Contests with multiplicative sabotage effect*

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Abstract

This paper investigates a two-player contest with a multiplicative sabotage effect, showing it can be converted into a standard Tullock contest with a nonlinear, endogenous cost function. We prove the existence and uniqueness of a pure strategy equilibrium. Our findings suggest that sabotage activities can be more pronounced when the productivity difference between players is small, and the more productive player might not necessarily undergo more attacks. Lazear and Rosen's (1981) first-best outcome is attainable for symmetric players if sabotage is sufficiently ineffective or costly. When it is unattainable, optimal pay dispersion induces positive sabotage only if sabotage is ineffective but relatively inexpensive. Optimal pay dispersion decreases with effectiveness and increases with the marginal cost of destructive effort, exhibiting a non-monotonic relationship with productive-effort effectiveness. This non-monotonicity contrasts with the monotonicity of the first best pay dispersion when sabotage is infeasible.

Keywords: Rank-Order Tournament; Sabotage; Interdependent Effects; First-Best; Optimal Pay Dispersion.

JEL classification: C72; M12; M52; J31; J33.

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1 Introduction

Rank-order tournament schemes, in which players are rewarded based on their relative performance, are widely adopted in many situations, such as political competitions, school admissions, and human resource management. Under those schemes, an agent's relative performance affects his successes in terms of being elected, admitted, or promoted (e.g. Baker et al., 1994; Eriksson, 1999; Carpenter et al., 2007; Franceschelli et al., 2010; Tran and Zeckhauser, 2012). One of the concerns for adopting such a scheme is that contestants have incentives to sabotage their competitors, and rampant sabotage could potentially prevent the society or institution from achieving optimal outcomes.

In political competitions, sabotage activities are widespread. For example, negative campaigning and gerrymandering are typically used by all parties. The former refers to the process of deliberately spreading negative information about opponents. The latter is a well-known practice carried out to establish an advantage for a particular party by manipulating district boundaries. As a part of war tactic, the term of sabotage is conceptualized to the activity of an individual or a group not directly linked to the military of the parties participating in the war, such as a secret agent sent to overseas operations, especially when those activities have the outcome of the destruction and demolition of vital facilities, such as equipment, arsenal, public health services or logistic routes. The more of the importance those vital facilities are, the higher of the destructive effects the sabotage activities will lead to.

In the private sector, sabotage is mostly done secretly. Nevertheless, its presence can be inferred from the declining popularity of the "stack ranking" scheme, a type of rank-order tournament scheme that was pioneered by GE in the 1980s and adopted by many large corporations, such as IBM, Ford, Microsoft, and Motorola in the 2000s. Jue (2012) stated that about two-thirds of companies that utilised this scheme ultimately abandoned it. Ovide (2013) claimed that the reason why Microsoft abandoned the scheme was that it created a cutthroat competitive environment in which workers often resorted to politicking and aggressive actions that were detrimental to the performance of their co-workers. The evidence suggests that in the presence of possible sabotage activity, using contest as a compensation scheme is suboptimal. However, in some cases, such as elections, a rank-order tournament scheme may be the only viable option.

The prevalence of rank-order tournaments sparked tremendous interest in the literature. The seminal work of Lazear and Rosen (1981) established that a rank-order tournament with an optimally designed prize structure (i.e. pay dispersion) would achieve the first best outcome when players cannot sabotage each other. Lazear (1989) extended the literature to incorporate sabotage into contests. He suggested that the pay dispersion should be lower in a rank-order tournament with sabotage than in that without it. Konrad (2000) further showed that even with sabotage, the first best can still be achieved when the number of contestants is rather large due to the free riding incentive among players.

The interaction between productive effort and sabotage effort (with the latter hereafter referred to as destructive effort) raises important issues with respect to equilibrium analysis and the optimal design of contests. In this paper, we follow Konrad (2000), Chen (2003), Münster (2007), and Gürtler and Münster (2010), and model destructive effort as an activity that directly reduces the effectiveness of other participants' efforts, thus decreasing their winning probability. Unlike Chen (2003) and Münster (2007), we model productive effort and destructive effort as multiplicative rather than additive in the production function. Therefore, the amount of harm inflicted by a player's destructive actions is directly proportional to their opponent's productive efforts.

Both negative campaigning and gerrymandering are corresponding examples of such cases. Negative campaigning hurts the reputation and credibility of opponents, which could significantly lower the marginal productivity of the opponents' productive effort and the damage is likely to be positively correlated with the level of the opponents' productive effort. Gerrymandering functions similarly. The incumbent party could either dilute the voting power of the opposing party's supporters across many districts, or concentrate the opposing party's voting power in one district to reduce the party's voting power in other districts. The former reduces the opposing party's winning probability in targeted districts while the latter reduces the number of districts that the opposing party can potentially win. Thus, the incumbent party increases the efficiency of its own productive effort and reduces the efficiency of its opposing party's productive effort.

To deal with the multidimensional nature of the strategy space due to the added sabotage activity, we propose a method that reduces it to a single dimensional problem in which players choose their effective composite efforts with endogenously determined cost functions. To apply this method, we need to restrict our attention to an environment with two players who could be asymmetric in two dimensions (i.e. their marginal costs of both productive and destructive efforts). This asymmetric setting is motivated by the observation that the costs of productive and destructive efforts might differ even among players with similar observed characteristics, such as educational attainment or experience. In our model, the effects of productive and destructive efforts are interdependent, which captures the situation in which a player's destructive effort becomes more damaging if his opponent puts in more productive effort.

We establish the existence and uniqueness of pure-strategy equilibrium for any given pay dispersion. The equilibrium analysis illustrates the necessary and sufficient conditions for either positive or zero sabotage at the optimum, and for achieving the first best outcome. The pay dispersion has two thresholds, which are ΔV_l and ΔV_h. When the pay dispersion is smaller than ΔV_l, no one sabotages in the equilibrium. When the dispersion is between ΔV_l and ΔV_h, only the low-destructive-cost player sabotages. When the dispersion is bigger than ΔV_h, both players sabotage. This result is consistent with Konrad (2000) who employed the Tullock rent seeking contest with the standard contest success function. He found that destructive effort reduces the effectiveness of its victim's productive effort, thereby reducing his probability of success.

We also show that both productive and destructive efforts (weakly) increase with the pay dispersion for both players. These results are consistent with those of Amegashie (2012) who studied a two-stage contest in which players chose their destructive efforts in stage 1 and their productive efforts in stage 2. In the analysis, he employed the standard Tullock rent-seeking contest with sabotage, which increased the victim's per unit cost of the productive effort.

Moreover, we find that the low-destructive-cost player always puts in (weakly) more destructive effort regardless of the differences in the effectiveness of the productive efforts between these two players, which diverges from the results of Chen (2003) and Münster (2007) who found that the low-productive-cost player tended to be the victim of more sabotage.[1] The difference is attributable to the difference in model specifications: The productive and destructive efforts are additive in their models but multiplicative in ours. In particular, Chen (2003) employed a rank-order tournament setup of Lazear and Rosen (1981), rather than the Tullock rent-seeking contest. In the setup, a player's destructive effort reduced his target's output. Münster (2007) also employed a rank-order tournament framework, but with at least three contestants. All in all, our results suggest that sabotage behavior can be sensitive to the model environment and exhibits quite diverse patterns across model environments.

We also conduct a comparative statics analysis of the equilibrium outcomes. In the analysis, we assume that the marginal cost of destructive effort is symmetric, while the marginal cost of the

[1] In addition, Corral et al. (2010) found the empirical evidence that more able teams may sabotage more in soccer games based on a rule change in Spanish football as a natural experiment, which is consistent with our findings.

productive effort can be asymmetric. Sabotage decreases with the effectiveness of the productive effort and the marginal costs of destructive effort, increases with the effectiveness of the destructive effort, and increases when players' productive effort costs converge.[2] The productive effort does not depend on the sabotage structural parameters. Thus, it exhibits the usual properties with respect to changes in its effectiveness and cost. In particular, both players' productive efforts increase when their productive effort costs converge, and they are single-peaked in the effectiveness of the productive effort.[3] The rationales behind this are as follows. The cost of effective composite effort increases with the marginal cost of the productive effort. When players' productive effort costs converge, their effective composite efforts increase in the reduced single dimensional problem, which leads to higher productive and destructive efforts if the substitution effect between these two types of efforts is secondary.

Our results differ from those of Chen (2003) who found that a player's destructive effort always decreases with his productive ability and increases with his opponent's productive ability, or equivalently increases with his marginal cost of productive effort and decreases with his opponent's marginal cost of productive effort. In our model the impact of a player's productive effort cost depends on the rank of the two players' marginal costs of productive effort. This divergence is due to the fact that productive and destructive efforts interact differently in these two environments. Our result suggests that contrary to conventional wisdom, sabotage can be more salient when players are more equal in their productive dimension in environments similar to ours.

Finally, we evaluate the optimal level of pay dispersion that maximises the principal's expected payoff. For tractability, we focus on a setting in which players are symmetric. In the setting, the two thresholds of pay dispersion are the same, i.e. $\Delta V_l = \Delta V_h = \Delta V_S$, and the equilibrium involves either both players sabotaging each other or neither sabotaging. The first best outcome is achievable if and only if the optimal pay dispersion of Lazear and Rosen (1981) does not trigger sabotage. This requires that either sabotage is sufficiently inefficient or costly. Whenever the first best outcome is not achievable, profit-maximizing principals have to balance the impacts of destructive effort with productive effort. On the one hand, the optimal pay dispersion can never exceed the optimal pay dispersion of Lazear and Rosen (1981), because a higher pay dispersion would induce less efficient productive effort and more destructive effort. This finding is consistent with Lazear (1989). On the

[2]Throughout the paper, by convergence, we mean that one player's productive effort cost moves closer to the other's while the latter is fixed.

[3]The non-monotonicity of productive effort with respect to its effectiveness and the intuition behind it has been thoroughly discussed by Wang (2010).

other hand, the optimal pay dispersion cannot be lower than the threshold level ΔV_S when the first best outcome is not achievable, as failing to do so reduces productive efforts but not destructive efforts.[4]

Whether it is optimal to set the pay dispersion at ΔV_S depends on the relative effectiveness of the productive vis-a-vis destructive effort in enhancing a player's winning probability, as well as on the marginal cost of the destructive effort. If the destructive effort is more effective than the productive effort, the damage caused by the increase in the destructive effort dominates the benefit caused by the increase in the productive effort. Therefore, any increases in the pay dispersion beyond ΔV_S are detrimental, and the optimal pay dispersion is the threshold value, ΔV_S. Nobody sabotages at the optimum. When the destructive effort is less effective than the productive effort, ΔV_S can still be the optimal pay dispersion if the sabotage cost is high but not high enough for the first best to be implementable. Once the sabotage cost is low enough, it becomes optimal for the principal to set the pay dispersion higher than ΔV_S, which induces sabotage at the optimum. The intuition is as follows. ΔV_S would be rather small if the sabotage cost is very low. While setting pay dispersion at such a low level can eliminate sabotage, it induces too little productive effort. The marginal productivity could be very high when the productive effort is low. Hence, while raising pay dispersion above ΔV_S induces sabotage, the gain from the increase in productive effort outweighs the loss from the increase in destructive effort.[5]

Interestingly, when sabotage does happen in the equilibrium, neither the optimal pay dispersion nor the optimal pay compression (i.e. the deviation of the optimal pay dispersion from its first best level) is monotonic in the effectiveness of the productive effort. In other words, the optimal pay dispersion and pay compression can be either higher or lower when the effectiveness of the productive effort increases. The non-monotonicity relationship when players sabotage each other is in contrast to the monotonic negative relationship between the first best pay dispersion and the effectiveness of the productive effort when sabotage is infeasible. This non-monotonicity relationship is due to the interaction between productive and destructive efforts.

[4] One should note that our model is much more specific than that of Lazear and Rosen (1981), if sabotage was removed from our model. Our point here is not about how general these results are, but rather, about the possible impact of introducing sabotage in the model.

[5] It should be noted that this interesting result showing that tolerating sabotage can sometimes be better than eliminating it can be partly attributable to the fact that the only instrument available for the principal in our setup is pay dispersion. If, in addition to the pay dispersion, the productive and destructive effort costs can be endogenously altered, the use of pay dispersion to induce the optimal outcome becomes less necessary; although obviously in such a case we must also consider the costs incurred in altering both productive and destructive effort costs.

Our paper differs from previous studies, with the exception of Konrad (2000), in that it assumes that productive and destructive efforts are interdependent. The interdependence between these two types of effort complicates the analysis and makes the existence of equilibrium less transparent. To reduce complexity, we transform the two-dimensional problem into a single-dimensional one. The transformed model is equivalent to a Tullock rent-seeking contest. Using the transformed model, we establish the existence of a unique pure-strategy equilibrium while allowing for players to be asymmetric in their efficiencies of both productive and destructive effort. This procedure resembles that of Arbatskaya and Mialon (2010) who studied multi-activity contests. We show that the technique of Arbatskaya and Mialon (2010) is applicable to contests with sabotage in our setting where the shocks to players' performance follow the Weibull (minimum) distribution. In comparison, based on the assumption that a unique symmetric equilibrium exists,[6] Lazear (1989) showed that the optimal pay dispersion should be lower when sabotage is an option than when it is not.[7] Chen (2003, 2005) and Münster (2007) characterised the equilibria when productive and destructive efforts are substitutes. Konrad (2000) established sufficient conditions for the existence of zero sabotage equilibria when the effects of productive and destructive efforts are interdependent.

Our post-transformation model is essentially a Tullock rent-seeking model, though with different forms of effort cost functions on different ranges of support. The existence and uniqueness of the pure strategy Nash equilibrium in a generalised Tullock contest model with linear effort cost function have long been established. Perez-Castrillo and Verdier (1992) characterised pure-strategy equilibria for symmetric Tullock contests with power-form impact functions. Szidarovsky and Okuguchi (1997) allowed for asymmetry across players and establish the existence of a unique non-symmetric pure-strategy Nash equilibrium for rent-seeking games under the assumption that contest success functions are twice differentiable, strictly increasing, and concave. Cornes and Hartley (2005) adopted methods from the literature on aggregative games, and gave a simplified proof of the same result. While they focused on a transformed model with convex effort cost functions, in our transformed model players' effort cost functions are convex in the lower range of the support, though they can be concave in the high range. Our paper shows that in a specific environment and functional form representing the interdependence between productive and destructive effort, the two-dimensional problem can be reduced to a single-dimensional problem, which allows us to analyze sabotage in a tractable manner.

[6]Lazear (1989) acknowledged that there is no guarantee that a unique interior solution exists.

[7]We will refer this result as pay compression in later discussion.

It has been shown in a variety of contexts that incorporating sabotage into contests generates significant impacts on the outcome of the competitions (e.g. Skaperdas and Grofman, 1995; Gürtler, 2015; Amegashie and Runkel, 2007; Amegashie, 2012; Gürtler, 2008; Gürtler and Münster, 2010; Ishida, 2012; Kräkel and Müller, 2012; Gürtler and Münster, 2013; Charness et al., 2014). Several recent empirical and experimental studies (e.g. Harbring and Irlenbusch, 2008, 2011; Balafoutas and Sutter, 2012; Dato and Nieken, 2014) also confirmed the presence of the productive and destructive effects.[8] The contemporary research regarding the effects of sabotage in different types of competitions and contests are summarised in several survey articles, the most recent ones being Amegashie (2015) and Chowdhury and Gürtler (2015). It is generally understood in the literature that a larger reward for the top performers provides players a stronger incentive to not only work harder but also exert more effort to sabotage competitors. In this paper, we further reveal how the existence of sabotage incentive shapes the optimal reward structure. In particular, our analysis illustrates how the equilibrium productive and destructive efforts as well as the optimal rewards respond to the structural parameters of the contest, including the effectiveness and the cost of both the productive and destructive efforts.

The remainder of the paper is organised as follows. Section 2 sets up the model using the framework of two asymmetric players. Section 3 carries out the equilibrium analysis for any given pay dispersion, and in particular establishes the existence and uniqueness of the pure-strategy equilibrium. Some comparative statics on equilibrium characterization are presented in Section 3. Section 4 examines the optimal design of pay dispersion. Section 5 concludes the paper.

2 The Model

Following Lazear and Rosen (1981) and Lazear (1989), our analysis focuses on a setting with one principal, and two players (contestants i and j). Total output, Q, is the sum of each player's output, i.e.

$$Q = q_i + q_j,$$

with

$$q_i = f(x_i) \cdot \phi(s_j) \cdot \varepsilon_i, \tag{1}$$

[8] Gürtler et al. (2013) also showed that the problem of reduction in productive effort due to the risk of being sabotaged by competitors is solvable by concealing intermediate information on the relative performance of players.

where x_i is the amount of productive effort of player i, s_j is the amount of destructive effort that player j inflicts on player i, and $\varepsilon_i \in (0, +\infty)$ is a random component that is assumed to follow a Weibull (minimum) distribution with $E(\varepsilon_i) = 1$ and a c.d.f of $F(\varepsilon_i) = 1 - \exp(-\varepsilon_i)$ following Hirshleifer and Riley (1992) and Fu and Lu (2012). Neither the productive effort, x_i, nor the destructive effort, s_i, is observable to the principal. Thus the principal will reward players based on the output q_i (or its rank) as these efforts are non-contractible.[9]

For tractability, throughout this paper we will consider the following specific functional forms; $f(x_i) = x_i^r$, and $\phi(s_i) = (1 + s_i)^{-\alpha}$, where $r \in (0, 1]$ measures the effectiveness of the productive effort and $\alpha \in (0, 1)$ measures the effectiveness of the destructive effort.[10] It should be noted that these functional forms and the multiplicative relationship between productive and destructive efforts are just a specific example of a more general interdependent relationship between these two types of efforts. We do not claim that our results apply to all more general cases. Nevertheless, working on the specific case makes the analysis tractable and allows us to illustrate some interesting insights which would otherwise be difficult to obtain. In addition, we impose the assumption of $\alpha + r <= 1$, which means that the "aggregate effectiveness" of productive and sabotage effort is considerably bounded. This assumption is to ensure the existence of equilibrium in the subsequent analysis and characterization.

The production function of player i can thus be written as

$$q_i = \frac{x_i^r}{(1 + s_j)^\alpha} \cdot \varepsilon_i. \tag{2}$$

Moreover, the disutility of effort is described by the cost function of player i,

$$C_i(x_i, s_i) = c_i x_i + k_i s_i,$$

where c_i is the marginal cost of the productive effort of player i, and k_i is the marginal cost of

[9] Instead of using the rank-order tournament as the incentive device, other types of contracts, such as piece rate, can also be used as incentive devices. While a piece rate contract may be optimal under some circumstances, a tournament contract may be optimal in other circumstances, particular in the case where players are intrinsically motivated by the desire to be ahead of their peer and the benefits obtained from winning the tournament (e.g. Sheremeta 2016). Although comparing the tournament contract and the piece rate contract is interesting, it is beyond the scope of this paper. In this paper, rather than designing an optimal labor contract, we focus on analysing the properties of the widely used rank-order tournament. Please see also Lazear and Rosen (1981) for discussions on optimal labor contracts.

[10] Lazear (1989) used a general form of production function. The cost of doing so is that he has to assume the existence and uniqueness of symmetric equilibrium.

destructive effort. If the marginal cost of the productive effort is normalised to 1, then the relative cost of the destructive effort is interpreted as $\frac{k_i}{c_i}$. For simplicity and tractability, we adopt a linear cost function.

Equations (1) and (2) reveal that in our environment, a player's destructive effort decreases the marginal productivity of his opponent's productive effort. This differs from Chen (2003) and Münster (2007) who assume that the productive and destructive efforts are additive in the production function. As a result, a player's destructive effort affects neither the marginal productivity of his opponent's productive effort nor his own productive effort. This modeling divergence generates different implications on interactions between productive and destructive efforts. With symmetric players, Propositions 1 and 3 will reveal that a player's productive and destructive efforts both (weakly) decrease with the marginal cost of the destructive effort and the marginal cost of the productive effort. In this sense, one type of effort complements the other type of effort. In Chen (2003) and Münster (2007)'s setting with two players, an increase in a player's marginal cost of destructive effort decreases his destructive effort but increases his productive effort, while a rise in a player's marginal cost of productive effort decreases his productive effort but increases his destructive effort. This means that a player's two dimensional efforts are substitutes.[11]

Under a rank-order contest scheme, the player with the highest output wins first prize, V_w, and the other receives second prize, V_l. Given the production function (2), from Hirshleifer and Riley (1992) and Fu and Lu (2012), i's probability of winning the first prize is

$$p_i(x_i, s_i, x_j, s_j) = \Pr(q_i > q_j) = \frac{x_i^r(1+s_i)^\alpha}{x_i^r(1+s_i)^\alpha + x_j^r(1+s_j)^\alpha}. \tag{3}$$

Equation (3) reveals that a player's destructive effort increases the marginal productivity of his own productive effort in terms of winning the contest.

3 Equilibrium Analysis

In the model, the two players simultaneously choose their productive and destructive efforts to maximize their payoffs. To solve the two-dimensional optimization problem, we first convert it to an equivalent single dimensional one.

[11] Details are provided in Section B.4 of the Online Appendix.

3.1 The Equivalent Single-Dimensional Optimization

Following Lazear and Rosen (1981), we assume players are risk neutral and express player j's problem as

$$\max_{(x_j,s_j)} EU_j = \frac{x_j^r(1+s_j)^\alpha}{x_i^r(1+s_i)^\alpha + x_j^r(1+s_j)^\alpha}\Delta V - (c_j x_j + k_j s_j), \tag{4}$$

where $\Delta V = V_w - V_l$. It is straightforward to see that player j's winning probability is fully characterised by the value of $x_j^r(1+s_j)^\alpha$. For the simplicity of notation, we denote

$$e_j \equiv x_j^r(1+s_j)^\alpha.$$

We call e_j the *effective composite effort* of player j. Obviously, the effective composite effort is zero when productive effort x_j is zero, but is not zero when sabotage effort s_j is zero (as long as the x_j is positive). In the multiplicative framework of our model, a player engaging in only sabotage effort would merely have zero output (by $x_j = 0$) and cannot win the rank-order contest.[12] Moreover, the probability that both players produce zero output will be zero because the Weibull (minimum) distribution on the residual term in our model has no mass points. Therefore, a player cannot win the contest with only sabotage effort, and he needs to exert some positive productive efforts.

The maximization problem can be solved in two steps. First, we solve for player $j's$ optimal choice of (x_j, s_j) that minimizes his cost C_j for any given level of winning probability determined by e_j. This step resembles Arbatskaya and Mialon (2010), which provides the endogenous cost function $C^*(e_j)$ of effective composite effort. In the second step, given the cost function $C^*(e_j)$ and the pay dispersion ΔV, we solve for player $j's$ choice of e_j that maximizes his expected utility.

In the following lemma, we can show the transformation of two-dimensional problem into the single-dimensional problem.

Lemma 1. *The expected utility maximization problem of (4) is transformed into the following problem,*

$$\max_{e_j} EU_j = \frac{e_j}{e_j + e_i}\Delta V - C_j^*(e_j)$$

where

$$C_j^*(e) = \begin{cases} \left(\left(\frac{r}{\alpha}\right)^{\frac{\alpha}{\alpha+r}} + \left(\frac{\alpha}{r}\right)^{\frac{r}{\alpha+r}} \right) c_j^{\frac{r}{\alpha+r}} k_j^{\frac{\alpha}{\alpha+r}} e^{\frac{1}{\alpha+r}} - k_j, & \text{if } e \geq B_j, \\ c_j e^{\frac{1}{r}}, & \text{if } e \in (0, B_j), \end{cases} \tag{5}$$

[12] A player cannot win the competition by only exerting destructive effort is a reasonable assumption. This is because even when one party wins an election by attacking the opponent, the winner still needs to demonstrate the voters what are his beliefs and what he hopes to achieve if being elected.

and $B_j = \left(\frac{rk_j}{\alpha c_j}\right)^r$. *Moreover, the cost functions* $C_j^*(e)$ *in (5) are differentiable everywhere, including the point of* $e = B_j$.

Proof. See Appendix A.1. □

Therefore, given the cost functions (5), player $i's$ and $j's$ problems can be respectively transformed into

$$\max_{e_i} EU_i = \frac{e_i}{e_i + e_j}\Delta V - C_i^*(e_i). \tag{6}$$

and

$$\max_{e_j} EU_j = \frac{e_j}{e_j + e_i}\Delta V - C_j^*(e_j). \tag{7}$$

Because the functional form of $C^*(e)$ depends on the range of e, the first order conditions of the maximization problems (6) and (7) will also depend on the range of e. Without loss of generality, we focus to solve player $j's$ problem.

If $e_j \geq B_j$, player j sabotages i. Substituting function (5) into equation (7) and solve for the first order condition yields

$$\frac{dEU_j}{de_j} = \frac{e_i}{(e_i + e_j)^2}\Delta V - \left(\frac{k_j}{\alpha}\right)^{\frac{\alpha}{\alpha+r}} \left(\frac{c_j}{r}\right)^{\frac{r}{\alpha+r}} e_j^{\frac{1-\alpha-r}{\alpha+r}} = 0. \tag{8}$$

If $e_j < B_j$, player j does not sabotage i, and his problem can be written as

$$\max_{e_j} EU_j = \frac{e_j}{e_j + e_i}\Delta V - c_j e_j^{\frac{1}{r}}.$$

The corresponding first order condition is

$$\frac{dEU_j}{de_j} = \frac{e_i}{(e_i + e_j)^2}\Delta V - \frac{c_j}{r} e_j^{\frac{1-r}{r}} = 0. \tag{9}$$

The solutions to the first order conditions of player i and j are the equilibrium (e_i^*, e_j^*). Given (e_i^*, e_j^*), we can find the equilibrium $(x_i^*, s_i^*, x_j^*, s_j^*)$ by solving the cost minimization problem as explained in the appendix. It will be shown that whether player j sabotages at equilibrium depends on the value of e_j.

3.2 Characterization of Equilibria

We firstly introduce the following notation. Let

$$\lambda_1 = \frac{(c_i^r + c_j^r)^2}{\alpha c_i^r c_j^r}, \qquad \lambda_2 = \frac{(k_i^\alpha c_i^r + k_j^\alpha c_j^r)^2}{\alpha k_i^\alpha k_j^\alpha c_i^r c_j^r},$$

and

$$\Delta V_l = \min(k_i, k_j) \cdot \lambda_1, \qquad \Delta V_h = \max(k_i, k_j) \cdot \lambda_2.$$

We present the comparison between ΔV_l and ΔV_h in the following lemma. The proof is relegated to the appendix.

Lemma 2. *(i)* $\Delta V_l < \Delta V_h$ *when* $k_i \neq k_j$*; (ii)* $\Delta V_l = \Delta V_h$ *when* $k_i = k_j$*.*

Proof. See Appendix A.2. □

Depending on the parameters, c_i, c_j, k_i, k_j, and the pay dispersion ΔV, the equilibrium outcomes could take one of the three forms: (1) if $\Delta V < \Delta V_l$, neither player sabotages; (2) if $\Delta V_l \leq \Delta V < \Delta V_h$, only one sabotages; and (3) if $\Delta V_h \leq \Delta V$, both players sabotage. These three plausible outcomes are established respectively in Propositions 1, 2 and 3.

Case I ($\Delta V < \Delta V_l$): No Sabotage

If the level of pay dispersion is relatively small such that $\Delta V < \Delta V_l$, the difference between winning and losing is less pronounced. A small reward softens the competition between the two players and makes sabotage not appealing for them. The level of productive effort exerted is positively related to the level of pay dispersion. The following proposition summarizes our results.

Proposition 1. *A pure strategy equilibrium with zero sabotage exists if and only if* $\Delta V < \Delta V_l$*. When such an equilibrium exists it is the unique pure strategy equilibrium. At the equilibrium, the player's productive effort and destructive effort are*

$$x_i^* = \frac{r\Delta V}{\alpha c_i \lambda_1}, \quad s_i^* = 0, \quad x_j^* = \frac{r\Delta V}{\alpha c_j \lambda_1}, \quad s_j^* = 0. \tag{10}$$

Proof. See Online Appendix A.3. □

This result shows that players do not always sabotage at the equilibrium, which is consistent with Konrad (2000).

<u>Case II ($\Delta V_l \leq \Delta V < \Delta V_h$): Only One Player Sabotages</u>

Lemma 2 implies that if the marginal cost of destructive effort differs between these two players, i.e. $k_i \neq k_j$, then $\Delta V_l \neq \Delta V_h$. Therefore, it is possible that only one of them sabotages if ΔV is between these two thresholds. Without loss of generality, we assume that $k_i > k_j$. We can show that there always exists an equilibrium such that only one player sabotages regardless of whether $c_i \geq c_j$ or $c_i < c_j$.

Proposition 2. *A pure strategy equilibrium in which only the low-destructive-cost player sabotages exists if and only if $\Delta V \in [\Delta V_l, \Delta V_h)$. When such an equilibrium exists it is the unique pure strategy equilibrium. With $k_i > k_j$, player i's equilibrium productive effort is uniquely determined by*

$$r^{1-\alpha}\alpha^{\alpha}c_i^{r+\alpha-1}c_j^{-r}k_j^{-\alpha}\Delta V \cdot (x_i^*)^{\alpha-1} = \left(1 + r^{-\alpha}\alpha^{\alpha}c_i^{r+\alpha}c_j^{-r}k_j^{-\alpha}(x_i^*)^{\alpha}\right)^2. \tag{11}$$

Player j's productive effort and these two players' destructive efforts are

$$x_j^* = \frac{c_i}{c_j}x_i^*, \quad s_i^* = 0, \quad s_j^* = \frac{\alpha c_i}{r k_j}x_i^* - 1. \tag{12}$$

Proof. See Online Appendix A.4. □

<u>Case III ($\Delta V \geq \Delta V_h$): Both Players Sabotage</u>

Finally, when the level of pay dispersion is sufficiently large such that $\Delta V \geq \Delta V_h$, winning becomes significantly more attractive than losing, and the competition between these two players becomes more intense. As a result, both players would have stronger incentives to sabotage their opponents. In this case, both players sabotage each other at the equilibrium. The following proposition summarizes this result.

Proposition 3. *A pure strategy equilibrium with strictly positive destructive efforts for both players exists if and only if $\Delta V \geq \Delta V_h$. When such an equilibrium exists it is the unique pure strategy equilibrium. At the equilibrium the productive effort and destructive effort are*

$$x_i^* = \frac{r\Delta V}{\alpha c_i \lambda_2}, \quad s_i^* = \frac{\Delta V}{k_i \lambda_2} - 1, \quad x_j^* = \frac{r\Delta V}{\alpha c_j \lambda_2}, \quad s_j^* = \frac{\Delta V}{k_j \lambda_2} - 1. \tag{13}$$

Proof. See Online Appendix A.5. □

The intuition behind these results is as follows. In the transformed single dimensional problem in which players choose their effective composite effort, their equilibrium composite effort must increase with the pay dispersion. Recall in the dimension reduction process, we show that both productive and destructive efforts will increase with the effective composite effort. Therefore, both players' productive and destructive efforts (weakly) increase with the pay dispersion. These results are consistent with those of Amegashie (2012) who studied a sequential game.

Moreover, our results show that the player with a relatively lower cost of destructive effort always conducts (weakly) more sabotage regardless of the difference between these two players' productive efforts. This result diverges from Chen (2003) and Münster (2007) who found that the player with a higher production ability is more vulnerable to sabotage. Chen (2003), in particular, found that a player's destructive effort always decreases with his productive ability and increases with his opponent's productive ability, or equivalently in the framework of our model, increases with his marginal cost of productive effort and decreases with his opponent's marginal cost of productive effort.[13] These divergences are attributable to the difference in the model specifications. While they assume that productive and destructive efforts are additive, we assume they are multiplicative in the form of production function. In settings where players' marginal costs of destructive efforts are symmetric, if these two types of efforts are additive, the higher productive-cost-player tends to rely more on destructive effort and less on productive effort. However, if these two types of efforts are multiplicative, they tend to (at least weakly) move in the same direction. The contrast in the predicted sabotage behaviour based on this theoretical analysis suggests that there can be diverse observable behavioural patterns in practice.

3.3 The Comparative Statics

In this part, we analyze how the equilibrium outcomes vary with the structural parameters, such as r (the effectiveness of the productive effort). α (the effectiveness of the destructive effort), and c_i and c_j (the marginal costs of productive effort). For simplicity, we conduct comparative statics analysis for settings in which players have the same marginal cost of destructive effort except

[13]Their papers adopt a setting with at least three players. In Sections B.8.3 and B.8.4 of the Online Appendix, we show that the same insight holds for two-player settings.

in Proposition 7 which examines the single player sabotage case,[14] i.e. $k_i = k_j = k$. Focusing on the case with $k_i = k_j = k$ is not as restrictive as it appears. This is because the marginal cost of the productive effort could still differ between players. Hence, we can still analyze the interaction between heterogeneous players by trading off between their productive and destructive efforts.

Let us denote the common marginal cost of destructive effort and threshold by k and ΔV^c, respectively. ΔV^c can be expressed as:

$$\Delta V^c = \frac{k(c_i^r + c_j^r)^2}{\alpha c_i^r c_j^r} = \frac{k}{\alpha}\left(\frac{c_i^r}{c_j^r} + \frac{c_j^r}{c_i^r} + 2\right). \tag{14}$$

We will fix the pay dispersion, ΔV, throughout the comparative static analysis. Propositions 1 and 3 state that a higher ΔV induces higher productive and destructive efforts, but ΔV does not affect ΔV^c shown in (14).

3.3.1 Changes in the Threshold ΔV^c

Let us first investigate the impact of our model parameters on the threshold value of ΔV^c. For a given pay dispersion, ΔV, it is possible that a no sabotage equilibrium (Case I) might shift to an equilibrium with mutual sabotage (Case III) as ΔV^c decreases, and vice versa.

Proposition 4. *Consider $r \in (0, 1]$, $\alpha \in (0, 1)$, $k > 0$, $c_i > 0$, $c_j > 0$. The following results regarding ΔV^c holds.*

(i) Fix α, k, c_i, c_j. ΔV^c increases with r when $c_i \neq c_j$, and is independent of r when $c_i = c_j$.

(ii) Fix r, k, c_i, c_j. ΔV^c decreases with α.

(iii) Fix α, r, c_i, c_j. ΔV^c increases with k.

(iv) Fix α, r, k. ΔV^c decreases when c_i and c_j converge.

Proof. See Appendix A.6. □

With a fixed ΔV, an increase in r (i.e. the effectiveness of the productive effort) raises the threshold ΔV^c if the marginal cost of the productive effort differs between these two players ($c_i \neq c_j$). As a result, the range of the pay dispersion ΔV that leads to the zero sabotage equilibrium expands, and an equilibrium in which both players sabotage might switch to an equilibrium in

[14] To have explicit solutions for x^* and s^*, we do not need to put any restrictions on c_i and c_j.

which no-one sabotages if $c_i \neq c_j$. The intuition for this result is that an increase in r encourages players to focus more on the productive effort than the destructive effort. If instead, we have $c_i = c_j$, equation (14) shows that the variation in r does not affect the threshold ΔV^c.

The impacts of α, the effectiveness of the destructive effort, are opposite to those of r. When α increases, the threshold ΔV^c decreases, and the range of the pay dispersion ΔV that produces the zero sabotage equilibrium shrinks. Consequently, an equilibrium in which no-one sabotages may switch to an equilibrium in which both players sabotage. Intuitively, an increase in α induces players to focus more on the destructive effort instead of the productive effort.

The impacts of k, the marginal cost of the destructive effort, is similar to that of r. An increase in k enlarges the range of the pay dispersion that leads to zero sabotage equilibrium. Hence, an equilibrium in which both players sabotage might switch to an equilibrium in which no-one sabotages as k increases.

The comparative statics analysis with respect to c_i and c_j is more interesting and less straightforward. If the two players initially have the same marginal cost of productive effort, then any changes that destroy this symmetry would expand the range of the pay dispersion where zero sabotage equilibrium exists. A larger gap in the marginal cost of the productive effort discourages sabotage. Similarly, when the gap in the marginal cost of productive effort is initially very big, a decrease in the gap lowers the threshold ΔV^c and thus shrinks the range of the pay dispersion where zero sabotage equilibrium exists. A narrowing gap provides players stronger motivations to sabotage each other. The intuition for this is as follows. When the difference between c_i and c_j is sufficiently large, the low cost player will find it more beneficial to increase the productive effort than to increase the destructive effort. On the contrary, the high cost player will not be able to increase his payoffs significantly even if he sabotages the low cost player.

3.3.2 Changes in the Equilibrium Productive and Destructive Efforts

We turn our attention to the changes in the equilibrium productive and destructive efforts in this part. With $k_i = k_j = k$, we can focus on the equilibria characterised in Propositions 1 and 3.

In this case, the optimal choices of x_i^* and x_j^* can be written as

$$x_i^* = \frac{r\Delta V}{c_i}\frac{c_i^r c_j^r}{(c_i^r + c_j^r)^2}, \tag{15}$$

$$x_j^* = \frac{r\Delta V}{c_j}\frac{c_i^r c_j^r}{(c_i^r + c_j^r)^2}. \tag{16}$$

The above two equations show that the productive effort increases with r for both players when $c_i = c_j$.

It is worth noting that when $k_i = k_j = k$ and the threshold is given by ΔV^c, Propositions 1 and 3 imply that

$$s^* = \begin{cases} 0 & \text{if } \Delta V < \Delta V^c \\ \frac{\alpha\Delta V}{k}\frac{c_i^r c_j^r}{(c_i^r+c_j^r)^2} - 1 & \text{if } \Delta V > \Delta V^c. \end{cases}$$

Hence, r does not affect their destructive efforts when $c_i = c_j$.

<u>The No Sabotage Equilibrium</u>

If $\Delta V < \Delta V^c$, Proposition 1 states that the zero sabotage equilibrium prevails ($s_i^* = s_j^* = 0$). In the following proposition, we analyze the impact of changes in r, c_i, and c_j on the equilibrium productive and destructive efforts. We define the unique solution of equation $1+x-(x-1)\ln(x) = 0$ as $\hat{c}$, and $\hat{r} = \ln(\hat{c})/\ln(\frac{\max\{c_i,c_j\}}{\min\{c_i,c_j\}})$.

Proposition 5. *Consider $r \in (0,1]$, $\alpha \in (0,1)$, $k > 0$, $c_i > 0$, $c_j > 0$ and $\Delta V < \Delta V^c$. In the unique pure strategy equilibrium with zero sabotage of Proposition 1, the following results hold.*

(i) Equilibrium productive efforts, x_i^* and x_j^*, do not depend on α and k.

(ii) There exists a cutoff $\hat{c} \approx 4.68$ such that when $\frac{\max\{c_i,c_j\}}{\min\{c_i,c_j\}} \leq \hat{c}$, both x_i^* and x_j^* increase with $r \in (0,1]$; and when $\frac{\max\{c_i,c_j\}}{\min\{c_i,c_j\}} \geq \hat{c}$, x_i^* and x_j^* increase with $r \in (0,\hat{r}]$ and decrease with $r \in (\hat{r},1]$, where $\hat{r}$ is smaller than 1 and decreases with $\frac{\max\{c_i,c_j\}}{\min\{c_i,c_j\}}$.

(iii) Both players' productive efforts drop with their own effort cost. The low-productive-cost player's productive effort decreases with the other player's marginal cost of productive effort, while the high-productive-cost player's productive effort increases with the other player's marginal cost of productive effort.

Proof. See Appendix A.7. □

Proposition 5 describes how productive effort changes with its marginal cost and the contest technology in a standard Tullock rent-seeking contest framework with no sabotage. In this setting, the low-productive-cost player is stronger than his opponent. Intuitively, the stronger player's winning probability increases with the gap in the marginal cost of productive efforts. As the gap widens, the stronger player becomes even stronger. To win the contest, he does not need to put in as much effort. However, a larger gap reduces the weaker player's winning probability, which provides him with less incentive to put in more effort.

The impact of changing r has been studied by Wang (2010) from a perspective of designing the optimal r^* that would induce the highest total effort in a Tullock contest without sabotage. Wang (2010) establishes that the total effort is single-peaked at r^*, which optimally balances the asymmetry in the players' productive costs. A larger ratio of $\frac{\max\{c_i,c_j\}}{\min\{c_i,c_j\}}$ implies that one player has a bigger comparative advantage in generating productive effort than the other. A ratio $\frac{\max\{c_i,c_j\}}{\min\{c_i,c_j\}}$ that is higher than $\hat{c}$ implies that the stronger player's comparative advantage in productive effort is substantial. In this case, a reduction in r^* (< 1) renders the productive effort less effective and makes the battle more balanced. Both players' effort is single peaked at r^* because they are simply proportional to the total effort. However, if $\frac{\max\{c_i,c_j\}}{\min\{c_i,c_j\}} < \hat{c}$, that is when the two players' costs of productive efforts are relatively homogeneous, the optimal r^* that strikes the optimal balance is greater than one. Thus, we have both players' productive effort increases in $r \in (0, 1]$.

The Mutual Sabotage Equilibrium

If $\Delta V > \Delta V^c$, both players sabotage in the equilibrium. Since equations (15) and (16) hold for both $\Delta V < \Delta V^c$ and $\Delta V > \Delta V^c$, the comparative statics of x_i^* and x_j^* are the same as in the zero sabotage equilibrium case, and we will focus on the destructive effort s^*. The expression $\Delta V^c = \frac{k(c_i^r+c_j^r)^2}{\alpha c_i^r c_j^r}$ implies that the impacts of all the structural parameters on s^* are reversed compared to their impacts on ΔV^c as having been discussed in Proposition 4.

Proposition 6. *In the equilibrium with mutual destructive efforts, the following results hold.*

(i) The comparative statics of productive efforts, x_i^* and x_j^*, are the same as in Proposition 5.

(ii) The equilibrium destructive effort, s^*, increases with α, and decreases with k.

(iii) s^* decreases with r when $c_i \neq c_j$, and does not depend on r when $c_i = c_j$.

(iv) s^* increases when players' marginal costs of productive efforts converge.

Proof. See Appendix A.8. □

Intuitively, the equilibrium destructive effort increases with its effectiveness and decreases with its marginal cost. Proposition 6 illustrates that the optimal destructive effort, s^*, decreases with the effectiveness of the productive effort r when its marginal cost differs between players. The rationale behind this is that at any given level of x and s, a rise in r enlarges the output gap between the two players if $c_i \neq c_j$, which decreases the effectiveness of, and hence the incentive to sabotage.

The comparative statics of the equilibrium destructive effort s^* with respect to c_i and c_j is more subtle. If players are symmetric in their marginal cost of productive effort ($c_i = c_j$), then any changes that break this symmetry should lower the equilibrium destructive effort. This implies that asymmetry in the marginal cost of productive effort discourages sabotage. Similarly, when players are initially asymmetric in their marginal cost of productive effort, then any changes in c_i or c_j that reduce the asymmetry encourage sabotage. The intuition behind this is that when c_i differs from c_j, the low-productive-cost player can respond by exerting higher productive effort and lowering destructive effort to increase his winning probability. In contrast, the high-productive-cost player would not be able to increase his payoff significantly even if he engages in sabotage. This observation is in line with our earlier insight obtained from the comparative statics analysis of the threshold ΔV^c.

In summary, sabotage decreases with the effectiveness of the productive effort and the marginal costs of destructive effort, and increases with the effectiveness of the destructive effort. Productive efforts do not depend on the sabotage structural parameters in our model, which means that they exhibit the usual properties with respect to changes in the effectiveness and costs of productive efforts. In particular, players' productive efforts are single-peaked in the effectiveness of the productive effort, which has been shown by Wang (2010).

In addition, players' productive and destructive efforts both increase as their productive effort costs converge. This relationship can be understood as follows. Recall that, in the reduced single dimensional problem, a player's endogenous cost of effective composite effort increases with his marginal cost of productive effort. Therefore, when the costs of productive effort converge, the effective composite effort increases as well. This raises both the productive and the destructive efforts if the substitution effect between these two efforts is not overly strong. This result differs from Chen (2003) who found that a player's destructive effort always decreases with his productive ability and increases with his opponent's productive ability, or equivalently increases with his marginal cost of productive effort and decreases with his opponent's marginal cost of productive effort. In our model, the impact of a player's productive effort cost depends on whether it is higher or lower than that of his opponent. This divergence arises because productive and destructive efforts interact differently in the two environments. These two types of effort are additive in Chen (2003), but are multiplicative in our model.

The Single Sabotage Equilibrium

In this part, we consider the case in which $k_i \neq k_j$ instead of $k_i = k_j$. Without loss of generality, we assume $k_i > k_j$ and focus our analysis on the impacts of k_j on the bidding strategy. According to Lemma 2, when $k_i \neq k_j$ the two thresholds of pay dispersion do not coincide. In the range of $\Delta V \in [\Delta V_l, \Delta V_h)$, the equilibrium is characterised in Proposition 2 with the implicit solutions from (11) to (12). The results of these comparative statics are summarised in Proposition 7.

Proposition 7. *With $k_i > k_j$, only player j sabotages at the equilibrium. The following results hold.*

(i) If $r + \alpha \leq 1$, the equilibrium productive effort from the player who does not sabotage, x_i^*, strictly decreases with c_i.

(ii) If $r \leq \alpha$, the equilibrium productive effort from the saboteur, x_j^*, strictly decreases with c_j.

(iii) For $r \in (0, 1)$ and $\alpha \in (0, 1)$, the equilibrium destructive effort from the saboteur, s_j^*, strictly decreases with k_j.

(iv) s_j^* is not always increasing when c_j converges to c_i.

Proof. See Appendix A.9. □

Except for part (*iv*), the results of Proposition 7 are similar to that of Propositions 5 and 6. However, part (*iv*) states that destructive effort does not always increase as the costs of productive efforts converge. This finding is in contrast to Proposition 6 (*iv*), which states that when both players sabotage, both of them raise their destructive efforts as their marginal costs of productive efforts converge. Consider a situation in which $r \leq \alpha$ and $c_j > c_i$. Proposition 7(*ii*) implies that as c_j decreases, x_j will increase. Since player i does not sabotage, an increase in x_i would trigger a proportional increase in s_j (Proposition 2), which can be quite damaging to himself. He thus might find it is to his benefit to lower his productive effort, which then leads to a lower destructive effort from player j. In the scenario of Proposition 6 where both players sabotage, s_j is still proportional to x_i. However, in this scenario x_i is instead a decreasing function of c_j, which equation (13) points out. With the assistance of sabotage, agent i finds that sabotaging more aggressively as a response to a decrease in opponent's cost c_j is in his best interest, which in turn leads to a higher s_j. Worrying about a rise in x_i would lead to a proportional increase in $j's$ destructive effort, s_j, player i might not have any incentive to raise his productive effort, x_i.

4 The Optimal Design of Pay Dispersion

In this section, we study the optimal design of pay dispersion ΔV that maximizes the principal's expected payoff. To be consistent with Lazear and Rosen (1981) and to facilitate comparison with their results, we assume in this section that the two players are symmetric and normalise their marginal costs of productive efforts to 1, i.e. $c_i = c_j = 1$ and $k_i = k_j = k$.

In this case, there exists only one threshold, $\Delta V_l = \Delta V_h = \Delta V_S$. As a result, in the equilibrium, either both players sabotage or none of them sabotages. The critical threshold ΔV_S can then be expressed as

$$\Delta V_S = \frac{4k}{\alpha}. \tag{17}$$

The characterizations of these symmetric equilibria are illustrated in Corollary 1.

Corollary 1. *When players are symmetric, then*

(i) if $\Delta V \leq \Delta V_S$, there is a unique symmetric pure strategy equilibrium, in which

$$\begin{aligned} x^* &= \frac{r}{4c}\Delta V, \qquad (18) \\ s^* &= 0. \end{aligned}$$

(ii) if $\Delta V > \Delta V_S$, there is a unique symmetric pure strategy equilibrium, in which

$$x^* = \frac{r}{4c}\Delta V, \tag{19}$$

$$s^* = \frac{\alpha}{4k}\Delta V - 1. \tag{20}$$

Proof. See Online Appendix A.10. □

Corollary 1 states that for any given level of pay dispersion ΔV, there always exists a unique symmetric equilibrium. By adjusting the level of pay dispersion, the principal can induce their players to exert an optimal level of destructive effort, which might or might not be zero.

Since the principal can always extract the entire surplus by choosing a losing prize such that a player's expected equilibrium payoff is zero,[15] it is implied that a profit-maximizing principal chooses an optimal pay dispersion that would maximize the social welfare.

Let us first define the first best allocation $(x_i^*, s_i^*, x_j^*, s_j^*)$ as the strategy profile that maximizes the social welfare, i.e.

$$(x_i^*, s_i^*, x_j^*, s_j^*) = \underset{(x_i,s_i,x_j,s_j)}{arg\,max}\, [E(q_i + q_j) - C(x_i, s_i) - C(x_j, s_j)].$$

Because sabotage reduces the total output and is costly to the players, the social welfare will be maximised only at

$$s_i^* = s_j^* = 0.$$

Recall that $E(\varepsilon_i) = 1$. Given the social optimal level of $s_i^* = s_j^* = 0$, the socially optimal choice of (x_i^*, x_j^*) should equalize the marginal social benefit of productive effort to its marginal social cost, i.e.

$$\frac{\partial f(x_i)}{\partial x_i}|_{s_i^*=0, s_j^*=0} = \frac{\partial C(x_i, 0)}{\partial x_i}, \quad i = 1, 2,$$

which implies that

$$x_i^* = x_j^* = r^{\frac{1}{1-r}}. \tag{21}$$

From (18) and given that $c = 1$, to induce players to exert the desired level of productive effort, the principal needs to set the prize at $4r^{\frac{r}{1-r}}$.

Let us define the critical value of ΔV on the first best allocation as

$$\Delta V^{FB} = 4r^{\frac{r}{1-r}}.$$

Before we proceed further, we first present the following Lemma that will be used in the later analysis.

[15] It is important to note that the losing prize established by the principal can be less than zero, and hence players are not subject to the limited liability constraints. For instance, in many contests, contestants need to pay a non-refundable participation fee.

Lemma 3. *(i)* $r^{\frac{r}{1-r}}$ *is a decreasing function of* $r \in (0,1)$*. (ii)* $\lim_{r\to 0^+} r^{\frac{r}{1-r}} = 1$.

Proof. See Appendix A.11. $\square$

Lemma 3 implies that ΔV^{FB} decreases with the effectiveness of the productive effort, r. We can now state the following proposition.

Proposition 8. *The first best allocation can be achieved through a symmetric contest, if and only if* $\Delta V^{FB} \leq \Delta V_S$*, i.e.* $k \geq \alpha r^{\frac{r}{1-r}}$*. In this case, the optimal prize is* $\Delta V^* = \Delta V^{FB}$*.*

Proof. See Appendix A.12. $\square$

Proposition 8 states that the first best allocation is still a possible equilibrium outcome even if sabotage can be used as an instrument when $\Delta V^{FB} \leq \Delta V_S$. Whether it is achievable or not depends on the effectiveness of the productive effort r, the marginal cost of destructive effort k, and the effectiveness of the destructive effort α.

Lemma 3 shows that $r^{\frac{r}{1-r}}$ decreases with r, and hence Proposition 8 implies that given k and α, the chance of achieving the first best allocation increases with r; and given r, the chance of achieving the first best allocation increases with k but decreases with α.

If $\Delta V^{FB} > \Delta V_S$, the equilibrium is given by Corollary 1 (ii) and the principal's maximization problem can be written as

$$\max_{\Delta V} E\pi = 2[x(\Delta V)]^r[1 + s(\Delta V)]^{-\alpha} - 2[x(\Delta V) + ks(\Delta V)]. \tag{22}$$

Substituting (19) and (20) into equation (22) yields

$$E\pi = 2\left(\left(\frac{r\Delta V}{4}\right)^r \left(\frac{4k}{\alpha \Delta V}\right)^\alpha - \left(\frac{r}{4} + \frac{\alpha}{4}\right)\Delta V + k\right),$$

and taking the first order derivative of $E\pi$ with respect to ΔV yields

$$\frac{dE\pi}{d\Delta V} = 2\left(r\left(\frac{r}{4}\right)^r \left(\frac{4k}{\alpha}\right)^\alpha \frac{1}{\Delta V^{(1-r)+\alpha}} - \alpha\left(\frac{r}{4}\right)^r \left(\frac{4k}{\alpha}\right)^\alpha \frac{1}{\Delta V^{(1-r)+\alpha}} - \left(\frac{r}{4} + \frac{\alpha}{4}\right)\right). \tag{23}$$

The first term of equation (23) reflects the positive impact of pay dispersion on the principal's profit from rising productive effort. The second term represents the negative impact of pay dispersion from rising destructive effort. The last term is the impact of the increased costs of productive and destructive efforts when the productive and destructive efforts increase.

If $\alpha \geq r$, then $\frac{dE\pi}{d\Delta V} < 0$. In this case, the principal will set the optimal pay dispersion $\Delta V^* = \Delta V_S$. In equilibrium, players do not sabotage their competitors. If $\alpha < r$, then the sign of $\frac{dE\pi}{d\Delta V}$ depends on whether the positive effect of rising productive effort dominates the negative effects of rising destructive effort and of the increased cost of both productive and destructive efforts. When $\alpha < r$, then $\frac{dE\pi}{d\Delta V} \leq 0, \quad \forall \quad \Delta V \geq \Delta V_S$ if

$$\left(\frac{r}{4}\right)^r \left(\frac{4k}{\alpha}\right)^\alpha (r-\alpha)\,\frac{1}{(\Delta V_S)^{(1-r)+\alpha}} - \left(\frac{r}{4}+\frac{\alpha}{4}\right) \leq 0, \tag{24}$$

Substituting equation (17) into inequality (24) yields

$$k \geq \alpha r^{\frac{r}{1-r}} \left(\frac{r-\alpha}{r+\alpha}\right)^{1/(1-r)}.$$

In this case, the optimal reward is $\Delta V^* = \Delta V_S = \frac{4k}{\alpha}$. At the optimum, there will be no sabotage.

If k is sufficiently small, $\frac{dE\pi}{d\Delta V}$ will be positive when $\Delta V = \Delta V_S$. A principal can increase its profit by increasing its pay dispersion ΔV. Hence, ΔV_S is not optimal anymore. As $\frac{dE\pi}{d\Delta V}$ decreases with ΔV when $\alpha < r$, there exists another optimal $\Delta V^* > \Delta V_S$ such that $\frac{dE\pi}{d\Delta V} = 0$. In this case, the optimal prize is

$$\Delta V^* = 4r^{\frac{r}{(1-r)+\alpha}} \left(\frac{k}{\alpha}\right)^{\frac{\alpha}{1-r+\alpha}} \left(\frac{r-\alpha}{r+\alpha}\right)^{\frac{1}{1-r+\alpha}},$$

which induces positive sabotage in equilibrium. It should be noted that ΔV^* is always smaller than ΔV^{FB}. One can verify that when $\alpha < r$ and $k < \alpha r^{\frac{r}{1-r}}$,

$$\Delta V^* = 4r^{\frac{r}{1-r+\alpha}} \left(\frac{k}{\alpha}\right)^{\frac{\alpha}{1-r+\alpha}} \left(\frac{r-\alpha}{r+\alpha}\right)^{\frac{1}{1-r+\alpha}} = \Delta V^{FB}.$$

The above results on the optimal pay dispersion are summarised in Proposition 9.

Proposition 9. *Suppose $\Delta V^{FB} > \Delta V_S$, i.e., $k < \alpha r^{\frac{r}{1-r}}$.*

(i) If $\alpha \geq r$, the optimal pay dispersion is $\Delta V^* = \Delta V_S$. At the optimum, neither player sabotages.

(ii) If $\alpha < r$ and $k \geq \alpha r^{\frac{r}{1-r}} \left(\frac{r-\alpha}{r+\alpha}\right)^{\frac{1}{1-r}}$, the optimal pay dispersion is

$$\Delta V^* = \Delta V_S.$$

At the optimum, neither player sabotages.

(iii) If $\alpha < r$ and $k < \alpha r^{\frac{r}{1-r}} \left(\frac{r-\alpha}{r+\alpha}\right)^{\frac{1}{1-r}}$, the optimal pay dispersion is

$$\Delta V^* = 4r^{\frac{r}{1-r+\alpha}} \left(\frac{k}{\alpha}\right)^{\frac{\alpha}{1-r+\alpha}} \left(\frac{r-\alpha}{r+\alpha}\right)^{\frac{1}{1-r+\alpha}} \in (\Delta V_S, \Delta V^{FB}).$$

At the optimum, both players sabotage.

Proposition 9 states that if the first best outcome is not achievable, i.e., when marginal cost of destructive effort is low ($k < \alpha r^{\frac{r}{1-r}}$), whether it is optimal to tolerate some sabotage actions depends on the effectiveness of the productive and the destructive effort (r and α), and the marginal cost of destructive effort k.

When $\alpha \geq r$, i.e., the player's destructive effort is quite effective in reducing his competitor's output, the optimal pay dispersion is the threshold value ΔV_S, and nobody sabotages. The rationale behind this is that when sabotage is very destructive, the principal will not tolerate any sabotage actions.

When $\alpha < r$, i.e., sabotage is not as effective, the total output always increases with ΔV as the sum of the first two terms of equation (23) is positive. However, whether it is profitable for the principal to increase ΔV depends on the difference between the gain in output and the cost of the increase in x and s. If the pay dispersion is set at $\Delta V_S = \frac{4k}{\alpha}$, the first two terms of equation (23) can be rewritten as $\left(\frac{r}{4}\right)^r \left(\frac{4k}{\alpha}\right)^{(r-1)}$, which decreases with k, approaches to ∞ as $k \to 0$ and approaches to 0 as $k \to \infty$. Therefore, the marginal gain in output from a small increase in ΔV beyond ΔV_S approaches to ∞ when k approaches 0. However, the marginal cost of rising x and s is fixed at $\frac{r+\alpha}{4}$. Consequently, it is optimal to set a pay dispersion above ΔV_S when k is very small. As k increases, the marginal gain in output decreases. The monotonicity of $\left(\frac{r}{4}\right)^r \left(\frac{4k}{\alpha}\right)^{(r-1)}$ implies that the marginal gain from output increase will eventually be smaller than the marginal cost. Hence, once k is large enough, it is optimal for the principal to set the pay dispersion at ΔV_S.

Based on Propositions 8 and 9, we can investigate how the structural parameters (k, α, r) affect the optimal pay dispersion.

Corollary 2. *The optimal pay dispersion ΔV^* increases with the marginal cost of sabotage k if $k < \alpha r^{\frac{r}{1-r}}$ and does not depend on k otherwise.*

Proof. See Online Appendix A.13. □

Corollary 2 implies that the optimal pay compression ($\Delta V^{FB} - \Delta V^*$) must weakly decrease with k as ΔV^{FB} does not depend on k.

Corollary 3. *The optimal pay dispersion ΔV^* decreases with α when $\alpha > kr^{-\frac{r}{1-r}}$ and otherwise does not depend on α.*

Proof. See Online Appendix A.14. □

Corollary 3 implies that the optimal pay compression ($\Delta V^{FB} - \Delta V^*$) must increase with the effectiveness of the destructive effort, as ΔV^{FB} does not depend on α. Corollaries 2 and 3 show the monotonic relationships between optimal pay dispersion and the effectiveness and marginal cost of the destructive effort, respectively. These results can help us infer the effectiveness and cost of the destructive effort based on the observed pay dispersion. For instance, the cost of sabotage is likely to be high or the effectiveness of sabotage is likely to be low if we observe a principal set a higher pay dispersion.

In the next corollary, we will investigate the impact of the effectiveness of the productive effort on the optimal pay dispersion and the optimal pay compression.

Corollary 4. *The relationship between ΔV^* and r is non-monotonic.*

Proof. See Online Appendix A.15. □

Corollary 4 implies that a higher r does not necessarily imply a higher optimal pay dispersion or lower equilibrium destructive effort. Figure 1 illustrates the relationship between ΔV^* and r when $k = 0.01$ and $\alpha = 0.04$. It shows that when $r_l < r < r_u$, ΔV^* firstly increases and then decreases with r. This nonlinear relationship also implies that the deviation from the first best pay dispersion, ($\Delta V^{FB} - \Delta V^*$) could be non-monotonic in r. Figure 2 demonstrates this point. This non-monotonicity contrasts with the monotonicity of the first best pay dispersion when sabotage is infeasible. The first best pay dispersion necessarily decreases with the effectiveness of the productive effort as revealed in Lemma 3.

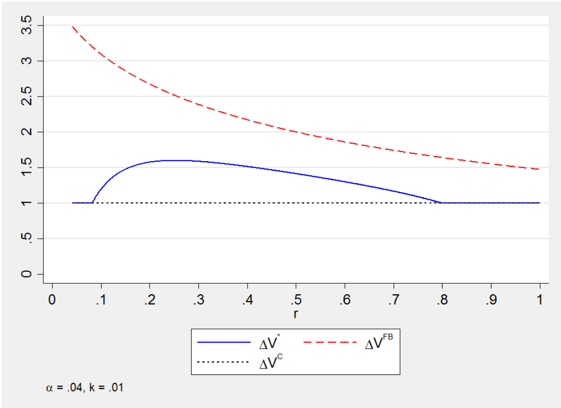

Figure 1: The relationship between optimal pay dispersion and r

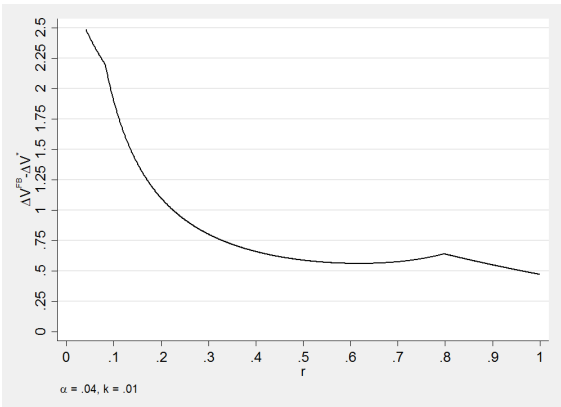

Figure 2: The relationship between $\Delta V^{FB} - \Delta V^{*}$ and r

5 Concluding Remarks

This paper complements the contest literature on sabotage by introducing interdependence between productive and destructive efforts. In particular, we assume the marginal damage caused by a player's destructive effort increases with his opponent's productive effort. Using a model with one principal and two players, we demonstrate the existence and uniqueness of pure strategy equilibrium for the different levels of pay dispersion. We establish two threshold values for the pay dispersion that regulate the equilibrium outcomes. If the pay dispersion is above the upper threshold, both players sabotage; if it is below the lower threshold, neither player sabotages; if it is between these two thresholds, only the more sabotage-efficient player sabotages. These two thresholds coincide if and only if both players share the same marginal cost of destructive effort. In comparison, with the substitutability of productive and destructive efforts, the existing literature often finds that the more productive players tend to undergo more sabotage-related attacks. Our equilibrium analysis reveals that this might not be the case when these two types of efforts are interdependent.

The comparative statics is conducted for the case in which players have the same marginal cost of destructive effort. Our analysis reveals that for a given pay dispersion, the equilibrium destructive effort increases with its effectiveness; and decreases with its marginal cost and the effectiveness of the productive effort. More interestingly, we find that the equilibrium destructive effort decreases with the gap between the two players' marginal costs of productive efforts. In other words, sabotage is more of a concern when both players' efficiencies in their productive efforts are quite similar. This is in contrast to the conventional wisdom that is established when the two-dimensional efforts are substitutes.

Based on the equilibrium relationship between the two types of efforts and the pay dispersion, we analyze the selection of the optimal pay dispersion for profit-maximizing principals. For tractability, we assume that players are symmetric. We first identify the necessary and sufficient conditions for achieving the first best outcome as described in Lazear and Rosen (1981). To satisfy these conditions, sabotage activities must be less destructive or very costly. Under these conditions, the pay compression that Lazear (1989) suggested is no longer required. If these conditions are violated, i.e. the first best is not implementable, then the pay compression result must hold.

When the first best outcome is not achievable, for given levels of effectiveness of the productive efforts, it is optimal for the principal to set an optimal pay dispersion that induces positive sabotage if and only if the destructive effort is less effective and its marginal cost is rather small. Otherwise, the pay dispersion should be set as high as possible without triggering any sabotage. Our analysis also suggests that zero sabotage in an organization might not indicate good management, as this can be a result of very effective sabotage activities. Similarly, observed sabotage activities might not necessarily be the signs of bad management or poor organizational performance, since positive sabotage can be the optimal result of a less effective destructive effort when its marginal cost is low.

A non-monotonic relationship is identified in general between the optimal pay dispersion and the effectiveness of the productive effort. A similar relationship is also discovered between the optimal pay compression and the effectiveness of the productive effort. First, these results illustrate the complexities of determining the direction in which the optimal pay dispersion should be adjusted when the effectiveness of the productive effort in an organization changes. Second, in general, the observed high level of sabotage cannot be viewed as an indication of a more (or less) effective productive effort. Third, this non-monotonicity of pay dispersion contrasts with the monotonicity of the first best pay dispersion when sabotage is infeasible, which is revealed by Lemma 3.

In this paper, we focused on a setting with two contestants. If a contest has more than two contestants, the fact that each individual could engage in either universal or individual-specific sabotage adds much more complexity to the entire analysis. We leave this to future work.

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Online Appendix for "Contests with multiplicative sabotage effect" by H. Liu, J. Lu, Y. E. Riyanto and Z. Wang

In this online appendix, we provide the proofs of Propositions 1 to 8 and Corollaries 1 to 4, as well as the analysis of a two-player environment with substitutable productive and sabotage effort.

A.1 Proof of Lemma 1

We consider the maximization problem of player j. For the simplicity, we temporarily drop the subscript j in the following parts. The transformed cost minimization problem can be expressed as,

$$\begin{aligned} \min_{x,s} C &= cx + ks \\ s.t. : x^r(1+s)^\alpha &= e, \\ x \geq 0, s &\geq 0. \end{aligned} \tag{25}$$

Let us denote $z = (1+s)^\alpha$, where $x = (e/z)^{\frac{1}{r}}$, $s = z^{\frac{1}{\alpha}} - 1$, and $z \geq 1$. We first solve the unrestricted minimization problem by ignoring the constraint $z \geq 1$. The first order condition for the unrestricted minimization problem can be written as

$$\frac{dC}{dz} = -\frac{c}{r} e^{\frac{1}{r}} z^{-\frac{1}{r}-1} + \frac{k}{\alpha} z^{\frac{1}{\alpha}-1} = 0. \tag{26}$$

The second order derivative is thus

$$\frac{d^2C}{dz^2} = \frac{c}{r}\left(\frac{1}{r}+1\right) e^{\frac{1}{r}} z^{-\frac{1}{r}-2} + \frac{1}{\alpha}\left(\frac{1}{\alpha}-1\right) k z^{\frac{1}{\alpha}-2}. \tag{27}$$

Because the second order derivative (27) is strictly positive if $r \in (0,1]$ and $\alpha \in (0,1)$, the objective function (25) is globally convex in z. Hence, the unrestricted minimization has a unique solution.

The solution of the first order condition (26) is

$$z = \left(\frac{\alpha c}{rk}\right)^{\frac{\alpha r}{\alpha+r}} (e)^{\frac{\alpha}{\alpha+r}} > 0.$$

Substituting this unique global minimizer into the objective function (25) yields

$$C_U^*(e) = \left(\left(\frac{r}{\alpha}\right)^{\frac{\alpha}{\alpha+r}} + \left(\frac{\alpha}{r}\right)^{\frac{r}{\alpha+r}}\right) c^{\frac{r}{\alpha+r}} k^{\frac{\alpha}{\alpha+r}} e^{\frac{1}{\alpha+r}} - k, \quad \forall e \geq 0.$$

Denote

$$\Phi = \left(\frac{\alpha c}{rk}\right)^{\frac{\alpha r}{\alpha+r}} (e)^{\frac{\alpha}{\alpha+r}}. \tag{28}$$

Because the objective function (25) is globally convex, the solution for the restricted cost minimization problem can be written as

$$z^* = \begin{cases} \Phi & \text{if } \Phi \geq 1, \\ 1 & \text{if } \Phi < 1. \end{cases}$$

Let us define the cost function when $z = 1$ (i.e. $s = 0$) as

$$C_R^*(e) = ce^{\frac{1}{r}}, \quad \forall e \geq 0.$$

Because $C_U^*(e)$ is the unrestricted cost function, and $C_R^*(e)$ the restricted one, $C_U^*(e) \leq C_R^*(e)$, $\forall e \geq 0$. When $\Phi = 1$, the solutions to both the unrestricted and the restricted minimization problems are z^*. Hence, $C_U^*(e)$ is tangent to $C_R^*(e)$ when $\Phi = 1$. That is

$$\left(\frac{\alpha c}{rk}\right)^{\frac{\alpha r}{\alpha+r}} (e)^{\frac{\alpha}{\alpha+r}} = 1,$$

which leads to the solution of the threshold level e_c,

$$e_c = \left(\frac{rk}{\alpha c}\right)^r > 0.$$

It is easy to figure out that $e < e_c$ if and only if $\Phi < 1$.

Therefore, by denoting $B_j = \left(\frac{rk_j}{\alpha c_j}\right)^r$, we can write the minimum cost function for the original problem (25) of player j as (using the subscript of j)

$$C_j^*(e) = \begin{cases} \left(\left(\frac{r}{\alpha}\right)^{\frac{\alpha}{\alpha+r}} + \left(\frac{\alpha}{r}\right)^{\frac{r}{\alpha+r}} \right) c_j^{\frac{r}{\alpha+r}} k_j^{\frac{\alpha}{\alpha+r}} e^{\frac{1}{\alpha+r}} - k_j & \text{if } e \geq B_j, \\ c_j e^{\frac{1}{r}}, & \text{if } e \in (0, B_j). \end{cases} \tag{29}$$

Given the cost function (29), the maximization problems of two players can be transformed into

$$\max_{e_i} EU_i = \frac{e_i}{e_i + e_j}\Delta V - C_i^*(e_i). \tag{30}$$

and

$$\max_{e_j} EU_j = \frac{e_j}{e_j + e_i}\Delta V - C_j^*(e_j). \tag{31}$$

Next, we will show that equation (29) is continuous at the connecting point of $e = B_j$. (For the two intervals of $e < B_j$ and $e > B_j$, it is trivially true that equation (29) is continuous.) That is, considering $e = B_j$, the limit value from LHS is equal to the limit value from RHS.

From the LHS of $e < B_j$, the value of $C_j^*(e_-)$ is

$$\begin{aligned} C_j^*(e_-) &= c_j \left(\left(\frac{rk_j}{\alpha c_j}\right)^r \right)^{\frac{1}{r}} \\ &= c_j \left(\frac{rk_j}{\alpha c_j}\right) \\ &= \frac{rk_j}{\alpha}. \end{aligned}$$

From the RHS of $e >= B_j$, the value of $C_j^*(e_+)$ is

$$
\begin{aligned}
C_j^*(e_+) &= \left(\left(\frac{r}{\alpha}\right)^{\frac{\alpha}{\alpha+r}} + \left(\frac{\alpha}{r}\right)^{\frac{r}{\alpha+r}}\right) c_j^{\frac{r}{\alpha+r}} k_j^{\frac{\alpha}{\alpha+r}} e^{\frac{1}{\alpha+r}} - k_j \\
&= \left(\left(\frac{r}{\alpha}\right)^{\frac{\alpha}{\alpha+r}} + \left(\frac{\alpha}{r}\right)^{\frac{r}{\alpha+r}}\right) c_j^{\frac{r}{\alpha+r}} k_j^{\frac{\alpha}{\alpha+r}} \left(\left(\frac{rk_j}{\alpha c_j}\right)^{r}\right)^{\frac{1}{\alpha+r}} - k_j \\
&= \left(\frac{r}{\alpha}\right)^{\frac{\alpha}{\alpha+r}} \left(\frac{\alpha+r}{r}\right) c_j^{\frac{r}{\alpha+r}} k_j^{\frac{\alpha}{\alpha+r}} \left(\frac{rk_j}{\alpha c_j}\right)^{\frac{r}{\alpha+r}} - k_j \\
&= \frac{rk_j}{\alpha}.
\end{aligned}
$$

Thus, it is straghtforward to see that $C_j^*(e_-) = C_j^*(e_+)$, and the function is continuous at the point of $e = B_j$.

Moreover, we will show that equation (29) is differentiable everywhere, including $e = B_j$. (For the two intervals of $e < B_j$ and $e > B_j$, it is trivially true that equation (29) is differentiable.) That is, considering $e = B_j$, the derivative approaching from LHS is equal to the derivative approaching from RHS.

From the LHS of $e < B_j$, the derivative of $C_j^*(\cdot)$ is

$$
\begin{aligned}
\left.\frac{dC_j^*}{de}\right|_{e_-} &= \frac{c_j}{r} e^{\frac{1}{r}-1} \\
&= \frac{c_j}{r}\left(\left(\frac{rk_j}{\alpha c_j}\right)^{r}\right)^{\frac{1-r}{r}} \\
&= \frac{c_j}{r}\left(\frac{rk_j}{\alpha c_j}\right)^{1-r} \\
&= r^{-r}\alpha^{r-1} c_j^{r} k_j^{1-r}.
\end{aligned}
$$

From the RHS of $e >= B_j$, the derivative of $C_j^*(\cdot)$ is

$$
\begin{aligned}
\left.\frac{dC_j^*}{de}\right|_{e_+} &= \left(\left(\frac{r}{\alpha}\right)^{\frac{\alpha}{\alpha+r}} + \left(\frac{\alpha}{r}\right)^{\frac{r}{\alpha+r}}\right) c_j^{\frac{r}{\alpha+r}} k_j^{\frac{\alpha}{\alpha+r}} \frac{1}{\alpha+r} e^{\frac{1-\alpha-r}{\alpha+r}} \\
&= \left(\frac{r}{\alpha}\right)^{\frac{\alpha}{\alpha+r}} \left(\frac{\alpha+r}{r}\right) c_j^{\frac{r}{\alpha+r}} k_j^{\frac{\alpha}{\alpha+r}} \frac{1}{\alpha+r} \left(\frac{rk_j}{\alpha c_j}\right)^{\frac{r(1-\alpha-r)}{\alpha+r}} \\
&= r^{\frac{\alpha}{\alpha+r}} r^{-1} r^{\frac{r(1-\alpha-r)}{\alpha+r}} \cdot \alpha^{\frac{-\alpha}{\alpha+r}} \alpha^{\frac{r(\alpha+r-1)}{\alpha+r}} c_j^{r} k_j^{1-r} \\
&= r^{-r}\alpha^{r-1} c_j^{r} k_j^{1-r}.
\end{aligned}
$$

Thus, it is straghtforward to see that $\left.\frac{dC_j^*}{de}\right|_{e_-} = \left.\frac{dC_j^*}{de}\right|_{e_+}$, and the function is therefore differentiable at the point of $e = B_j$ as well.

In addition, with the assumption of $\alpha + r <= 1$, it is easy to see that in equation (29), the transformed cost $C^*(e)$ is a globally convex function, on whether $e < B_j$ or $e \geq B_j$. Thus, the maximization problems of (30) and (31) are concave functions accordingly, which ensures the existence of solutions of e_i and e_j on the two players, and it can be further analyzed with the characterization of equilibria.

Q.E.D.

A.2 Proof of Lemma 2

It is clear that $k_i = k_j$ yields $\Delta V_l = \Delta V_h$. We now turn to the second part. Without loss of generality, we assume $k_i > k_j > 0$. In this case, we have $\Delta V_l = \frac{k_j}{\alpha}\frac{(c_i^r+c_j^r)^2}{c_i^r c_j^r}$ and $\Delta V_h = \frac{k_i}{\alpha}\frac{(k_i^\alpha c_i^r + k_j^\alpha c_j^r)^2}{k_i^\alpha k_j^\alpha c_i^r c_j^r}$, and hence

$$\begin{aligned}
&\Delta V_h - \Delta V_l \\
=&\frac{k_i}{\alpha}\frac{(k_i^\alpha c_i^r + k_j^\alpha c_j^r)^2}{k_i^\alpha k_j^\alpha c_i^r c_j^r} - \frac{k_j}{\alpha}\frac{(c_i^r + c_j^r)^2}{c_i^r c_j^r} \\
=&\frac{1}{\alpha k_i^\alpha k_j^\alpha c_i^r c_j^r}\left[k_i(k_i^\alpha c_i^r + k_j^\alpha c_j^r)^2 - k_i^\alpha k_j^{\alpha+1}(c_i^r + c_j^r)^2\right] \\
=&\frac{1}{\alpha k_i^\alpha k_j^\alpha c_i^r c_j^r}\left[k_i(k_i^{2\alpha} c_i^{2r} + 2k_i^\alpha k_j^\alpha c_i^r c_j^r + k_j^{2\alpha} c_j^{2r}) - k_i^\alpha k_j^{\alpha+1}(c_i^{2r} + 2c_i^r c_j^r + c_j^{2r})\right] \\
=&\frac{1}{\alpha k_i^\alpha k_j^\alpha c_i^r c_j^r}\left[k_i^\alpha k_j^{\alpha+1} c_i^{2r}\left(\left(\frac{k_i}{k_j}\right)^{\alpha+1} - 1\right) + 2k_i^\alpha k_j^\alpha c_i^r c_j^r(k_i - k_j) + k_i^\alpha k_j^{\alpha+1} c_j^{2r}\left(\left(\frac{k_i}{k_j}\right)^{1-\alpha} - 1\right)\right].
\end{aligned}$$

Since $k_i > k_j > 0$, and $\alpha \in (0,1)$, all the three terms in the bracket are positive. Hence, ΔV_h is strictly greater than ΔV_l when $k_i > k_j$. Similarly $\Delta V_l < \Delta V_h$ holds when $k_i < k_j$.

Q.E.D.

A.3 Proof of Proposition 1

For the pure strategy equilibrium in which neither of the players sabotages the other one, the ranges of e_i and e_j are

$$e_i < B_i = \left(\frac{rk_i}{\alpha c_i}\right)^r, \quad e_j < B_j = \left(\frac{rk_j}{\alpha c_j}\right)^r,$$

and the cost functions of player i and j are in a similar form as equation (29). Therefore, the corresponding first order conditions are

$$\begin{aligned}
\frac{dEU_i}{de_i} &= \frac{e_j}{(e_i + e_j)^2}\Delta V - \frac{c_i}{r}e_i^{\frac{1-r}{r}} = 0, \\
\frac{dEU_j}{de_j} &= \frac{e_i}{(e_i + e_j)^2}\Delta V - \frac{c_j}{r}e_j^{\frac{1-r}{r}} = 0.
\end{aligned}$$

The solutions of e_i and e_j are

$$e_i^* = \left(\frac{r\Delta V}{c_i} \frac{c_i^r c_j^r}{(c_i^r + c_j^r)^2} \right)^r, \tag{32}$$

$$e_j^* = \left(\frac{r\Delta V}{c_j} \frac{c_i^r c_j^r}{(c_i^r + c_j^r)^2} \right)^r. \tag{33}$$

In this case, it is required that $e_i^* < B_i$ and $e_j^* < B_j$, which are

$$\left(\frac{r\Delta V}{c_i} \frac{c_i^r c_j^r}{(c_i^r + c_j^r)^2} \right)^r < \left(\frac{rk_i}{\alpha c_i} \right)^r, \quad \left(\frac{r\Delta V}{c_j} \frac{c_i^r c_j^r}{(c_i^r + c_j^r)^2} \right)^r < \left(\frac{rk_j}{\alpha c_j} \right)^r.$$

Therefore, the sufficient and necessary condition on ΔV for the case that neither player sabotages the other is

$$\Delta V < \min(k_i, k_j) \cdot \frac{(c_i^r + c_j^r)^2}{\alpha c_i^r c_j^r}. \tag{34}$$

As neither player sabotages the other in this equilibrium, it yields

$$s_i^* = s_j^* = 0.$$

So it is shown that $e_i = x_i^r(1 + s_i)^\alpha = x_i^r$ and $e_j = x_j^r(1 + s_j)^\alpha = x_j^r$. Since e_i^* and e_j^* are derived in (32) and (33), we have

$$x_i^* = \frac{r\Delta V}{c_i} \frac{c_i^r c_j^r}{(c_i^r + c_j^r)^2}, \quad x_j^* = \frac{r\Delta V}{c_j} \frac{c_i^r c_j^r}{(c_i^r + c_j^r)^2}.$$

The solutions of x_i^* and x_j^* in (10) are thus solved from these two equations.

For the uniqueness of equilibrium, please refer to the proofs of Proposition 2.

Q.E.D.

A.4 Proof of Proposition 2

With reference to the inequality

$$\Delta V < \min(k_i, k_j) \cdot \frac{(c_i^r + c_j^r)^2}{\alpha c_i^r c_j^r},$$

it is shown that under the condition of $k_i > k_j$, the range of ΔV for the equilibrium in which neither player sabotages is

$$\Delta V < \frac{k_j}{\alpha} \frac{(c_i^r + c_j^r)^2}{c_i^r c_j^r},$$

and the range of ΔV for the equilibrium in which both players sabotage each other is

$$\Delta V \geq \frac{k_i}{\alpha}\frac{(k_i^\alpha c_i^r + k_j^\alpha c_j^r)^2}{k_i^\alpha k_j^\alpha c_i^r c_j^r}.$$

It is already demonstrated that $\frac{k_i}{\alpha}\frac{(k_i^\alpha c_i^r+k_j^\alpha c_j^r)^2}{k_i^\alpha k_j^\alpha c_i^r c_j^r} > \frac{k_j}{\alpha}\frac{(c_i^r+c_j^r)^2}{c_i^r c_j^r}$ with the assumption of $k_i > k_j$, which is true for either $c_i \geq c_j$ or $c_i < c_j$.

We next establish a sufficient and necessary condition for an equilibrium in which only the player with lower sabotage marginal cost sabotages the other:

$$\Delta V \in [k_j\lambda_1, k_i\lambda_2)\,. \tag{35}$$

Since $k_i > k_j$, it indicates that player j has a relatively lower cost of sabotage effort. So we firstly verify the existence of an equilibrium in which only player j sabotages player i while player i does not sabotage. Then the corresponding ranges of e_i and e_j are

$$e_i < \left(\frac{rk_i}{\alpha c_i}\right)^r, \tag{36}$$

$$e_j \geq \left(\frac{rk_j}{\alpha c_j}\right)^r. \tag{37}$$

Therefore, we need to consider the following first order conditions

$$\frac{dEU_i}{de_i} = \frac{e_j}{(e_i+e_j)^2}\Delta V - \frac{c_i}{r}e_i^{\frac{1-r}{r}} = 0,$$
$$\frac{dEU_j}{de_j} = \frac{e_i}{(e_i+e_j)^2}\Delta V - \left(\frac{k_j}{\alpha}\right)^{\frac{\alpha}{\alpha+r}}\left(\frac{c_j}{r}\right)^{\frac{r}{\alpha+r}} e_j^{\frac{1-\alpha-r}{\alpha+r}} = 0,$$

and then e_i is expressed as

$$e_i = \left(\frac{rk_j}{\alpha}\right)^{\frac{\alpha r}{\alpha+r}} c_i^{-r} c_j^{\frac{r^2}{\alpha+r}} e_j^{\frac{r}{\alpha+r}}. \tag{38}$$

By substituting e_i into the previous first order conditions, we have

$$\begin{aligned}(\Delta V)^{\frac{1}{2}} = {} & \left(\frac{k_j}{\alpha}\right)^{\frac{\alpha+\alpha r}{2(\alpha+r)}}\left(\frac{1}{r}\right)^{\frac{r-\alpha r}{2(\alpha+r)}} c_i^{-\frac{r}{2}} c_j^{\frac{r+r^2}{2(\alpha+r)}} e_j^{\frac{1-\alpha}{2(\alpha+r)}} \\ & + \left(\frac{k_j}{\alpha}\right)^{\frac{\alpha-\alpha r}{2(\alpha+r)}}\left(\frac{1}{r}\right)^{\frac{r+\alpha r}{2(\alpha+r)}} c_i^{\frac{r}{2}} c_j^{\frac{r-r^2}{2(\alpha+r)}} e_j^{\frac{1+\alpha}{2(\alpha+r)}}.\end{aligned} \tag{39}$$

Equation (39) is an implicit solution of e_j as a function of ΔV. Then it is necessary to check that with this implicit solution, the requirement (37) is satisfied. From equation (39), it can be easily shown that $\frac{d(\Delta V)^{\frac{1}{2}}}{de_j} > 0$, and hence we can derive a lower bound of ΔV if $e_j \geq \left(\frac{rk_j}{\alpha c_j}\right)^r$,

$$\begin{aligned}(\Delta V)^{\frac{1}{2}} \geq& \left(\frac{k_j}{\alpha}\right)^{\frac{\alpha+\alpha r}{2(\alpha+r)}} \left(\frac{1}{r}\right)^{\frac{r-\alpha r}{2(\alpha+r)}} c_i^{-\frac{r}{2}} c_j^{\frac{r+r^2}{2(\alpha+r)}} \left(\frac{rk_j}{\alpha c_j}\right)^{\frac{r-\alpha r}{2(\alpha+r)}} \\ &+ \left(\frac{k_j}{\alpha}\right)^{\frac{\alpha-\alpha r}{2(\alpha+r)}} \left(\frac{1}{r}\right)^{\frac{r+\alpha r}{2(\alpha+r)}} c_i^{\frac{r}{2}} c_j^{\frac{r-r^2}{2(\alpha+r)}} \left(\frac{rk_j}{\alpha c_j}\right)^{\frac{r+\alpha r}{2(\alpha+r)}} \\ =& \left(\frac{k_j}{\alpha}\right)^{\frac{1}{2}} \frac{c_i^r + c_j^r}{c_i^{\frac{r}{2}} c_j^{\frac{r}{2}}},\end{aligned}$$

or equivalently, $\Delta V \geq \frac{k_j}{\alpha}\frac{(c_i^r+c_j^r)^2}{c_i^r c_j^r}$. Therefore, given the range of ΔV in (35), the requirement on e_j in (37) is satisfied.

From (38), we have

$$e_j = \left(\frac{rk_j}{\alpha}\right)^{-\alpha} c_i^{\alpha+r} c_j^{-r} e_i^{\frac{\alpha+r}{r}},$$

and substituting it into the first order conditions yields

$$\begin{aligned}(\Delta V)^{\frac{1}{2}} =& \left(\frac{k_j}{\alpha}\right)^{\frac{\alpha}{2}} \left(\frac{1}{r}\right)^{\frac{1-\alpha}{2}} c_i^{\frac{1-\alpha-r}{2}} c_j^{\frac{r}{2}} e_i^{\frac{1-\alpha}{2r}} \\ &+ \left(\frac{k_j}{\alpha}\right)^{-\frac{\alpha}{2}} \left(\frac{1}{r}\right)^{\frac{1+\alpha}{2}} c_i^{\frac{1+\alpha+r}{2}} c_j^{-\frac{r}{2}} e_i^{\frac{1+\alpha}{2r}}.\end{aligned} \tag{40}$$

Equation (40) is an implicit solution of e_i as a function of ΔV. Then it is necessary to check that with this implicit solution, the requirement (36) is satisfied. From equation (40), it can be easily shown that $\frac{d(\Delta V)^{\frac{1}{2}}}{de_i} > 0$, and hence we can derive an upper bound of ΔV if $e_i < \left(\frac{rk_i}{\alpha c_i}\right)^r$,

$$\begin{aligned}(\Delta V)^{\frac{1}{2}} <& \left(\frac{k_j}{\alpha}\right)^{\frac{\alpha}{2}} \left(\frac{1}{r}\right)^{\frac{1-\alpha}{2}} c_i^{\frac{1-\alpha-r}{2}} c_j^{\frac{r}{2}} \left(\frac{rk_i}{\alpha c_i}\right)^{\frac{1-\alpha}{2}} \\ &+ \left(\frac{k_j}{\alpha}\right)^{-\frac{\alpha}{2}} \left(\frac{1}{r}\right)^{\frac{1+\alpha}{2}} c_i^{\frac{1+\alpha+r}{2}} c_j^{-\frac{r}{2}} \left(\frac{rk_i}{\alpha c_i}\right)^{\frac{1+\alpha}{2}} \\ =& \left(\frac{k_i}{\alpha}\frac{(k_i^\alpha c_i^r + k_j^\alpha c_j^r)^2}{k_i^\alpha k_j^\alpha c_i^r c_j^r}\right)^{\frac{1}{2}},\end{aligned}$$

or equivalently, $\Delta V < \frac{k_i}{\alpha}\frac{(k_i^\alpha c_i^r+k_j^\alpha c_j^r)^2}{k_i^\alpha k_j^\alpha c_i^r c_j^r}$. Therefore, given the range of ΔV in (35), the requirement on e_i in (36) is satisfied.

Therefore, under the assumption of $k_i > k_j$ as well as the range of ΔV shown in (35), the requirements on e_i and e_j for the existence of an equilibrium are both satisfied, and the implicit solutions of e_j and e_i are illustrated in equations (39) and (40), respectively, as the effective composite effort in equilibrium.

Since the equations (39) and (40) are both continuous in the corresponding ranges of e_i and e_j

as shown in (36) and (37), by the Intermediate Value Theorem it can be shown that the solutions for the effective composite efforts e_i^* and e_j^* exist. It is thus verified that there exists an equilibrium such that only player j sabotages player i.

In addition, we can verify the uniqueness of the equilibrium. With $k_i > k_j$, if there exists an equilibrium such that neither player sabotages the other, then the necessary condition on the range of ΔV is

$$\Delta V < k_j \lambda_1.$$

This necessary condition is established from inequality (34).

Moreover, if there exists an equilibrium such that both players sabotage each other, then the necessary condition on the range of ΔV is

$$\Delta V \geq k_i \lambda_2.$$

This necessary condition is established from inequality (53).

However, with the range of ΔV in (35), neither of these inequalities can be satisfied. Therefore, the two categories of above-mentioned equilibria cannot exist given condition (35).

To complete the proof, we next establish that there exists no equilibrium in which only the player with higher marginal sabotage conducts positive sabotage effort. Suppose on the contrary that with $k_i > k_j$, there exists an equilibrium in which only player i sabotages player j, which is equivalent to a situation where the player with a relatively larger marginal cost of sabotage exerts sabotage effort. Then the requirements of e_i and e_j are

$$e_i \geq \left(\frac{rk_i}{\alpha c_i}\right)^r, \tag{41}$$

$$e_j < \left(\frac{rk_j}{\alpha c_j}\right)^r, \tag{42}$$

and the corresponding first order conditions with respect to e_i and e_j are

$$\frac{dEU_i}{de_i} = \frac{e_j}{(e_i+e_j)^2} \cdot \Delta V - \left(\frac{k_i}{\alpha}\right)^{\frac{\alpha}{\alpha+r}} \left(\frac{c_i}{r}\right)^{\frac{r}{\alpha+r}} e_i^{\frac{1-\alpha-r}{\alpha+r}} = 0,$$
$$\frac{dEU_j}{de_j} = \frac{e_i}{(e_i+e_j)^2} \cdot \Delta V - \left(\frac{c_j}{r}\right) e_j^{\frac{1-r}{r}} = 0,$$

from which it can be shown that

$$e_j = \left(\frac{rk_i}{\alpha}\right)^{\frac{\alpha r}{\alpha+r}} c_i^{\frac{r^2}{\alpha+r}} c_j^{-r} e_i^{\frac{r}{\alpha+r}}. \tag{43}$$

Substituting (43) into the first order conditions yields

$$
\begin{aligned}
(\Delta V)^{\frac{1}{2}} &= \left(\frac{1}{r}\right)^{\frac{r-\alpha r}{2(\alpha+r)}} \left(\frac{k_i}{\alpha}\right)^{\frac{\alpha+\alpha r}{2(\alpha+r)}} c_i^{\frac{r+r^2}{2(\alpha+r)}} c_j^{-\frac{r}{2}} e_i^{\frac{1-\alpha}{2(\alpha+r)}} \\
&+ \left(\frac{1}{r}\right)^{\frac{r+\alpha r}{2(\alpha+r)}} \left(\frac{k_i}{\alpha}\right)^{\frac{\alpha-\alpha r}{2(\alpha+r)}} c_i^{\frac{r-r^2}{2(\alpha+r)}} c_j^{\frac{r}{2}} e_i^{\frac{1+\alpha}{2(\alpha+r)}},
\end{aligned}
$$

and it can be shown that $\frac{d(\Delta V)^{\frac{1}{2}}}{de_i} > 0$. From the requirement on e_i in (41), it derives a corresponding lower bound,

$$
\begin{aligned}
(\Delta V)^{\frac{1}{2}} &\geq \left(\frac{1}{r}\right)^{\frac{r-\alpha r}{2(\alpha+r)}} \left(\frac{k_i}{\alpha}\right)^{\frac{\alpha+\alpha r}{2(\alpha+r)}} c_i^{\frac{r+r^2}{2(\alpha+r)}} c_j^{-\frac{r}{2}} \left(\frac{rk_i}{\alpha c_i}\right)^{\frac{1-\alpha}{2(\alpha+r)}} \\
&+ \left(\frac{1}{r}\right)^{\frac{r+\alpha r}{2(\alpha+r)}} \left(\frac{k_i}{\alpha}\right)^{\frac{\alpha-\alpha r}{2(\alpha+r)}} c_i^{\frac{r-r^2}{2(\alpha+r)}} c_j^{\frac{r}{2}} \left(\frac{rk_i}{\alpha c_i}\right)^{\frac{1+\alpha}{2(\alpha+r)}} \\
&= \left(\frac{k_i}{\alpha}\right)^{\frac{1}{2}} \frac{c_i^r + c_j^r}{c_i^{\frac{r}{2}} c_j^{\frac{r}{2}}},
\end{aligned}
$$

or equivalently,

$$
\Delta V \geq \frac{k_i}{\alpha} \frac{(c_i^r + c_j^r)^2}{c_i^r c_j^r}. \tag{44}
$$

Likewise, from the requirement on e_j in (42), it yields an upper bound of ΔV,

$$
(\Delta V)^{\frac{1}{2}} < \left(\frac{k_j}{\alpha}\right)^{\frac{1}{2}} \frac{k_i^\alpha c_i^r + k_j^\alpha c_j^r}{k_i^{\frac{\alpha}{2}} k_j^{\frac{\alpha}{2}} c_i^{\frac{r}{2}} c_j^{\frac{r}{2}}},
$$

or equivalently,

$$
\Delta V < \frac{k_j}{\alpha} \frac{(k_i^\alpha c_i^r + k_j^\alpha c_j^r)^2}{k_i^\alpha k_j^\alpha c_i^r c_j^r}. \tag{45}
$$

Comparing the right-hand-side of (44) and (45) shows

$$
\begin{aligned}
&\frac{k_i}{\alpha}\frac{(c_i^r+c_j^r)^2}{c_i^r c_j^r}-\frac{k_j}{\alpha}\frac{(k_i^\alpha c_i^r+k_j^\alpha c_j^r)^2}{k_i^\alpha k_j^\alpha c_i^r c_j^r}\\
=&\frac{1}{\alpha k_i^\alpha k_j^\alpha c_i^r c_j^r}[k_i^{\alpha+1}k_j^\alpha(c_i^r+c_j^r)^2-k_j(k_i^\alpha c_i^r+k_j^\alpha c_j^r)^2]\\
=&\frac{1}{\alpha k_i^\alpha k_j^\alpha c_i^r c_j^r}[k_i^{\alpha+1}k_j^\alpha(c_i^{2r}+2c_i^r c_j^r+c_j^{2r})-k_j(k_i^{2\alpha}c_i^{2r}+2k_i^\alpha k_j^\alpha c_i^r c_j^r+k_j^{2\alpha}c_j^{2r})]\\
=&\frac{k_i^{2\alpha}k_j c_i^{2r}\left(\left(\frac{k_i}{k_j}\right)^{1-\alpha}-1\right)+2k_i^\alpha k_j^\alpha c_i^r c_j^r(k_i-k_j)+k_j^{2\alpha+1}c_j^{2r}\left(\left(\frac{k_i}{k_j}\right)^{\alpha+1}-1\right)}{\alpha k_i^\alpha k_j^\alpha c_i^r c_j^r}.
\end{aligned}
$$

Since $k_i > k_j$ and $\alpha \in (0,1)$, all three terms in the numerator are positive. Alternatively speaking, it is verified that under the condition of $k_i > k_j$, the two inequalities (44) and (45) are contradictory. Note that this result holds for either $c_i \geq c_j$ or $c_i < c_j$.

Therefore, with the condition $k_i > k_j$, the equilibrium is unique, and in the equilibrium only player j sabotages player i, or equivalently only the player with a relatively lower marginal cost of sabotage makes a positive level of sabotage effort. Moreover, the relationship between the two players' marginal cost of productive effort (c_i and c_j) does not play a critical rule.

With the condition $k_i > k_j$, we can determine the implicit solutions of productive and sabotage effort. It is already shown that $s_i^* = 0$ and $s_j^* > 0$ in equilibrium. We can start with the optimization problem of player i. Given pay dispersion ΔV and player j's choice (x_j, s_j), player i's problem can be written as

$$\max_{(x_i,s_i)} EU_i = \frac{x_i^r(1+s_i)^\alpha}{x_i^r(1+s_i)^\alpha + x_j^r(1+s_j)^\alpha}\cdot\Delta V - (c_i x_i + k_i s_i),$$

and with $s_i = 0$, it can be updated as

$$\max_{(x_i)} EU_i = \frac{x_i^r}{x_i^r + x_j^r(1+s_j)^\alpha}\cdot\Delta V - c_i x_i. \tag{46}$$

The first order condition for (46) is thus

$$\frac{dEU_i}{dx_i} = \frac{rx_i^{r-1}(x_i^r + x_j^r(1+s_j)^\alpha) - x_i^r(rx_i^{r-1})}{(x_i^r + x_j^r(1+s_j)^\alpha)^2}\cdot\Delta V - c_i = 0,$$

or alternatively,

$$rx_i^{r-1}x_j^r(1+s_j)^\alpha\Delta V = c_i(x_i^r + x_j^r(1+s_j)^\alpha)^2. \tag{47}$$

For player j, given pay dispersion ΔV and player i's choice (x_i, s_i), player j's problem can be written as (with $s_i = 0$)

$$\max_{(x_j,s_j)} EU_j = \frac{x_j^r(1+s_j)^\alpha}{x_i^r + x_j^r(1+s_j)^\alpha}\cdot\Delta V - (c_j x_j + k_j s_j). \tag{48}$$

The first order conditions for (48) with respect to x_j and s_j are thus

$$\frac{dEU_j}{dx_j} = \frac{rx_j^{r-1}(1+s_j)^\alpha(x_i^r + x_j^r(1+s_j)^\alpha) - x_j^r(1+s_j)^\alpha(rx_j^{r-1}(1+s_j)^\alpha)}{(x_i^r + x_j^r(1+s_j)^\alpha)^2} \cdot \Delta V - c_j = 0,$$
$$\frac{dEU_j}{ds_j} = \frac{\alpha x_j^r(1+s_j)^{\alpha-1}(x_i^r + x_j^r(1+s_j)^\alpha) - x_j^r(1+s_j)^\alpha(\alpha x_j^r(1+s_j)^{\alpha-1})}{(x_i^r + x_j^r(1+s_j)^\alpha)^2} \cdot \Delta V - k_j = 0,$$

or alternatively,

$$rx_i^r x_j^{r-1}(1+s_j)^\alpha \Delta V = c_j(x_i^r + x_j^r(1+s_j)^\alpha)^2, \tag{49}$$
$$\alpha x_i^r x_j^r(1+s_j)^{\alpha-1}\Delta V = k_j(x_i^r + x_j^r(1+s_j)^\alpha)^2. \tag{50}$$

From equations (47), (49) and (50), we have

$$x_j = \frac{c_i x_i}{c_j}, \qquad s_j = \frac{\alpha c_i x_i}{r k_j} - 1.$$

Substituting these equations into (47) give those implicit solutions as in (11) and (12). Although the explicit solutions for productive and sabotage effort are not obtainable, those equations provide a characterization of the equilibrium in which only one player sabotages. Q.E.D.

A.5 Proof of Proposition 3

For the pure strategy equilibrium in which both players sabotage each other, the ranges of e_i and e_j are

$$e_i \geq B_i = \left(\frac{rk_i}{\alpha c_i}\right)^r, \quad e_j \geq B_j = \left(\frac{rk_j}{\alpha c_j}\right)^r,$$

and the cost functions of player i and j are in the similar form as equation (29). Therefore, the corresponding first order conditions are

$$\frac{dEU_i}{de_i} = \frac{e_j}{(e_i+e_j)^2}\Delta V - \left(\frac{k_i}{\alpha}\right)^{\frac{\alpha}{\alpha+r}} \left(\frac{c_i}{r}\right)^{\frac{r}{\alpha+r}} e_i^{\frac{1-\alpha-r}{\alpha+r}} = 0,$$
$$\frac{dEU_j}{de_j} = \frac{e_i}{(e_i+e_j)^2}\Delta V - \left(\frac{k_j}{\alpha}\right)^{\frac{\alpha}{\alpha+r}} \left(\frac{c_j}{r}\right)^{\frac{r}{\alpha+r}} e_j^{\frac{1-\alpha-r}{\alpha+r}} = 0.$$

The solutions are

$$e_i^* = \left(\frac{\alpha}{k_i}\right)^\alpha \left(\frac{r}{c_i}\right)^r \left(\frac{k_i^\alpha k_j^\alpha c_i^r c_j^r}{(k_i^\alpha c_i^r + k_j^\alpha c_j^r)^2}\Delta V\right)^{\alpha+r}, \tag{51}$$
$$e_j^* = \left(\frac{\alpha}{k_j}\right)^\alpha \left(\frac{r}{c_j}\right)^r \left(\frac{k_i^\alpha k_j^\alpha c_i^r c_j^r}{(k_i^\alpha c_i^r + k_j^\alpha c_j^r)^2}\Delta V\right)^{\alpha+r}. \tag{52}$$

In this case, it is required that $e_i^* \geq B_i$ and $e_j^* \geq B_j$, that is,

$$\left(\frac{\alpha}{k_i}\right)^\alpha \left(\frac{r}{c_i}\right)^r \left(\frac{k_i^\alpha k_j^\alpha c_i^r c_j^r}{(k_i^\alpha c_i^r + k_j^\alpha c_j^r)^2}\Delta V\right)^{\alpha+r} \geq \left(\frac{rk_i}{\alpha c_i}\right)^r,$$
$$\left(\frac{\alpha}{k_j}\right)^\alpha \left(\frac{r}{c_j}\right)^r \left(\frac{k_i^\alpha k_j^\alpha c_i^r c_j^r}{(k_i^\alpha c_i^r + k_j^\alpha c_j^r)^2}\Delta V\right)^{\alpha+r} \geq \left(\frac{rk_j}{\alpha c_j}\right)^r.$$

Therefore, the sufficient and necessary condition on ΔV for the equilibrium characterized by both players sabotaging each other is

$$\Delta V \geq \max(k_i, k_j) \cdot \frac{(k_i^\alpha c_i^r + k_j^\alpha c_j^r)^2}{\alpha k_i^\alpha k_j^\alpha c_i^r c_j^r}. \tag{53}$$

Recall that $z_i^* = \Phi_i = \left(\frac{\alpha c_i}{rk_i}\right)^{\frac{\alpha r}{\alpha+r}} e_i^{\frac{\alpha}{\alpha+r}}$ and $z_j^* = \Phi_j = \left(\frac{\alpha c_j}{rk_j}\right)^{\frac{\alpha r}{\alpha+r}} e_j^{\frac{\alpha}{\alpha+r}}$ following the definition of Φ in (28). With the solutions of e_i^* and e_j^* in (51) and (52), we have

$$(1+s_i)^\alpha = \left(\frac{\alpha c_i}{rk_i}\right)^{\frac{\alpha r}{\alpha+r}} \left(\frac{\alpha}{k_i}\right)^{\frac{\alpha^2}{\alpha+r}} \left(\frac{r}{c_i}\right)^{\frac{\alpha r}{\alpha+r}} \left(\frac{k_i^\alpha k_j^\alpha c_i^r c_j^r}{(k_i^\alpha c_i^r + k_j^\alpha c_j^r)^2}\Delta V\right)^\alpha,$$
$$(1+s_j)^\alpha = \left(\frac{\alpha c_j}{rk_j}\right)^{\frac{\alpha r}{\alpha+r}} \left(\frac{\alpha}{k_j}\right)^{\frac{\alpha^2}{\alpha+r}} \left(\frac{r}{c_j}\right)^{\frac{\alpha r}{\alpha+r}} \left(\frac{k_i^\alpha k_j^\alpha c_i^r c_j^r}{(k_i^\alpha c_i^r + k_j^\alpha c_j^r)^2}\Delta V\right)^\alpha,$$

and hence,

$$s_i^* = \frac{\alpha \Delta V}{k_i} \frac{k_i^\alpha k_j^\alpha c_i^r c_j^r}{(k_i^\alpha c_i^r + k_j^\alpha c_j^r)^2} - 1,$$
$$s_j^* = \frac{\alpha \Delta V}{k_j} \frac{k_i^\alpha k_j^\alpha c_i^r c_j^r}{(k_i^\alpha c_i^r + k_j^\alpha c_j^r)^2} - 1.$$

The solutions of s_i^* and s_j^* in (13) are thus pinned down.

Moreover, we have $e_i^* = (x_i^*)^r(1+s_i^*)^\alpha$ and $e_j^* = (x_j^*)^r(1+s_i^*)^\alpha$ by their definitions. Substituting the solutions of s_i^* and s_j^* yields

$$(x_i^*)^r \left(\frac{\alpha \Delta V}{k_i} \frac{k_i^\alpha k_j^\alpha c_i^r c_j^r}{(k_i^\alpha c_i^r + k_j^\alpha c_j^r)^2}\right)^\alpha = \left(\frac{\alpha}{k_i}\right)^\alpha \left(\frac{r}{c_i}\right)^r \left(\frac{k_i^\alpha k_j^\alpha c_i^r c_j^r}{(k_i^\alpha c_i^r + k_j^\alpha c_j^r)^2}\Delta V\right)^{\alpha+r},$$
$$(x_j^*)^r \left(\frac{\alpha \Delta V}{k_j} \frac{k_i^\alpha k_j^\alpha c_i^r c_j^r}{(k_i^\alpha c_i^r + k_j^\alpha c_j^r)^2}\right)^\alpha = \left(\frac{\alpha}{k_j}\right)^\alpha \left(\frac{r}{c_j}\right)^r \left(\frac{k_i^\alpha k_j^\alpha c_i^r c_j^r}{(k_i^\alpha c_i^r + k_j^\alpha c_j^r)^2}\Delta V\right)^{\alpha+r},$$

and hence,

$$x_i^* = \frac{r\Delta V}{c_i} \frac{k_i^\alpha k_j^\alpha c_i^r c_j^r}{(k_i^\alpha c_i^r + k_j^\alpha c_j^r)^2},$$
$$x_j^* = \frac{r\Delta V}{c_j} \frac{k_i^\alpha k_j^\alpha c_i^r c_j^r}{(k_i^\alpha c_i^r + k_j^\alpha c_j^r)^2}.$$

The solutions of x_i^* and x_j^* in (13) are thus pinned down.

For the uniqueness of equilibrium, please refer to the proofs of Proposition 2.

Q.E.D.

A.6 Proof of Proposition 4

Parts (ii) and (iv) are straightforward.

Part (i): When $c_i \neq c_j$, the partial derivative of ΔV^c from (14) with respect to r is

$$\begin{aligned}\frac{\partial \Delta V^c}{\partial r} &= \frac{k}{\alpha} \frac{2(c_i^r + c_j^r)(c_i^r \ln c_i + c_j^r \ln c_j)c_i^r c_j^r - (c_i^r + c_j^r)^2(c_i^r c_j^r \ln c_i + c_i^r c_j^r \ln c_j)}{c_i^{2r} c_j^{2r}} \\ &= \frac{k}{\alpha} \frac{(c_i^r + c_j^r)c_i^r c_j^r (c_i^r - c_j^r)(\ln c_i - \ln c_j)}{c_i^{2r} c_j^{2r}}.\end{aligned}$$

Therefore, $\frac{\partial \Delta V^c}{\partial r} > 0$. On the other hand, if $c_i = c_j$, clearly the variation of the parameter r does not influence ΔV^c by (14).

Part (iii): The partial derivative of ΔV^c from (14) with respect to c_i is

$$\frac{\partial \Delta V^c}{\partial c_i} = \frac{kr}{\alpha c_i} \left(\frac{c_i^r}{c_j^r} - \frac{c_j^r}{c_i^r} \right),$$

and the partial derivative of ΔV^c with respect to c_j is

$$\frac{\partial \Delta V^c}{\partial c_j} = \frac{kr}{\alpha c_j} \left(\frac{c_j^r}{c_i^r} - \frac{c_i^r}{c_j^r} \right).$$

Therefore, if $c_i > c_j$, we have $\frac{\partial \Delta V^c}{\partial c_i} > 0$ and $\frac{\partial \Delta V^c}{\partial c_j} < 0$.

Note ΔV^c is minimized only if $c_i = c_j$, which means that any change in c_i or c_j would increase ΔV^c.

Q.E.D.

A.7 Proof of Proposition 5

Part (i) is straightforward. We now consider part (ii), i.e. the impact of r. According to Nti (2004) and Wang (2010), there exists a cutoff $\hat{c} \approx 4.68$ such that if $\frac{\max\{c_i, c_j\}}{\min\{c_i, c_j\}} \leq \hat{c}$, the total effort $x_i^* + x_j^*$ increases with $r \in (0, 1]$; and if $\frac{\max\{c_i, c_j\}}{\min\{c_i, c_j\}} \geq \hat{c}$, there exists an $\hat{r}$ (< 1), which decreases with

$\frac{\max\{c_i,c_j\}}{\min\{c_i,c_j\}}$, such that the total effort $x_i^* + x_j^*$ increases with r on $(0,\hat{r}]$ and decreases with r on $(\hat{r}, 1]$; Note both x_i^* and x_j^* are simply proportional to their sum. Therefore, both x_i^* and x_j^* satisfy the same property.

We now turn to the impact of c_i and c_j. It is sufficient to check the partial derivatives with one parameter only. For instance, consider the partial derivatives with respect to c_i:

$$\begin{aligned}
\frac{\partial x_i^*}{\partial c_i} &= r\Delta V c_j^r \frac{(r-1)c_i^{r-2}(c_i^r + c_j^r)^2 - 2c_i^{r-1}(c_i^r + c_j^r) r c_i^{r-1}}{(c_i^r + c_j^r)^4} \\
&= r\Delta V c_j^r \frac{-c_i^{2r-2}(r+1) - c_i^{r-2}c_j^r(1-r)}{(c_i^r + c_j^r)^3} < 0, \\
\frac{\partial x_j^*}{\partial c_i} &= r\Delta V c_j^{r-1} \frac{r c_i^{r-1}(c_i^r + c_j^r)^2 - 2c_i^r(c_i^r + c_j^r) r c_i^{r-1}}{(c_i^r + c_j^r)^4} \\
&= r^2 \Delta V c_i^{r-1} c_j^{r-1} \frac{c_j^r - c_i^r}{(c_i^r + c_j^r)^3}.
\end{aligned}$$

Since $\frac{\partial x_i^*}{\partial c_i} < 0$, we can see that when c_i increases, the optimal productive effort of this player x_i^* will decrease. Furthermore, $\frac{\partial x_j^*}{\partial c_i} > 0$ if $c_i < c_j$, and $\frac{\partial x_j^*}{\partial c_i} < 0$ if $c_i > c_j$.

Q.E.D.

A.8 Proof of Proposition 6

The proof of part (i) is similar with the proof in A.7.

For part (ii), the partial derivative of s^* with respect to α is

$$\frac{\partial s^*}{\partial \alpha} = \frac{\Delta V}{k} \frac{c_i^r c_j^r}{(c_i^r + c_j^r)^2} > 0,$$

and hence, the equilibrium sabotage effort s^* increases with α. The partial derivative of s^* with respect to k is

$$\frac{\partial s^*}{\partial k} = -\frac{\alpha \Delta V}{k^2} \frac{c_i^r c_j^r}{(c_i^r + c_j^r)^2} < 0,$$

and hence, the equilibrium sabotage effort s^* decreases with k.

For part (iii), the partial derivative of s^* with respect to r is

$$\begin{aligned}
\frac{\partial s^*}{\partial r} &= \frac{\alpha \Delta V}{k} \frac{(c_i^r c_j^r \ln c_i + c_i^r c_j^r \ln c_j)(c_i^r + c_j^r)^2 - c_i^r c_j^r 2(c_i^r + c_j^r)(c_i^r \ln c_i + c_j^r \ln c_j)}{(c_i^r + c_j^r)^4} \\
&= \frac{\alpha \Delta V}{k} \frac{c_i^r c_j^r (\ln c_i + \ln c_j)(c_i^r + c_j^r) - c_i^r c_j^r (2c_i^r \ln c_i + 2c_j^r \ln c_j)}{(c_i^r + c_j^r)^3} \\
&= \frac{\alpha \Delta V}{k} \frac{c_i^r c_j^r (c_i^r - c_j^r)(\ln c_j - \ln c_i)}{(c_i^r + c_j^r)^3}.
\end{aligned}$$

On the one hand, if $c_i = c_j$, then $\frac{\partial s^*}{\partial r} = 0$ and hence the change of parameter r does not affect the equilibrium sabotage effort s^*. On the other hand, if $c_i \neq c_j$, it can be verified that $\frac{\partial s^*}{\partial r} < 0$, so the equilibrium sabotage effort s^* decreases with r.

For part (iv), consider the partial derivatives of s^* with respect to c_i and c_j:

$$\frac{\partial s^*}{\partial c_i} = \frac{\alpha r \Delta V}{k} \frac{c_i^{r-1} c_j^r (c_j^r - c_i^r)}{(c_i^r + c_j^r)^3},$$
$$\frac{\partial s^*}{\partial c_j} = \frac{\alpha r \Delta V}{k} \frac{c_i^r c_j^{r-1} (c_i^r - c_j^r)}{(c_i^r + c_j^r)^3}.$$

If $c_i > c_j$, then $\frac{\partial s^*}{\partial c_i} < 0$ and $\frac{\partial s^*}{\partial c_j} > 0$, and hence s^* decreases with c_i and increases with c_j. If $c_i = c_j$, then the increasing of c_i leads to $\frac{\partial s^*}{\partial c_i} < 0$ and the increasing of c_j leads to $\frac{\partial s^*}{\partial c_j} < 0$. So with the condition of $c_i = c_j$, any increasing of c_i or c_j would decrease the equilibrium sabotage effort s^*.

Q.E.D.

A.9 Proof of Proposition 7

For part (i), we consider the effect on x_i of changing marginal cost of production c_i in this equilibrium. From equation (11), we have

$$\alpha k_j^{-1} c_j^{-r} c_i^{r+\alpha-1} \left(\frac{\alpha x_i}{r k_j}\right)^{\alpha-1} \Delta V = \left(1 + c_j^{-r} c_i^{r+\alpha} \left(\frac{\alpha x_i}{r k_j}\right)^{\alpha}\right)^2. \tag{54}$$

Denote $\theta = \frac{\alpha x_i}{r k_j}$, $\Sigma_1 = \alpha k_j^{-1} c_j^{-r} c_i^{r+\alpha-1} \theta^{\alpha-1} \Delta V$, and $\Sigma_2 = \left(1 + c_j^{-r} c_i^{r+\alpha} \theta^{\alpha}\right)^2$. Therefore given the parameters, Σ_1 is strictly decreasing with θ because $\alpha \in (0, 1)$ while Σ_2 is strictly increasing with θ. As θ approaches to 0, Σ_1 will approach to $+\infty$ and Σ_2 will approach to 0. So there will be a single crossing point of $\Sigma_1 = \Sigma_2$ when drawing the two curves of $[\Sigma_1, \theta]$ and $[\Sigma_2, \theta]$ based on equation (54), which gives the solution of θ.

Next, with the condition $r + \alpha - 1 \leq 0$, we aim to examine the effect on x_i of changing c_i. With $r + \alpha - 1 \leq 0$, if c_i increases, the curve of $[\Sigma_1, \theta]$ shifts to the left because of the term $c_i^{r+\alpha-1}$ and the curve of $[\Sigma_2, \theta]$ shifts to the left because of the term $c_i^{r+\alpha}$. It means that the intersection of the two curves also shifts to the left, which leads to a decreasing of the solution of θ.[16] Alternatively, an increase in c_i (holding the other parameters as unchanged) leads to a decreasing in x_i.

For part (ii), we consider the effect on x_j of changing marginal cost of production c_j in this equilibrium. Rewrite equation (54) based on equation (12) as

$$\alpha k_j^{-1} c_i^{r} c_j^{-r+\alpha-1} \left(\frac{\alpha x_j}{r k_j}\right)^{\alpha-1} \Delta V = \left(1 + c_i^{r} c_j^{-r+\alpha} \left(\frac{\alpha x_j}{r k_j}\right)^{\alpha}\right)^2. \tag{55}$$

[16]For the equality $r + \alpha - 1 = 0$, the curve of $[\Sigma_1, \theta]$ is not affected when c_i increases, while the curve of $[\Sigma_2, \theta]$ still shifts to the left. Therefore, the result of a decrease in θ still holds.

Denote $\zeta = \frac{\alpha x_j}{r k_j}$, $\Sigma_1 = \alpha k_j^{-1} c_i^r c_j^{-r+\alpha-1} \zeta^{\alpha-1} \Delta V$, and $\Sigma_2 = \left(1 + c_i^r c_j^{\alpha-r} \zeta^{\alpha}\right)^2$. Therefore given the parameters, Σ_1 is strictly decreasing with ζ because $\alpha \in (0,1)$ while Σ_2 is strictly increasing with ζ. Similarly, as ζ approaches to 0, Σ_1 will approach to $+\infty$ and Σ_2 will approach to 0; as a result, there will be a single crossing point of $\Sigma_1 = \Sigma_2$ when drawing the two curves of $[\Sigma_1, \zeta]$ and $[\Sigma_2, \zeta]$ based on equation (55), which gives the solution of ζ and thus x_j.

Next, with the condition $r - \alpha \leq 0$, we aim to examine the effect on x_j of changing c_j. With $r - \alpha \leq 0$, if c_j increases, the curve of $[\Sigma_1, \zeta]$ shifts to the left because of the term $c_j^{-r+\alpha-1}$ and the curve of $[\Sigma_2, \zeta]$ shifts to the left because of the term $c_j^{\alpha-r}$. It means that the intersection of the two curves also shifts to the left, which leads to a decreasing of the solution of ζ.[17] Alternatively, the equilibrium productive effort x_j^* strictly decreases with c_j, holding the other parameters as unchanged.

For part *(iii)*, we consider the effect on s_j of changing marginal cost of sabotage k_j in this equilibrium. Rewrite equation (11) as

$$\alpha k_j^{-1} \left(\frac{c_i}{c_j}\right)^r \left(\frac{\alpha c_i x_i}{r k_j}\right)^{\alpha-1} \Delta V = \left(1 + \left(\frac{c_i}{c_j}\right)^r \left(\frac{\alpha c_i x_i}{r k_j}\right)^{\alpha}\right)^2. \tag{56}$$

Denote $\kappa = \frac{\alpha c_i x_i}{r k_j}$, $\Sigma_1 = \alpha k_j^{-1} \left(\frac{c_i}{c_j}\right)^r \kappa^{\alpha-1} \Delta V$, and $\Sigma_2 = \left(1 + \left(\frac{c_i}{c_j}\right)^r \kappa^{\alpha}\right)^2$. Therefore given the parameters, Σ_1 is strictly decreasing with κ because $\alpha \in (0,1)$ while Σ_2 is strictly increasing with κ. As κ approaches to 0, Σ_1 will approach to $+\infty$ and Σ_2 will approach to 0. So there will be a single crossing point of $\Sigma_1 = \Sigma_2$ when drawing the two curves of $[\Sigma_1,\ \kappa]$ and $[\Sigma_2,\ \kappa]$ based on equation (56), which gives the solution of κ. According to equation (12), $s_j = \kappa - 1$ and thus the solution of s_j is determined.

Next, we aim to examine the effect on s_j of changing k_j. If k_j increases, the curve of $[\Sigma_2,\ \kappa]$ is not affected, while the curve of $[\Sigma_1,\ \kappa]$ shifts to the left directly because of the term k_j^{-1}. It means that the intersection of the two curves also shifts to the left, which leads to a decreasing of the solution of κ and thus s_j. Alternatively, the equilibrium sabotage effort s_j^* strictly decreases with k_j, holding the other parameters as unchanged.

For part *(iv)*, we can prove it by contradiction. Based on equation (11) in Proposition 2, it yields

$$r^{1-\alpha} \alpha^{\alpha} k_j^{-\alpha} \Delta V \left(\frac{c_i}{c_j}\right)^r (c_i x_i)^{\alpha-1} = \left(1 + r^{-\alpha} \alpha^{\alpha} k_j^{-\alpha} \left(\frac{c_i}{c_j}\right)^r (c_i x_i)^{\alpha}\right)^2. \tag{57}$$

[17] For the equality $r - \alpha = 0$, the curve of $[\Sigma_2, \zeta]$ is not affected when c_j increases, while the curve of $[\Sigma_1, \zeta]$ still shifts to the left. Therefore, the result of a decrease in ζ still holds.

Denote these terms:

$$\begin{aligned}
A_1 &= r^{1-\alpha}\alpha^{\alpha}k_j^{-\alpha}\Delta V, \\
A_2 &= r^{-\alpha}\alpha^{\alpha}k_j^{-\alpha}, \\
w &= \left(\frac{c_i}{c_j}\right)^r, \\
y &= c_i x_i.
\end{aligned}$$

Now suppose $c_j < c_i$ so the two players differ in their marginal costs of productive effort and thus $w > 1$. Consider a small positive change in c_j (i.e., $dc_j > 0$), and it leads that w decreases to the value of 1. Since c_i remains unchanged, it remains to see how y changes (or x_i changes) with respect to w as w gets closer to 1, that is, as c_j converges to c_i.

Equation (57) is updated as

$$A_1 w y^{\alpha-1} = (1 + A_2 w y^{\alpha})^2 . \tag{58}$$

Taking partial derivative with respect to w on both sides of (58) gives

$$A_1 y^{\alpha-1} + A_1 w(\alpha-1)y^{\alpha-2}\frac{dy}{dw} = 2(1 + A_2 w y^{\alpha})\left(A_2 y^{\alpha} + A_2 w\alpha y^{\alpha-1}\frac{dy}{dw}\right),$$

and thus

$$\begin{aligned}
\frac{dy}{dw} &= \frac{A_1 y^{\alpha-1} - 2A_2 y^{\alpha}(1 + A_2 w y^{\alpha})}{2A_2 w\alpha y^{\alpha-1}(1 + A_2 w y^{\alpha}) + A_1 w(1-\alpha)y^{\alpha-2}} \\
&= \frac{A_1 y^{\alpha-1} - 2A_2 y^{\alpha} - 2A_2^2 w y^{2\alpha}}{2A_2 w\alpha y^{\alpha-1} + 2A_2^2 w^2\alpha y^{2\alpha-1} + A_1 w(1-\alpha)y^{\alpha-2}},
\end{aligned} \tag{59}$$

Let

$$\begin{aligned}
K_1 &= A_1 y^{\alpha-1} - 2A_2 y^{\alpha} - 2A_2^2 w y^{2\alpha}, \\
K_2 &= 2A_2 w\alpha y^{\alpha-1} + 2A_2^2 w^2\alpha y^{2\alpha-1} + A_1 w(1-\alpha)y^{\alpha-2}.
\end{aligned}$$

So equation (59) can be expressed as $\frac{dy}{dw} = \frac{K_1}{K_2} = 0$.

To determine whether y is maximum or minimum with the first order condition, we need to check the sign of second order derivative at the point of $\frac{dy}{dw} = 0$:

$$\frac{d^2y}{dw^2} = \frac{\left(A_1(\alpha-1)y^{\alpha-2}\frac{dy}{dw} - 2A_2\alpha y^{\alpha-1}\frac{dy}{dw} - 2A_2^2y^{2\alpha} - 4A_2^2w\alpha y^{2\alpha-1}\frac{dy}{dw}\right)[K_2]}{[K_2]^2}$$

$$\frac{-\left(2A_2\alpha y^{\alpha-1} + 2A_2w\alpha(\alpha-1)y^{\alpha-2}\frac{dy}{dw} + 4A_2^2w\alpha y^{2\alpha-1} + 2A_2^2w^2\alpha(2\alpha-1)y^{2\alpha-2}\frac{dy}{dw}\right)}{}$$

$$\frac{\left(+A_1(1-\alpha)y^{\alpha-2} + A_1w(1-\alpha)(\alpha-2)y^{\alpha-3}\frac{dy}{dw}\right)}{}$$

$$= \frac{(-2A_2^2y^{2\alpha})[K_2] - \left(2A_2\alpha y^{\alpha-1} + 4A_2^2w\alpha y^{2\alpha-1} + A_1(1-\alpha)y^{\alpha-2}\right)[K_1]}{[K_2]^2}, \tag{60}$$

where equation (60) is obtained given that $\frac{dy}{dw} = 0$.

Since it is already shown $\frac{d^2y}{dw^2} < 0$ at the point of $\frac{dy}{dw} = 0$ with $\alpha \in (0,1)$ according to (60), the value of y is maximized at that point if it is solvable.

With equation (58), we can rewrite equation (59) as

$$\begin{aligned}\frac{dy}{dw}w &= \frac{(1+A_2wy^\alpha)^2 - 2A_2wy^\alpha(1+A_2wy^\alpha)}{2A_2w\alpha y^{\alpha-1}(1+A_2wy^\alpha) + A_1w(1-\alpha)y^{\alpha-2}} \\ &= \frac{(1+A_2wy^\alpha)(1-A_2wy^\alpha)}{2A_2w\alpha y^{\alpha-1}(1+A_2wy^\alpha) + A_1w(1-\alpha)y^{\alpha-2}}.\end{aligned}$$

When c_j converges to c_i, the value of w is equal to 1. Suppose that y is maximized at $w = 1$ which leads to $\frac{dy}{dw}|_{w=1} = 0$. Therefore, at $w = 1$ we can have

$$A_2y^\alpha = 1. \tag{61}$$

When $w = 1$, equation (58) is written as

$$A_1y^{\alpha-1} = (1 + A_2y^\alpha)^2,$$

which leads to

$$A_1y^{\alpha-1} = 4. \tag{62}$$

Based on equations (61) and (62), the result is that

$$\frac{A_2^{\alpha-1}}{A_1^\alpha} = \frac{1}{4^\alpha},$$

which cannot be true in general given the definitions of A_1 and A_2.

Therefore, with proof by contradiction, the assumption of $\frac{dy}{dw}|_{w=1} = 0$ cannot hold, and thus $y = c_ix_i$ is not maximized when c_j converges to c_i, which shows that x_i is is not always increasing when c_j converges to c_i. With equation (12) in Proposition 2, it shows that s_j^* is not always increasing when c_j converges to c_i.

Q.E.D.

A.10 Proof of Corollary 1

(i) It is already shown that the sufficient and necessary condition of ΔV for the equilibrium in which neither of the players sabotages the other is the inequality (34). For the analysis of the model with symmetric players, it can be changed with $c_i = c_j = c$ and $k_i = k_j = k$ for $i \neq j$, which yields the inequality $\Delta V < \frac{4k}{\alpha}$. Furthermore, with $\Delta V = \frac{4k}{\alpha}$, the players make zero sabotage efforts in the equilibrium as well. Hence, the sufficient and necessary condition is $\Delta V \leq \frac{4k}{\alpha}$.

Given that $\Delta V \leq \frac{4k}{\alpha}$ in the model with $c_i = c_j = c$ and $k_i = k_j = k$, it can be shown from (32) and (33) that in the equilibrium, for $i = 1, 2$,

$$e_i = \left(\frac{r}{4c} \cdot \Delta V\right)^r. \tag{63}$$

As it is already given that $e_i = x_i^r (1 + s_i)^\alpha$ for the determination of the winning probability and that $s_i = 0$ in the equilibrium with zero sabotage efforts, a simple substitution from (63) shows that, for $i = 1, 2$,

$$x_i = \frac{r}{4c} \cdot \Delta V,$$

which is equation (18).

(ii) It is shown that the sufficient and necessary condition of ΔV for the equilibrium in which both players sabotage each other is the inequality (53). For the analysis regarding symmetric players, it can be changed with $c_i = c_j = c$ and $k_i = k_j = k$ for $i \neq j$, yielding the inequality $\Delta V > \frac{4k}{\alpha}$ as the condition for the existence of the unique symmetric pure strategy equilibrium.

Given the condition $\Delta V > \Delta V_S$ with $c_i = c_j = c$ and $k_i = k_j = k$, the symmetric equilibrium is analyzed with the value of e from (51) and (52),

$$e = \left(\frac{\alpha}{k}\right)^\alpha \left(\frac{r}{c}\right)^r \left(\frac{1}{4}\Delta V\right)^{\alpha + r}. \tag{64}$$

With the definition of Φ in (28), substituting it into (64) yields

$$\Phi = \left(\frac{\alpha}{4k}\Delta V\right)^\alpha. \tag{65}$$

In the equilibrium with both players making positive sabotage efforts, it is shown that $\Phi = (1 + s)^\alpha$. Hence, the expression of $s(\Delta V)$ in (19) is easily derived from (65). Furthermore, by definition, $e \equiv x^r (1 + s)^\alpha$, so from the solutions of $s(\Delta V)$ in (19) and e in (64), the expression of $x(\Delta V)$ in (19) is easily derived.

Q.E.D.

A.11 Proof of Lemma 3

Let $\xi = r^{\frac{r}{1-r}}$. We have $\ln \xi = \frac{r}{1-r} \ln r$, and hence

$$(\ln \xi)' = (1-r)^{-2}\Psi,$$

where $\Psi = \ln r + (1-r)$. Ψ increases with r and reaches 0 when $r = 1$. Thus

$$(\ln \xi)' \leq 0,$$

which means that $r^{\frac{r}{1-r}}$ decreases with r.

By l'Hôpital's rule, we have $\lim_{r \to 0^+} \ln \xi = \lim_{r \to 0^+} \frac{r}{1-r} \ln r = \lim_{r \to 0^+} \frac{\ln r}{r^{-1}} = \lim_{r \to 0^+} \frac{r^{-1}}{-r^{-2}} = 0$. Thus $\lim_{r \to 0^+} r^{\frac{r}{1-r}} = 1$.

Q.E.D.

A.12 Proof of Proposition 8

(1) Proof for the "if"part: According to Corollary 1 (i), if $\Delta V^{FB} \leq \Delta V_S$, a rank-order contest with $\Delta V = \Delta V^{FB}$ induces zero sabotage, and productive efforts are

$$x_i = x_j = \frac{r}{4}\Delta V^{FB} = r^{\frac{1}{1-r}},$$

which is the first best allocation.

(2) Proof for the "only if"part: It suffices to show that if $\Delta V = \Delta V^{FB} > \Delta V_S$, then the first best allocation cannot be achieved through a symmetric rank-order contest. Now suppose that $\Delta V^{FB} > \Delta V_S$. According to Corollary 1 (ii), for $\Delta V = \Delta V^{FB} > \Delta V_S$, there is a unique symmetric pure-strategy equilibrium with the induced optimal productive efforts and sabotage efforts shown as (19). Under the condition of $\Delta V^{FB} > \Delta V_S$, it can be shown that the optimal sabotage effort $s^*(\Delta V) > 0$ in this case, which says that the first best allocation cannot be achieved.

Q.E.D.

A.13 Proof of Corollary 2

When $k \geq \alpha r^{\frac{r}{1-r}}$, Proposition 8 states that

$$\Delta V^* = \Delta V^{FB} = 4r^{\frac{r}{1-r}},$$

which is independent of k.

When $k < \alpha r^{\frac{r}{1-r}}$, we need to consider two cases: $\alpha \geq r$ *and* $\alpha < r$.

Case I: $\alpha \geq r$.

In this case, Proposition 9 states that

$$\Delta V^* = \Delta V_S = \frac{4k}{\alpha},$$

which increases with k.

Case II: $\alpha < r$.

In this case, Proposition 9 states that when $k \in [\alpha r^{\frac{r}{1-r}} \left(\frac{r-\alpha}{r+\alpha}\right)^{\frac{1}{1-r}}, \alpha r^{\frac{r}{1-r}})$,

$$\Delta V^* = \Delta V_S = \frac{4k}{\alpha},$$

which increases with k.

When $k < \alpha r^{\frac{r}{1-r}} \left(\frac{r-\alpha}{r+\alpha}\right)^{\frac{1}{1-r}}$,

$$\Delta V^* = 4r^{\frac{r}{1-r+\alpha}} \left(\frac{k}{\alpha}\right)^{\frac{\alpha}{1-r+\alpha}} \left(\frac{r-\alpha}{r+\alpha}\right)^{\frac{1}{1-r+\alpha}},$$

which increases with k.

Since ΔV^* is continuous in k, the result of Corollary 2 holds.

Q.E.D.

A.14 Proof of Corollary 3

When $\alpha \leq kr^{-\frac{r}{1-r}}$, Proposition 8 states that

$$\Delta V^* = \Delta V^{FB} = 4r^{\frac{r}{1-r}},$$

which does not depend on α.

When $\alpha > kr^{-\frac{r}{1-r}}$, i.e. $k < \alpha r^{\frac{r}{1-r}}$, we need to consider two cases: $\alpha \geq r$ or $\alpha < r$.

Case I: $\alpha \geq r$.

In this case, Proposition 9 states that

$$\Delta V^* = \Delta V_S = \frac{4k}{\alpha},$$

which decreases with α.

Case II: $\alpha < r$.

In this case, Proposition 9 states that when $k \in [\alpha r^{\frac{r}{1-r}} \left(\frac{r-\alpha}{r+\alpha}\right)^{1/(1-r)}, \alpha r^{\frac{r}{1-r}})$, we have

$$\Delta V^* = \Delta V_S = \frac{4k}{\alpha},$$

which decreases with α.

When $k < \alpha r^{\frac{r}{1-r}} \left(\frac{r-\alpha}{r+\alpha}\right)^{\frac{1}{1-r}}$, we have

$$\Delta V^* = 4r^{\frac{r}{1-r+\alpha}} \left(\frac{k}{\alpha}\right)^{\frac{\alpha}{1-r+\alpha}} \left(\frac{r-\alpha}{r+\alpha}\right)^{\frac{1}{1-r+\alpha}}.$$

A monotonic transformation leads to that

$$\ln \Delta V^* = \ln 4 + \frac{r}{1-r+\alpha}\ln r + \frac{\alpha}{1-r+\alpha}\ln\left(\frac{k}{\alpha}\right) + \frac{1}{1-r+\alpha}\ln\left(\frac{r-\alpha}{r+\alpha}\right).$$

Taking the first order derivative yields

$$\begin{aligned}\frac{d\ln\Delta V^*}{d\alpha} =& -(r\ln r)\frac{1}{(1-r+\alpha)^2} + \left[\frac{1}{1-r+\alpha} - \frac{\alpha}{(1-r+\alpha)^2}\right]\ln\left(\frac{k}{\alpha}\right)\\ & -\frac{\alpha}{1-r+\alpha}\frac{\alpha}{k}\frac{k}{\alpha^2} + \left[-\frac{1}{(1-r+\alpha)^2}\right]\ln\left(\frac{r-\alpha}{r+\alpha}\right)\\ & + \frac{1}{1-r+\alpha}\frac{r+\alpha}{r-\alpha}\left[\frac{-1}{r+\alpha} - \frac{r-\alpha}{(r+\alpha)^2}\right]\\ =& -\frac{\Psi}{(1-r+\alpha)^2},\end{aligned}$$

where

$$\begin{aligned}\Psi =& (r\ln r + 1 - r + \alpha) - (1-r)\ln\left(\frac{k}{\alpha}\right)\\ & + \ln\left(\frac{r-\alpha}{r+\alpha}\right) + (1-r+\alpha)\left(\frac{1}{r-\alpha} + \frac{1}{r+\alpha}\right)\\ =& \{[r\ln r + (1-r) + \alpha] - \ln(r+\alpha)\} - (1-r)\ln\left(\frac{k}{\alpha}\right)\\ & + \left\{\ln(r-\alpha) + [(1-r)+\alpha]\frac{1}{r-\alpha}\right\} + (1-r+\alpha)\frac{1}{r+\alpha}.\end{aligned} \tag{66}$$

Because $d(r\ln r - r)/dr = \ln r \leq 0$ if $r \leq 1$ and $r\ln r - r = -1$ when $r = 1$, we have

$$r\ln r + 1 - r + \alpha \geq \alpha.$$

Subtracting $\ln(r+\alpha)$ from both sides of the above inequality yields

$$[r\ln r + (1-r) + \alpha] - \ln(r+\alpha) \geq \alpha - \ln(1+\alpha).$$

The right hand side of the inequality is increasing in α and reaches its minimum 0 when $\alpha = 0$. Therefore, the left hand side of the inequality, which is also the first term of expression (66), is non-negative.

The second term of expression (66), i.e. $\left[-(1-r)\ln\left(\frac{k}{\alpha}\right)\right]$, is also positive because in this case $\frac{k}{\alpha} < r^{\frac{r}{1-r}} < 1$, and $1 - r \geq 0$.

To examine the sign of the third term of expression (66), let us define

$$\tilde{r} \equiv r - \alpha \qquad \text{where } \alpha \in [0, r).$$

The third term of expression (66) can then be rewritten as

$$\varphi(\tilde{r}) = \ln \tilde{r} + (1 - \tilde{r})\frac{1}{\tilde{r}}.$$

Since $\varphi'(\tilde{r}) \leq 0$, $\varphi(\tilde{r})$ reaches its minimum $\varphi(r) = \ln r + (1-r)\frac{1}{r}$ at $\tilde{r} = r$. Consequently, $\varphi(r)$ reaches its minimum $\varphi(1) = 0$ at $r = 1$. Therefore,

$$\ln(r - \alpha) + (1 - r + \alpha)\frac{1}{r - \alpha} \geq 0.$$

Clearly, the last term of expression (66), $(1 - r + \alpha)\frac{1}{r+\alpha}$ is positive $\forall r \in (0, 1]$ and $\alpha \in (0, r)$. Therefore, we have $\Psi \geq 0$ and thus

$$\frac{d \ln \Delta V^*}{d\alpha} \leq 0,$$

which means that ΔV^* decreases with α.

Since ΔV^* is continuous in α and the thresholds adopted above for k are differentiable in α, the above results mean that Corollary 3 holds.

Q.E.D.

A.15 Proof of Corollary 4

When $k \geq \alpha r^{\frac{r}{1-r}}$, Proposition 8 states that

$$\Delta V^* = \Delta V^{FB} = 4r^{\frac{r}{1-r}},$$

which decreases with r according to Lemma 3.

When $k < \alpha r^{\frac{r}{1-r}}$, we need to consider two cases: $\alpha \geq r$ or $\alpha < r$.

Case I: $\alpha \geq r$.

Proposition 9 states that

$$\Delta V^* = \Delta V_S = \frac{4k}{\alpha},$$

which does not depend r.

Case II: $\alpha < r$.

Proposition 9 states that when $k \in [\alpha r^{\frac{r}{1-r}} \left(\frac{r-\alpha}{r+\alpha}\right)^{\frac{1}{1-r}}, \alpha r^{\frac{r}{1-r}})$, we have

$$\Delta V^* = \Delta V_S = \frac{4k}{\alpha},$$

which does not depend r.

When $k \leq \alpha r^{\frac{r}{1-r}} \left(\frac{r-\alpha}{r+\alpha}\right)^{\frac{1}{1-r}}$, Proposition 9 states that

$$\Delta V^* = 4r^{\frac{r}{1-r+\alpha}} \left(\frac{k}{\alpha}\right)^{\frac{\alpha}{1-r+\alpha}} \left(\frac{r-\alpha}{r+\alpha}\right)^{\frac{1}{1-r+\alpha}},$$

and ΔV^* is non-monotonic as illustrated by Figure 1.

Figure 1 demonstrates the relationship between ΔV^* and r when $k = 0.01$ and $\alpha = 0.04$. For the selected parameters, the first best is not achievable regardless of the value of r. The two kink points (r_l, r_u) correspond to the two solutions of r for the following equation

$$k = \alpha \cdot r^{\frac{r}{1-r}} \left(\frac{r - \alpha}{r + \alpha} \right)^{\frac{1}{1-r}}.$$

The figure shows that $\Delta V^* = \Delta V_S$ if $r < r_l$ or $r \in (r_u, \hat{r})$ where $\hat{r}$ is the solution of $k = \alpha \cdot \hat{r}^{\frac{\hat{r}}{1-\hat{r}}}$. In these two regions, the equilibrium sabotage effort is 0. When $r_l < r < r_u$, ΔV^* first increases with r and then decreases with r. In this region, $\Delta V^* > \Delta V_S$, and the sabotage effort must be positive.

Q.E.D.

B Two-Player Environment with Substitutable Productive and Sabotage Effort

In this part, we study a variation of our two-player model where productive and destructive effort are additive in the production function, which is in the spirit of the Chen and Münster's environment with two players. We find that the player who is more efficient in making productive effort receives more sabotage. This finding confirms that the contrast between multiplicative and additive plays an important role in the predicted differences in players' sabotage behavior between our model and Chen and Münster's models. We also present comparative statics in a symmetric setting, which shows that a rise in the marginal cost of productive effort lowers productive effort but raises destructive effort; and a rise in the marginal cost of destructive effort lowers destructive effort but raises productive effort. In this sense, the two dimensions of effort are substitutable.

B.1 Two-Player Model with Substitutability

With one principal and two workers (contestants i and j) in the model, we define x_i as the level of productive effort exerted by worker i, and s_j as the level of sabotage effort exerted by worker j targeting worker i. The output function of worker i with substitutability between two types of efforts is

$$\tilde{y_i} = x_i^r - s_j^\alpha + \varepsilon_i,$$

where ε_i is random productive shock following a distribution function of $F(\varepsilon_i) = 1 - exp(-\varepsilon_i)$ with the property of $E(\varepsilon_i) = 1$. The output function for $\tilde{y_j}$ is defined in a similar way.

Given the choices of productive effort and sabotage effort from the two workers, the probability of worker i winning the contest is denoted by $p_i(x_i, s_i, x_j, s_j) = \Pr(\tilde{y_i} > \tilde{y_j})$. Thus, we can consider an equivalent function for worker i in determining the winning probability:

$$y_i = x_i^r + s_i^\alpha + \varepsilon_i. \tag{67}$$

Moreover, the definitions of prizes V_w, V_l and ΔV are the same as in the manuscript, where we have $\Delta V = V_w - V_l$. The disutility of effort is described by the cost function as

$$C_i(x_i, s_i) = c_i x_i + k_i s_i, \tag{68}$$

where c_i and k_i are the marginal cost of worker i exerting productive effort and sabotage effort, respectively. We focus on the case that both workers have the same marginal cost of sabotage effort with the assumption $k_i = k_j = k$.

Without loss of generality, it is assumed that $c_i < c_j$. We want to examine whether the player with higher ability in productive activities (represented by lower marginal cost c_i) will be sabotaged more aggressively in this two-player setting.

B.2 Equilibrium Analysis

Given the pay dispersion ΔV, worker j's choice (x_j, s_j) and the functions in (67) and (68), the expected utility maximization problem of worker i is written as

$$\max_{(x_i, s_i)} EU_i = \frac{x_i^r + s_i^\alpha}{(x_i^r + s_i^\alpha) + (x_j^r + s_j^\alpha)} \Delta V - (c_i x_i + k s_i) \,. \tag{69}$$

Taking the first order conditions of (69) with respect to x_i and s_i yields

$$\frac{dEU_i}{dx_i} = \frac{r x_i^{r-1}(x_i^r + s_i^\alpha + x_j^r + s_j^\alpha) - r x_i^{r-1}(x_i^r + s_i^\alpha)}{(x_i^r + s_i^\alpha + x_j^r + s_j^\alpha)^2} \Delta V - c_i = 0,$$

$$\frac{dEU_i}{ds_i} = \frac{\alpha s_i^{\alpha-1}(x_i^r + s_i^\alpha + x_j^r + s_j^\alpha) - \alpha s_i^{\alpha-1}(x_i^r + s_i^\alpha)}{(x_i^r + s_i^\alpha + x_j^r + s_j^\alpha)^2} \Delta V - k = 0,$$

and they can be transformed to

$$\frac{(x_j^r + s_j^\alpha) r x_i^{r-1} \Delta V}{(x_i^r + s_i^\alpha + x_j^r + s_j^\alpha)^2} = c_i, \tag{70}$$

$$\frac{(x_j^r + s_j^\alpha) \alpha s_i^{\alpha-1} \Delta V}{(x_i^r + s_i^\alpha + x_j^r + s_j^\alpha)^2} = k. \tag{71}$$

Similarly, the first order conditions with respect to x_j and s_j from worker j's maximization problem are

$$\frac{(x_i^r + s_i^\alpha) r x_j^{r-1} \Delta V}{(x_i^r + s_i^\alpha + x_j^r + s_j^\alpha)^2} = c_j, \tag{72}$$

$$\frac{(x_i^r + s_i^\alpha) \alpha s_j^{\alpha-1} \Delta V}{(x_i^r + s_i^\alpha + x_j^r + s_j^\alpha)^2} = k. \tag{73}$$

From (70) and (72), we have

$$\frac{x_j^r + s_j^\alpha}{x_i^r + s_i^\alpha} = \frac{c_i x_i^{1-r}}{c_j x_j^{1-r}}. \tag{74}$$

From (71) and (73) (as well as (74)), we have

$$\frac{x_j^r + s_j^\alpha}{x_i^r + s_i^\alpha} = \frac{s_i^{1-\alpha}}{s_j^{1-\alpha}} = \frac{c_i x_i^{1-r}}{c_j x_j^{1-r}}. \tag{75}$$

Consider equation (75) and we can prove that $s_i < s_j$ by contradiction.

Suppose instead $s_i > s_j$, then by the first equality in equation (75) it yields $x_j^r + s_j^\alpha > x_i^r + s_i^\alpha$. Thus we have $x_j > x_i$ (otherwise the second inequality cannot hold). However, with the condition of $c_i < c_j$, this means that $c_i x_i^{1-r} < c_j x_j^{1-r}$; and according to second equality in (75) it shows $s_i < s_j$. This contradicts against $s_i > s_j$ in the assumption! So the result of $s_i < s_j$ is proved by contradiction.

Given $s_i < s_j$, from the first equality in (75) we have $x_j^r + s_j^\alpha < x_i^r + s_i^\alpha$. Thus, it is easy to see $x_i > x_j$.

B.3 Summary of Results

Therefore, with the assumption of $c_i < c_j$ and $k_i = k_j = k$ in this model with additivity rather than multiplicity, we can have the following results in the equilibrium:

(1) $x_i > x_j$. It means that the worker with lower marginal cost of productive effort will exert higher level of productive effort.

(2) $s_i < s_j$. It means that the worker with lower marginal cost of productive effort will exert lower level of sabotage effort. Alternatively, it indicates that the worker who has higher ability in productive activities (with lower marginal cost) is making more productive effort and actually subject to higher level of sabotage from his opponent.

(3) $x_i^r + s_i^\alpha > x_j^r + s_j^\alpha$. It means that the worker who has higher ability in productive activities will have a higher chance of winning the contest in equilibrium.

B.4 Comparative Statics of Effort in Symmetric Setting

With symmetric players, we assume $c_i = c_j = c$ and $k_i = k_j = k$. The equilibrium conditions of (70), (71), (72) and (73) reduce to

$$\begin{aligned} \frac{r x^{r-1} \Delta V}{4(x^r + s^\alpha)} &= c, \\ \frac{\alpha s^{\alpha-1} \Delta V}{4(x^r + s^\alpha)} &= k. \end{aligned} \tag{76}$$

We can simplify these two equations to determine x:

$$\begin{aligned}\frac{rs^{1-\alpha}}{\alpha x^{1-r}} &= \frac{c}{k}, \\ x &= \left(\frac{r}{\alpha c}\right)^{\frac{1}{1-r}} \left(ks^{1-\alpha}\right)^{\frac{1}{1-r}}. \end{aligned} \tag{77}$$

Substituting (77) into (76), we have

$$\alpha \Delta V = 4ks^{1-\alpha} \left(\left(\frac{r}{\alpha c}\right)^{\frac{r}{1-r}} \left(ks^{1-\alpha}\right)^{\frac{r}{1-r}} + s^{\alpha} \right) \tag{78}$$

Thus, a higher k must lead to a lower s according to equation (78). Furthermore, since s is lower, we must have that the term $ks^{1-\alpha}$ is higher so that equation (78) holds, which means that x will be higher based on equation (77) given the other parameters are unchanged.

In summary, we can show that a higher k leads to higher x but lower s.

Alternatively, we can express (77) as

$$x = \left(\frac{r}{\alpha}\right)^{\frac{1}{1-r}} \left(\frac{ks^{1-\alpha}}{c}\right)^{\frac{1}{1-r}}. \tag{79}$$

Then based on equation (78), a higher c leads to a higher s (or otherwise the equality cannot hold). And since s is higher, we must have that the term $\frac{ks^{1-\alpha}}{c}$ is lower so that the equality hold, which in turn leads to a lower x according to equation (79).

In summary, we can show that a higher c leads to lower x but higher s.