar{\theta}\alpha^*(\bar{\theta}) + \underline{c} > 0. \end{aligned}$$

What is left is to show that $p^{*'}(\theta) < 0$. Notice that from (47) and the fact that $H(c^*(\theta), \alpha^*(\theta)) < 1$, we have

$$p^{*'}(\theta) = c^{*'}(\theta) + x^{*'}(\theta) = c^{*'}(\theta)[1 - H(c^*(\theta), \alpha^*(\theta))] < 0.$$

\square

Proof of Proposition 5: Recall that the absolute value of the first-stage transfer is increasing in the second-stage cutoff (as shown in (47)), and that the second-stage cutoff increasing in the

agent's R&D efficiency is necessary for IC in the class of deterministic mechanisms (Theorem 1). Therefore, limited liability imposes an upper bound on the second-stage cutoff, denoted as $c(K)$, and this bound $c(K)$ is obviously increasing in K . If $c(K) \geq c_0$ —i.e., K is even larger than the highest possible default remedy (which is paid by the most efficient type when he defaults), which means that $K \geq r^*(\underline{\theta})$ —then the optimal deterministic mechanism must be the same as the one without limited liability; however, if $c(K) < c_0$, then bunching arises. Specifically, let $\tilde{\theta}$ be the unique cutoff such that⁴⁸ $c^*(\tilde{\theta}) = c(K)$ —i.e., the default remedy by type $\tilde{\theta}$, which is $r^*(\tilde{\theta})$ in the optimal mechanism without limited liability, equals K —then the optimal deterministic mechanism is a pooling one when $\theta \leq \tilde{\theta}$. More precisely, for all $\theta \leq \tilde{\theta}$, the optimal deterministic mechanism always takes the form $\{p^*(\tilde{\theta}), r^*(\tilde{\theta})\}$; for all $\theta > \tilde{\theta}$, the optimal deterministic mechanism is the same as the one without limited liability. Because $c(K)$ is increasing in K , obviously $\tilde{\theta}$ is decreasing in K . \square

Proofs for Section 5

In this subsection, we derive the optimal mechanism for the pure adverse selection case. As usual, we start from the second stage.

Stage Two

Assume that the agent truthfully reported his type θ in stage one, and his realized cost is c . Let $\tilde{\pi}_p(\theta, \hat{c}, c)$ be his expected payoff in stage two if he reports \hat{c} . Then

$$\tilde{\pi}_p(\theta, \hat{c}, c) = \tilde{y}(\theta, \hat{c}) - \tilde{p}(\theta, \hat{c})c. \quad (48)$$

The envelope theorem yields

$$\frac{d\tilde{\pi}_p(\theta, c, c)}{dc} = \frac{\partial \tilde{\pi}_p(\theta, \hat{c}, c)}{\partial c} \Big|_{\hat{c}=c} = -\tilde{p}(\theta, c).$$

Thus

$$\tilde{\pi}_p(\theta, c, c) = \tilde{\pi}_p(\theta, \bar{c}, \bar{c}) + \int_c^{\bar{c}} \tilde{p}(\theta, s) ds. \quad (49)$$

It is standard to establish that the second-stage IC is equivalent to that (49) holds and $\tilde{p}(\theta, c)$ is decreasing in c for any fixed θ . Note that the agent will still truthfully report his second-stage type c on the off-equilibrium path. That is, if the agent misreported his type in stage one as $\hat{\theta}$, then he will still truthfully report c in stage two. The reason is similar to that in our main analysis of the mixed problem.

Note that here we only consider IC in stage two. As will be shown later, at the optimum the agent's second-stage IR is satisfied.

⁴⁸If $\tilde{\theta} > \bar{\theta}$, then set $\tilde{\theta} = \bar{\theta}$.

Stage One

We need to consider both IC and IR in stage one. If the agent with type θ reports his type in stage one as $\hat{\theta}$, his expected payoff is

$$\pi_p(\hat{\theta}, \theta) = \tilde{x}(\hat{\theta}) - \theta \tilde{\alpha}(\hat{\theta}) + \int_{\underline{c}}^{\bar{c}} \tilde{\pi}_p(\hat{\theta}, c, c) h(c, \tilde{\alpha}(\hat{\theta})) dc. \quad (50)$$

The first term on the right-hand side is the payment, the second term is the agent's investment cost, and the last term is his expected profit from the second stage. Substitute (49) into (50)

$$\begin{aligned} \pi_p(\hat{\theta}, \theta) &= \tilde{x}(\hat{\theta}) - \theta \tilde{\alpha}(\hat{\theta}) + \int_{\underline{c}}^{\bar{c}} \left(\tilde{\pi}_p(\hat{\theta}, \bar{c}, \bar{c}) + \int_c^{\bar{c}} p(\hat{\theta}, s) ds \right) h(c, \tilde{\alpha}(\hat{\theta})) dc \\ &= \tilde{x}(\hat{\theta}) - \theta \tilde{\alpha}(\hat{\theta}) + \tilde{\pi}_p(\hat{\theta}, \bar{c}, \bar{c}) + \int_{\underline{c}}^{\bar{c}} \int_c^{\bar{c}} p(\hat{\theta}, s) h(c, \tilde{\alpha}(\hat{\theta})) ds dc. \end{aligned}$$

There is no loss of generality to assume that $\tilde{\pi}_p(\theta, \bar{c}, \bar{c}) = 0$ for all θ . The reason is that we can always define $\check{x}(\theta) = \tilde{x}(\theta) + \tilde{\pi}_p(\theta, \bar{c}, \bar{c})$. Such change does not affect the first-stage expected payoff $\pi_p(\hat{\theta}, \theta)$, and thus the agent's incentive remains the same.

Envelope theorem leads to

$$\frac{d\pi_p(\theta, \theta)}{d\theta} = \frac{\partial \pi_p(\hat{\theta}, \theta)}{\partial \theta} \Big|_{\hat{\theta}=\theta} = -\tilde{\alpha}(\theta),$$

thus

$$\pi_p(\theta, \theta) = \pi_p(\bar{\theta}, \bar{\theta}) + \int_{\theta}^{\bar{\theta}} \tilde{\alpha}(s) ds. \quad (51)$$

Routine procedure leads to the observation that IC and IR in stage one are equivalent to $\tilde{\alpha}(\theta)$ being decreasing in θ , $\pi_p(\bar{\theta}, \bar{\theta}) \geq 0$, and that (51) holds.

The Principal's Objective

The expected social cost can be written as

$$\begin{aligned} SC_p &= \int_{\underline{\theta}}^{\bar{\theta}} \left(\theta \tilde{\alpha}(\theta) + \int_{\underline{c}}^{\bar{c}} [\tilde{p}(\theta, c)c + (1 - \tilde{p}(\theta, c))c_0] h(c, \tilde{\alpha}(\theta)) dc \right) g(\theta) d\theta \\ &= \int_{\underline{\theta}}^{\bar{\theta}} \left(\theta \tilde{\alpha}(\theta) + \int_{\underline{c}}^{\bar{c}} \tilde{p}(\theta, c)(c - c_0) h(c, \tilde{\alpha}(\theta)) dc \right) g(\theta) d\theta + c_0. \end{aligned}$$

The principal's total cost is the sum of the expected social cost and the agent's expected profit:

$$\begin{aligned} TC_p &= SC_p + \int_{\underline{\theta}}^{\bar{\theta}} \left(\pi(\bar{\theta}, \bar{\theta}) + \int_{\theta}^{\bar{\theta}} \tilde{\alpha}(s) ds \right) g(\theta) d\theta \\ &= \int_{\underline{\theta}}^{\bar{\theta}} \left(J(\theta) \tilde{\alpha}(\theta) + \int_{\underline{c}}^{\bar{c}} \tilde{p}(\theta, c) (c - c_0) h(c, \tilde{\alpha}(\theta)) dc \right) g(\theta) d\theta + \pi(\bar{\theta}, \bar{\theta}) + c_0, \end{aligned}$$

where $J(\theta) = \theta + \frac{G(\theta)}{g(\theta)}$.

Now the principal's problem can be written as

$$\min_{\tilde{\alpha}(\theta), \tilde{p}(\theta, c)} TC_p$$

subject to

$$\tilde{\alpha}(\theta) \geq 0, \forall \theta; \tag{52}$$

$$\tilde{\alpha}(\theta) \text{ is decreasing in } \theta; \tag{53}$$

$$\pi_p(\bar{\theta}, \bar{\theta}) \geq 0; \tag{54}$$

$$0 \leq \tilde{p}(\theta, c) \leq 1, \forall c, \forall \theta; \tag{55}$$

$$\tilde{p}(\theta, c) \text{ is decreasing in } c, \forall \theta. \tag{56}$$

Call this Problem (M).

Obviously, $\pi_p(\bar{\theta}, \bar{\theta}) = 0$ at the optimum. We minimize the objective function pointwisely. That is, we choose $\tilde{\alpha}$ and $\tilde{p}(\theta, c)$ to minimize

$$\tilde{\alpha} J(\theta) + \int_{\underline{c}}^{\bar{c}} \tilde{p}(\theta, c) (c - c_0) h(c, \tilde{\alpha}) dc \tag{57}$$

for fixed θ .

Notice that there are two unknowns to pin down; one is $\tilde{\alpha}(\theta)$, and the other is $\tilde{p}(\theta, c)$. First fix $\tilde{\alpha}$ and solve for the optimal $\tilde{p}(\theta, c)$ for this fixed $\tilde{\alpha}$, and then varying across all possible $\tilde{\alpha} \geq 0$ will give the optimal $\tilde{\alpha}(\theta)$. The optimal $\tilde{p}(\theta, c)$ for any fixed $\tilde{\alpha}$ and any fixed θ is very simple. In fact, to minimize the integrand, we should set

$$\tilde{p}^*(\theta, c) = \begin{cases} 1, & \text{if } c \leq c_0, \\ 0, & \text{if } c > c_0. \end{cases}$$

Then

$$\begin{aligned} \int_{\underline{c}}^{\bar{c}} \tilde{p}^*(\theta, c)(c - c_0)h(c, \tilde{\alpha})dc &= \int_{\underline{c}}^{c_0} (c - c_0)h(c, \tilde{\alpha})dc = \int_{\underline{c}}^{c_0} (c - c_0)dH(c, \tilde{\alpha}) \\ &= (c - c_0)H(c, \tilde{\alpha})|_{c=\underline{c}}^{c_0} - \int_{\underline{c}}^{c_0} H(c, \tilde{\alpha})dc = - \int_{\underline{c}}^{c_0} H(c, \tilde{\alpha})dc. \end{aligned}$$

Now (57) becomes

$$\tilde{\alpha}J(\theta) - \int_{\underline{c}}^{c_0} H(c, \tilde{\alpha})dc. \quad (58)$$

Notice that such $\tilde{p}^*(\theta, c)$ satisfies constraints (55) and (56) in Problem (M). Hence the next thing to do is to choose $\tilde{\alpha}$ to minimize the above term. In fact, it is easy to show that there is a unique $\tilde{\alpha}^*(\theta)$ that minimizes (58), as the function is strictly convex in $\tilde{\alpha}$, which is characterized by

$$J(\theta) - \int_{\underline{c}}^{c_0} H_2(c, \tilde{\alpha}^*(\theta))dc \geq 0, \text{ with equality when } \tilde{\alpha}^*(\theta) = 0. \quad (59)$$

We have $\tilde{\alpha}^*(\theta)$ is decreasing in θ (so this will imply IC), and $\tilde{\alpha}^*(\theta) \leq \alpha^{FB}(\theta)$ with equality only when $\theta = \underline{\theta}$. The proof is relegated to the subsection entitled ‘‘Derivations’’ (Derivation 1).

Henceforth, let $\tilde{\alpha}^*(\theta)$ denote the optimal solution. Now, because $\tilde{\alpha}^*(\theta)$ is nonnegative and decreasing in θ , all the constraints in Problem (M) are satisfied. Therefore, we have indeed found the optimal solution to Problem (M).

Implementation

Recall that there is no loss of generality to assume that $\tilde{\pi}_p(\theta, \bar{c}, \bar{c}) = 0$ for all θ . By (49) and

$$\tilde{p}^*(\theta, c) = \begin{cases} 1, & \text{if } c \leq c_0, \\ 0, & \text{if } c > c_0, \end{cases}$$

we have

$$\tilde{\pi}_p(\theta, c, c) = \tilde{\pi}_p(\theta, \bar{c}, \bar{c}) + \int_c^{\bar{c}} \tilde{p}^*(\theta, s)ds = \begin{cases} c_0 - c, & \text{if } c \leq c_0, \\ 0, & \text{if } c > c_0. \end{cases}$$

Thus

$$\tilde{y}^*(\theta, c) = \tilde{p}^*(\theta, c)c + \tilde{\pi}_p(\theta, c, c) = \begin{cases} c_0, & \text{if } c \leq c_0, \\ 0, & \text{if } c > c_0. \end{cases}$$

This implies that the second-stage mechanism is independent of the agent’s first-stage type. Now we proceed to characterize the first-stage payment rule.

By (51),

$$\pi_p(\theta, \theta) = \pi_p(\bar{\theta}, \bar{\theta}) + \int_{\theta}^{\bar{\theta}} \tilde{\alpha}^*(s) ds = \int_{\theta}^{\bar{\theta}} \tilde{\alpha}^*(s) ds.$$

Recall that

$$\pi_p(\theta, \theta) = \tilde{x}^*(\theta) - \theta \tilde{\alpha}^*(\theta) + \int_{\underline{c}}^{\bar{c}} \tilde{\pi}_p(\theta, c, c) h(c, \tilde{\alpha}^*(\theta)) dc.$$

Hence

$$\begin{aligned} \tilde{x}^*(\theta) &= \int_{\theta}^{\bar{\theta}} \tilde{\alpha}^*(s) ds + \theta \tilde{\alpha}^*(\theta) - \int_{\underline{c}}^{\bar{c}} \tilde{\pi}_p(\theta, c, c) h(c, \tilde{\alpha}^*(\theta)) dc \\ &= \int_{\theta}^{\bar{\theta}} \tilde{\alpha}^*(s) ds + \theta \tilde{\alpha}^*(\theta) + \int_{\underline{c}}^{c_0} (c - c_0) h(c, \tilde{\alpha}^*(\theta)) dc \\ &= \int_{\theta}^{\bar{\theta}} \tilde{\alpha}^*(s) ds + \theta \tilde{\alpha}^*(\theta) - \int_{\underline{c}}^{c_0} H(c, \tilde{\alpha}^*(\theta)) dc. \end{aligned}$$

It is easy to obtain that $\tilde{x}^*(\theta) \leq 0$ and is increasing in θ . The derivation is in the next subsection, entitled ‘‘Derivations’’ (Derivation 2).

Derivations

Derivation 1: Define

$$\tilde{\phi}(\tilde{\alpha}, \theta) = \tilde{\alpha} J(\theta) - \int_{\underline{c}}^{c_0} H(c, \tilde{\alpha}) dc, \quad (\tilde{\alpha}, \theta) \in [0, +\infty) \times [\underline{\theta}, \bar{\theta}].$$

We need to minimize $\tilde{\phi}(\tilde{\alpha}, \theta)$. The first-order partial derivative with respect to $\tilde{\alpha}$ is $\frac{\partial \tilde{\phi}}{\partial \tilde{\alpha}} = J(\theta) - \int_{\underline{c}}^{c_0} H_2(c, \tilde{\alpha}) dc$, and the second-order partial derivative $\frac{\partial^2 \tilde{\phi}}{\partial \tilde{\alpha}^2} = - \int_{\underline{c}}^{c_0} H_{22}(c, \tilde{\alpha}) dc > 0$. Hence, $\tilde{\phi}(\tilde{\alpha}, \theta)$ is strictly convex in $\tilde{\alpha}$ so that there exists a unique $\tilde{\alpha} \geq 0$ minimizing $\tilde{\phi}(\tilde{\alpha}, \theta)$, denoted as $\tilde{\alpha}^*(\theta)$. The fact that $\tilde{\alpha}^*(\theta)$ is decreasing in θ follows from the observation that $\tilde{\phi}(\tilde{\alpha}, \theta)$ has increasing difference in $(\tilde{\alpha}, \theta)$. Finally, a comparison between (13) and (59) implies that $\tilde{\alpha}^*(\theta) \leq \alpha^{FB}(\theta)$ with equality only when $\theta = \underline{\theta}$. (Note that $\alpha^{FB}(\theta) > 0$ for all θ by Assumption 0.) \square

Derivation 2: Notice first that

$$\begin{aligned} \tilde{x}^*(\bar{\theta}) &= \bar{\theta} \tilde{\alpha}^*(\bar{\theta}) - \int_{\underline{c}}^{c_0} H(c, \tilde{\alpha}^*(\bar{\theta})) dc = J(\bar{\theta}) \tilde{\alpha}^*(\bar{\theta}) - \int_{\underline{c}}^{c_0} H(c, \tilde{\alpha}^*(\bar{\theta})) dc - \frac{G(\bar{\theta})}{g(\bar{\theta})} \tilde{\alpha}^*(\bar{\theta}) \\ &\leq J(\bar{\theta}) \cdot 0 - \int_{\underline{c}}^{c_0} H(c, 0) dc - \frac{G(\bar{\theta})}{g(\bar{\theta})} \tilde{\alpha}^*(\bar{\theta}) \leq 0, \end{aligned}$$

where the first inequality comes from the fact that $\tilde{\alpha}^*(\bar{\theta})$ minimizes the function $\tilde{\alpha} J(\bar{\theta}) - \int_{\underline{c}}^{c_0} H(c, \tilde{\alpha}) dc$.

Therefore, what is left is to show that $\tilde{x}^{*'}(\theta) \geq 0$. In fact,

$$\begin{aligned}\tilde{x}^{*'}(\theta) &= -\tilde{\alpha}^*(\theta) + \tilde{\alpha}^*(\theta) + \theta\tilde{\alpha}^{*'}(\theta) - \tilde{\alpha}^{*'}(\theta) \int_{\underline{c}}^{c_0} H_2(c, \tilde{\alpha}^*(\theta))dc \\ &= \tilde{\alpha}^{*'}(\theta) \left(\theta - \int_{\underline{c}}^{c_0} H_2(c, \tilde{\alpha}^*(\theta))dc \right).\end{aligned}$$

Then,

$$\int_{\underline{c}}^{c_0} H_2(c, \tilde{\alpha}^*(\theta))dc \geq \int_{\underline{c}}^{c_0} H_2(c, \alpha^{FB}(\theta))dc = \theta,$$

as $\tilde{\alpha}^*(\theta) \leq \alpha^{FB}(\theta)$. Thus we have $\theta - \int_{\underline{c}}^{c_0} H_2(c, \tilde{\alpha}^*(\theta))dc \leq 0$, so $\tilde{x}^{*'}(\theta) \geq 0$ as $\tilde{\alpha}^*(\theta)$ is decreasing in θ . \square

Derivation 3: We only need to show that $\tilde{p}^*(\underline{\theta}) > 0$ as $\tilde{p}^*(\theta)$ is increasing in θ . In fact,

$$\begin{aligned}\tilde{p}^*(\underline{\theta}) &= c_0 + \tilde{x}^*(\underline{\theta}) = c_0 + \int_{\underline{\theta}}^{\bar{\theta}} \tilde{\alpha}^*(s)ds + \underline{\theta}\tilde{\alpha}^*(\underline{\theta}) - \int_{\underline{c}}^{c_0} H(c, \tilde{\alpha}^*(\underline{\theta}))dc \\ &> c_0 + \int_{\underline{\theta}}^{\bar{\theta}} \tilde{\alpha}^*(s)ds + \underline{\theta}\tilde{\alpha}^*(\underline{\theta}) - (c_0 - \underline{c}) = \int_{\underline{\theta}}^{\bar{\theta}} \tilde{\alpha}^*(s)ds + \underline{\theta}\tilde{\alpha}^*(\underline{\theta}) + \underline{c} > 0.\end{aligned}$$

\square

9 Appendix B

Proof of Lemma A1: From (18), because $H_{22}(c, \alpha) < 0$ when $c \in (\underline{c}, \bar{c})$, $\alpha(\hat{\theta}, \theta)$ is decreasing in θ for any fixed $\hat{\theta}$ and strictly decreasing if and only if $\alpha(\hat{\theta}, \theta) > 0$. Therefore, for any $\hat{\theta}$, there exists a unique $\gamma(\hat{\theta})$ such that $\alpha(\hat{\theta}, \theta) > 0$ if and only if $\theta < \gamma(\hat{\theta})$; and $\alpha(\hat{\theta}, \theta) = 0$ if and only if $\theta \geq \gamma(\hat{\theta})$ (temporarily ignore the situation in which $\gamma(\hat{\theta}) \notin [\underline{\theta}, \bar{\theta}]$). When $\theta < \gamma(\hat{\theta})$, (18) holds with equality. By the implicit function theorem, $\alpha(\hat{\theta}, \theta)$ is a differentiable function of θ . When $\theta > \gamma(\hat{\theta})$, $\alpha(\hat{\theta}, \theta) = 0$ so that it is also differentiable in θ . As a result, the function $\alpha(\hat{\theta}, \theta)$ is differentiable in θ except when $\theta = \gamma(\hat{\theta})$. Because $\gamma(\hat{\theta})$ may not fall in $[\underline{\theta}, \bar{\theta}]$, $\alpha(\hat{\theta}, \cdot)$ is differentiable everywhere except possibly at one point. The continuity of $\alpha(\hat{\theta}, \theta)$ is obvious. \square

Proof of Lemma A2: Differentiability follows directly from Lemma A1: By Lemma A1, for any $\hat{\theta}$, $\pi(\hat{\theta}, \theta)$ is differentiable in θ except possibly at one point. Now we show the other two properties.

When $\theta < \gamma(\hat{\theta})$, $\frac{\partial \alpha(\hat{\theta}, \theta)}{\partial \theta}$ exists, and (18) holds with equality. Thus

$$\frac{\partial \alpha(\hat{\theta}, \theta)}{\partial \theta} \left(-\theta + \int_{\underline{c}}^{\bar{c}} p(\hat{\theta}, c)H_2(c, \alpha(\hat{\theta}, \theta))dc \right) = 0. \quad (60)$$

When $\theta > \gamma(\hat{\theta})$, $\alpha(\hat{\theta}, \theta) = 0$ so that $\frac{\partial \alpha(\hat{\theta}, \theta)}{\partial \theta} = 0$. Therefore, the above equality still holds.

Notice that $\pi(\hat{\theta}, \theta)$ is not differentiable only at $\theta = \gamma(\hat{\theta})$ (note that when $\gamma(\hat{\theta}) \notin [\underline{\theta}, \bar{\theta}]$, $\pi(\hat{\theta}, \theta)$ is differentiable everywhere). When $\theta \neq \gamma(\hat{\theta})$,

$$\frac{\partial \pi(\hat{\theta}, \theta)}{\partial \theta} = \left(-\theta + \int_{\underline{c}}^{\bar{c}} p(\hat{\theta}, c) H_2(c, \alpha(\hat{\theta}, \theta)) dc \right) \frac{\partial \alpha(\hat{\theta}, \theta)}{\partial \theta} - \alpha(\hat{\theta}, \theta) = -\alpha(\hat{\theta}, \theta).$$

Note that for any fixed $\hat{\theta}$, $\pi(\hat{\theta}, \theta)$ is continuous in θ and $0 \leq \alpha(\hat{\theta}, \theta) \leq \alpha(\hat{\theta}, \underline{\theta}) < +\infty$. Therefore, $\alpha(\hat{\theta}, \theta) \in [0, \alpha(\hat{\theta}, \underline{\theta})]$. Thus when $\theta \leq \gamma(\hat{\theta})$, $\pi(\hat{\theta}, \theta)$ is Lipschitz continuous in θ . Because $\pi(\hat{\theta}, \theta)$ is a constant when $\theta \geq \gamma(\hat{\theta})$ and $\pi(\hat{\theta}, \theta)$ is continuous in θ , we have $\pi(\hat{\theta}, \theta)$ is Lipschitz continuous over the whole domain of θ .⁴⁹ \square

Proof of Lemma A3: Consider the function

$$g(t) = \theta_h \beta^*(t) - \theta_l \gamma^*(t) + \int_{\underline{c}}^t [H(c, \gamma^*(t)) - H(c, \beta^*(t))] dc, t \in [\underline{c}, \bar{c}],$$

where $\beta^*(t)$ is the (unique) maximizer of

$$\tau_1(\beta) = -\theta_h \beta + \int_{\underline{c}}^t H(c, \beta) dc, \beta \geq 0,$$

and $\gamma^*(t)$ is the (unique) maximizer of

$$\tau_2(\gamma) = -\theta_l \gamma + \int_{\underline{c}}^t H(c, \gamma) dc, \gamma \geq 0.$$

That is,

$$-\theta_h + \int_{\underline{c}}^t H_2(c, \beta^*(t)) dc \leq 0, \text{ with equality if } \beta^*(t) > 0,$$

and

$$-\theta_l + \int_{\underline{c}}^t H_2(c, \gamma^*(t)) dc \leq 0, \text{ with equality if } \gamma^*(t) > 0.$$

We need to show that $g(t)$ is increasing in $[\underline{c}, \bar{c}]$. Note that $g(t)$ is a continuous function. By Assumption 0, $\theta_h < \int_{\underline{c}}^{c_0} H_2(c, 0) dc$. Then there exist \hat{t}_1 and \hat{t}_2 with $\underline{c} < \hat{t}_1 < \hat{t}_2 < c_0$, such that: $\beta^*(t) = 0$ when $t \leq \hat{t}_2$, $\beta^*(t) > 0$ when $t > \hat{t}_2$; $\gamma^*(t) = 0$ when $t \leq \hat{t}_1$, $\gamma^*(t) > 0$ when $t > \hat{t}_1$. Therefore, $g(t) = 0$ when $t \leq \hat{t}_1$. When $t \in (\hat{t}_1, \hat{t}_2]$,

$$g(t) = -\theta_l \gamma^*(t) + \int_{\underline{c}}^t (H(c, \gamma^*(t)) - H(c, 0)) dc,$$

where

$$-\theta_l + \int_{\underline{c}}^t H_2(c, \gamma^*(t)) dc = 0.$$

⁴⁹Here we ignore the situation in which $\gamma(\hat{\theta})$ is not in $[\underline{\theta}, \bar{\theta}]$. If this is the case, then obviously $\pi(\hat{\theta}, \theta)$ is Lipschitz continuous, as now it is differentiable everywhere and the derivative is bounded.

When $t \in (\hat{t}_2, \bar{c}]$,

$$g(t) = \theta_h \beta^*(t) - \theta_l \gamma^*(t) + \int_{\underline{c}}^t (H(c, \gamma^*(t)) - H(c, \beta^*(t))) dc,$$

where

$$-\theta_h + \int_{\underline{c}}^t H_2(c, \beta^*(t)) dc = 0, \text{ and } -\theta_l + \int_{\underline{c}}^t H_2(c, \gamma^*(t)) dc = 0.$$

Therefore, it suffices to show that $g(t)$ is strictly increasing in $(\hat{t}_1, \bar{c}]$. To see this, note that $g(t)$ is continuous in this interval and differentiable in $(\hat{t}_1, \hat{t}_2) \cup (\hat{t}_2, \bar{c})$.⁵⁰ Taking derivative with respect to t : When $t \in (\hat{t}_1, \hat{t}_2)$,

$$g'(t) = H(t, \gamma^*(t)) - H(t, 0) > 0.$$

When $t \in (\hat{t}_2, \bar{c})$,

$$g'(t) = H(t, \gamma^*(t)) - H(t, \beta^*(t)) > 0,$$

because $\gamma^*(t) > \beta^*(t)$ when $\beta^*(t) > 0$.

Thus $g(t)$ is increasing in $[\underline{c}, \bar{c}]$. This completes the proof of Lemma A3. \square

Proof of Lemma A4: Recall that the optimal solution $\alpha^*(\theta)$ to Problem (O-R-D) is strictly positive, which implies that $c^*(\theta) \in (\hat{c}(\theta), c_0)$. Therefore, $c^*(\theta)$ satisfies the first-order condition:

$$\frac{\partial \widetilde{TC}(c^*(\theta), \theta)}{\partial \hat{c}} = 0.$$

Take derivative with respect to θ on both sides of the above equation:

$$\frac{\partial^2 \widetilde{TC}(c^*(\theta), \theta)}{\partial \hat{c}^2} \cdot \frac{dc^*(\theta)}{d\theta} + \frac{\partial^2 \widetilde{TC}(c^*(\theta), \theta)}{\partial \theta \partial \hat{c}} = 0. \quad (61)$$

Because $\hat{c} = c^*(\theta)$ is the optimal solution, $\frac{\partial^2 \widetilde{TC}(c^*(\theta), \theta)}{\partial \hat{c}^2} \geq 0$. It then follows from (61) that $\frac{\partial^2 \widetilde{TC}(c^*(\theta), \theta)}{\partial \theta \partial \hat{c}} > 0$ implies $\frac{dc^*(\theta)}{d\theta} < 0$. \square

Proof of Observation 1: Because $\int_{\underline{c}}^{c^*} H_{22}(c, \alpha) dc < 0$, we only need to show that

$$H_{22}(c^*, \alpha^*) \frac{\partial \alpha(c^*, \theta)}{\partial \theta} + \frac{\partial \alpha(c^*, \theta)}{\partial \theta} \frac{\partial \alpha(c^*, \theta)}{\partial \hat{c}} \int_{\underline{c}}^{c^*} H_{222}(c, \alpha^*) dc \geq 0. \quad (62)$$

⁵⁰ $\beta^*(t) = 0$ when $t \in (\hat{t}_1, \hat{t}_2)$, which is trivially differentiable. When $t \in (\hat{t}_2, \bar{c})$, it is differentiable by the implicit function theorem. $\gamma^*(t)$ is differentiable when $t \in (\hat{t}_1, \bar{c})$ by the implicit function theorem.

Using (43) and (44), we have

$$\begin{aligned} & H_{22}(c^*, \alpha^*) \frac{\partial \alpha(c^*, \theta)}{\partial \theta} + \frac{\partial \alpha(c^*, \theta)}{\partial \theta} \frac{\partial \alpha(c^*, \theta)}{\partial \hat{c}} \int_{\underline{c}}^{c^*} H_{222}(c, \alpha^*) dc \\ &= \frac{\partial \alpha(c^*, \theta)}{\partial \theta} \left(H_{22}(c^*, \alpha^*) - \frac{H_2(c^*, \alpha^*)}{\int_{\underline{c}}^{c^*} H_{22}(c, \alpha^*) dc} \int_{\underline{c}}^{c^*} H_{222}(c, \alpha^*) dc \right). \end{aligned}$$

Recall that $\frac{\partial \alpha(c^*, \theta)}{\partial \theta} < 0$, so it suffices to show that

$$H_{22}(c^*, \alpha^*) - \frac{H_2(c^*, \alpha^*)}{\int_{\underline{c}}^{c^*} H_{22}(c, \alpha^*) dc} \int_{\underline{c}}^{c^*} H_{222}(c, \alpha^*) dc \leq 0.$$

Notice that $H_2(c^*, \alpha^*) > 0$; thus the above inequality is equivalent to

$$\frac{H_{22}(c^*, \alpha^*)}{H_2(c^*, \alpha^*)} \leq \frac{\int_{\underline{c}}^{c^*} H_{222}(c, \alpha^*) dc}{\int_{\underline{c}}^{c^*} H_{22}(c, \alpha^*) dc},$$

which is implied by Assumption 3. \square

Proof of Observation 3: $B_1 < 0$ is equivalent to

$$(c^* - c_0) \left(H_{12}(c^*, \alpha^*) - \frac{H_{22}(c^*, \alpha^*) \cdot H_2(c^*, \alpha^*)}{\int_{\underline{c}}^{c^*} H_{22}(c, \alpha^*) dc} \right) < \left(\frac{G(\theta)}{g(\theta)} \right)' H_2(c^*, \alpha^*).$$

Because $c^* < c_0$, the above inequality is equivalent to

$$H_{12}(c^*, \alpha^*) - \frac{H_{22}(c^*, \alpha^*) \cdot H_2(c^*, \alpha^*)}{\int_{\underline{c}}^{c^*} H_{22}(c, \alpha^*) dc} > -\frac{H_2(c^*, \alpha^*)}{c_0 - c^*} \left(\frac{G(\theta)}{g(\theta)} \right)'. \quad (63)$$

Recall that $\alpha^* < \alpha^{FB}(\underline{\theta})$ because $\theta > \underline{\theta}$, and that $H_{22} \leq 0$. Therefore, by Assumption 2,

$$-\frac{H_2(c^*, \alpha^*)}{c_0 - c^*} \left(\frac{G(\theta)}{g(\theta)} \right)' < -\frac{H_2(c^*, \alpha^{FB}(\underline{\theta}))}{c_0 - c^*} \left(\frac{G(\theta)}{g(\theta)} \right)' < -\frac{H_2(c^*, \alpha^{FB}(\underline{\theta}))}{c_0 - \underline{c}} \left(\frac{G(\theta)}{g(\theta)} \right)'.$$

Thus, to show (63), it suffices to show that

$$H_{12}(c^*, \alpha^*) - \frac{H_{22}(c^*, \alpha^*) \cdot H_2(c^*, \alpha^*)}{\int_{\underline{c}}^{c^*} H_{22}(c, \alpha^*) dc} \geq -\frac{H_2(c^*, \alpha^{FB}(\underline{\theta}))}{c_0 - \underline{c}} \left(\frac{G(\theta)}{g(\theta)} \right)'.$$

Notice that

$$-\frac{H_2(c^*, \alpha^{FB}(\underline{\theta}))}{c_0 - \underline{c}} \left(\frac{G(\theta)}{g(\theta)} \right)' \leq -\frac{MN}{c_0 - \underline{c}}.$$

Thus, it suffices to show that

$$H_{12}(c^*, \alpha^*) - \frac{H_{22}(c^*, \alpha^*) \cdot H_2(c^*, \alpha^*)}{\int_{\underline{c}}^{c^*} H_{22}(c, \alpha^*) dc} \geq -\frac{MN}{c_0 - \underline{c}},$$

which is implied by Assumption 4. \square

References

- Arozamena, L. and Cantillon, E. “Investment Incentives in Procurement Auctions.” *The Review of Economic Studies*, Vol. 71 (2004), pp. 1-18.
- Asseyer, A. “Optimal Monitoring in Dynamic Procurement Contracts.” Working Paper, Humboldt-Universität zu Berlin, 2015.
- Athey, S. and Segal, I. “An Efficient Dynamic Mechanism.” *Econometrica*, Vol. 81 (2013), pp. 2463-2485.
- Bag, P. K. “Optimal Auction Design and R&D.” *European Economic Review*, Vol. 41 (1997), pp. 1655-1674.
- Baron, D. P. and Besanko, D. “Regulation and Information in a Continuing Relationship.” *Information Economics and Policy*, Vol. 1 (1984), pp. 267-302.
- Baron, D. P. and Myerson, R. B. “Regulating a Monopolist with Unknown Costs.” *Econometrica*, Vol. 50 (1982), pp. 911-930.
- Bergemann, D. and Välimäki, J. “The Dynamic Pivot Mechanism.” *Econometrica*, Vol. 78 (2010), pp. 771-789.
- Board, S. “Bidding into the Red: A Model of Post-Auction Bankruptcy.” *The Journal of Finance*, Vol. 62 (2007), pp. 2695-2723.
- Burguet, R., Ganuza, J. J., and Hauk, E. “Limited Liability and Mechanism Design in Procurement.” *Games and Economic Behavior*, Vol. 76 (2012), pp. 15-25.
- Courty, P. and Li, H. “Sequential Screening.” *The Review of Economic Studies*, Vol. 67 (2000), pp. 697-717.
- Dasgupta, S. “Competition for Procurement Contracts and Underinvestment.” *International Economic Review*, Vol. 57 (1990), pp. 841-865.
- Eső, P. and Szentes, B. “Optimal Information Disclosure in Auctions and the Handicap Auction.” *The Review of Economic Studies*, Vol. 74 (2007), pp. 705-731.

- Eső, P. and Szentes, B. “Dynamic Contracting: An Irrelevance Result.” *Theoretical Economics*, Vol. 12 (2017), pp. 109-139.
- Gershkov, A. and Perry, M. “Dynamic Contracts with Moral Hazard and Adverse Selection.” *The Review of Economic Studies*, Vol. 79 (2012), pp. 268-306.
- Halac, M., Kartik, N., and Liu, Q. “Optimal Contracts for Experimentation.” *The Review of Economic Studies*, Vol. 83 (2016), pp. 1040-1091.
- Hendricks, K., Porter, R. H., and Boudreau, B. “Information, Returns, and Bidding Behavior in OCS Auctions: 1954-1969.” *The Journal of Industrial Economics*, Vol. 35 (1987), pp. 517-542.
- Krähmer, D. and Strausz, R. “Optimal Procurement Contracts with Pre-Project Planning.” *The Review of Economic Studies*, Vol. 78 (2011), pp. 1015-1041.
- Laffont, J. J. and Martimort, D. *The Theory of Incentives: The Principal-Agent Model*. Princeton NJ: Princeton University Press, 2002.
- Laffont, J. J. and Tirole, J. “Using Cost Observation to Regulate Firms.” *The Journal of Political Economy*, Vol. 94 (1986), pp. 614-641.
- Lewis, G. and Bajari, P. “Procurement Contracting with Time Incentives: Theory and Evidence.” *The Quarterly Journal of Economics*, Vol. 126 (2011), pp. 1173-1211.
- Lewis, G. and Bajari, P. “Moral Hazard, Incentive Contracts, and Risk: Evidence from Procurement.” *Review of Economic Studies*, Vol. 81 (2014), pp. 1201-1228.
- Lichtenberg, F. “The Private R&D Investment Response to Federal Design and Technical Competitions.” *The American Economic Review*, Vol. 78 (1988), pp. 550-559.
- Milgrom, P. and Segal, I. “Envelope Theorems for Arbitrary Choice Sets.” *Econometrica*, Vol. 70 (2002), pp. 583-601.
- Myerson, R. B. “Multistage Games with Communication.” *Econometrica*, Vol. 54 (1986), pp. 323-58.
- Nyiri, L., Osimo, D., Özcivelek, R., Centeno, C., and Cabrera, M. “Public Procurement for the Promotion of R&D and Innovation in ICT.” European Commission, Institute for Prospective Technological Studies, 2007.
- Parlane, S. “Procurement Contracts under Limited Liability.” *The Economic and Social Review*, Vol. 34 (2003), pp. 1-21.
- Pavan, A., Segal, I., and Toikka, J. “Dynamic Mechanism Design: A Myersonian Approach.” *Econometrica*, Vol. 82 (2014), pp. 601-653.

Piccione, M. and Tan, G. "Cost-Reducing Investment, Optimal Procurement and Implementation by Auctions." *International Economic Review*, Vol. 37 (1996), pp. 663-685.

Rob, R. "The Design of Procurement Contracts." *The American Economic Review*, Vol. 76 (1986), pp. 378-389.

Rogerson, W. P. "Profit Regulation of Defense Contractors and Prizes for Innovation." *The Journal of Political Economy*, Vol. 97 (1989), pp. 1284-1305.

Tan, G. "Entry and R&D in Procurement Contracting." *Journal of Economic Theory*, Vol. 58 (1992), pp. 41-60.

Waehrer, K. "A Model of Auction Contracts with Liquidated Damages." *Journal of Economic Theory*, Vol. 67 (1995), pp. 531-555.

Zheng, C. Z. "High Bids and Broke Winners." *Journal of Economic Theory*, Vol. 100 (2001), pp. 129-171.